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An alternative measure of economic inequality under the Lorenz curve framework in analogue to the index of refraction of geometrical optics

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Abstract

Index of refraction is found to be a good measure of economic inequality within the Lorenz curve framework. It has origin in geometrical optics, where it measures bending of a ray of light passing from one transparent medium into another. As light refracts according to characteristics of different media, so also Lorenz curve does according to concentration of wealth or income in different strata. In line with this analogy, first I compute refractive index for each stratum under the Lorenz curve framework to evaluate condition in each and then simply add all to propose an overall measure for the whole framework. The latter appears to be pro transfer-sensitive and equivalent to the measures based on length of the Lorenz curve. Also, it is related to transfer-neutral Gini coefficient by quadratic equation. The applicability of the approach is tested utilising data on distribution of income or consumption from the WDI 2014. Results are lively and remarkable. While an index value of less than 1.00 represents an 'anomalous refraction' in optics, such a condition of inequality appears to be too common for many of us in reality. In contrast to that, in some countries, the refractive index of the richest group exceeds that of Diamond (2.42), where an index value of 1.00 depicts an ideal condition that is enviable. Although the preliminary exercise is done with grouped data, it can be extended vividly to the case when the Lorenz curve is continuous.

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1. Introduction

Index of refraction has origin in geometrical optics, which deals with the propagation of light by geometrical means and establishes some fundamental principles on refraction of light and the law by which it is governed, such as Snell's law, etc. (Mazumdar 1983). Whenever a ray of light proceeds from one homogeneous transparent medium into another, its path is bent at the junction of these two media and this bending of ray is called refraction of light. Index of refraction or refractive index is a quantity, which measures the extent of bending of a ray of light in the aforesaid conditions (Jenkins and White 1981, and Mazumdar 1983). Such a concept is akin to that of the Gini coefficient under the Lorenz curve framework, as the latter measures the extent to which the distribution of income or consumption expenditure among individuals or groups within an economy deviates from a perfectly equal distribution. If we consider the unit square of the Lorenz curve framework superimposing the ideas of geometrical optics on it, we realise that in case of an ideal condition, light (or equivalently the Lorenz curve) passes diagonally without refraction. In the presence of inequality, however, it deviates from the hypothetical line of absolute equality and is seen to refract while passing from one stratum into another. The sole objective of this paper is to introduce a new measure of economic inequality in analogue to the index of refraction for each stratum under the Lorenz curve framework as well as for the complete framework. Consequently, I use simple mathematical tools (following Snell's law) to measure refractive index for each stratum as a measure of inequality associated with it with respect to the ideal condition, and treat a simple summation of those for all the strata as an overall measure of inequality for the whole Lorenz curve framework. The exercise is done for the World Bank member countries utilising data on distribution of income or consumption from the World Development Indicators (WDI) 2014. In this context it is to be mentioned that although the indices of refraction and the overall measure of inequality are computed for fewer groups, the exercise can be extended vividly to the cases when number of groups or individuals is sufficiently large or when the Lorenz curve is continuous.

Further, it is to be noted that although literature on alternative and intuitively simpler derivations of Gini coefficient has grown exponentially over the years, any previous attempt to assimilate the idea of refraction of light with that of inequality based on Lorenz curve is not known. Popular survey papers by Xu (2004), and Yitzhaki and Schechtman (2013) do not reveal presence of any study on the approach under discussion. However, it is observed that after aggregation of the refractive indices for all the strata, the overall index becomes equivalent to a standardised measure that can be expressed as a ratio of length of the observed Lorenz curve to that in the ideal condition, as proposed by Amato (1968) and Kakwani (1980). This linkage between the measures based on the index of refraction and the length of the Lorenz curve puts the present research in advantageous position. This is obvious, as Kakwani (1980) discussed about transfer-sensitivity property and proved that unlike the Gini coefficient, the measure based on the length of the Lorenz curve is more sensitive to transfers at the lower levels of income, making it particularly applicable to problems such as measuring the intensity of poverty. Subramanian (2015, 2010) made it clear that the transfer-neutral Gini coefficient is a linear convex combination of two measures which are anti transfer-sensitive and pro transfer-sensitive respectively. According to him, the pro transfer-sensitivity of the latter is reminiscent of a similarly ‘left-wing’ inequality measure derived from the Lorenz curve, which is based on the length (rather than area, as in the case of the Gini coefficient) of the Lorenz curve, as advanced by Amato (1968), Kakwani (1980) as well as the one based on index of refraction as proposed by this author in Majumder (2014) and in the present paper[1]. Further, one may realise that the proposed measure goes beyond the above-mentioned conceptual advantages in its practical application, as it is: (i) applicable in parts (for each stratum or income groups) and as whole (for the whole Lorenz curve framework), (ii) additive (leading to the overall measure, which is related to the Gini coefficient or the one based on the length of the Lorenz curve), (iii) interpretable as per the scientific propositions of both economics and optics, and (iv) ornamentally comparable with the refractive indices of the precious gemstones, etc.

The paper is organised as follows. Section 2 derives the methods of computing refractive index for each stratum as well as the overall measure for the whole Lorenz curve framework. Section 3 presents refractive indices for some selected countries. Section 4 presents the overall measure of inequality (say, Optical Inequality Index or OI index) and explores its relationship with the Gini coefficient. Section 5 discusses transfer sensitive properties of the OI index. Section 6 presents conclusion followed by references.

2. Methods of computing the index of refraction and the overall inequality measure

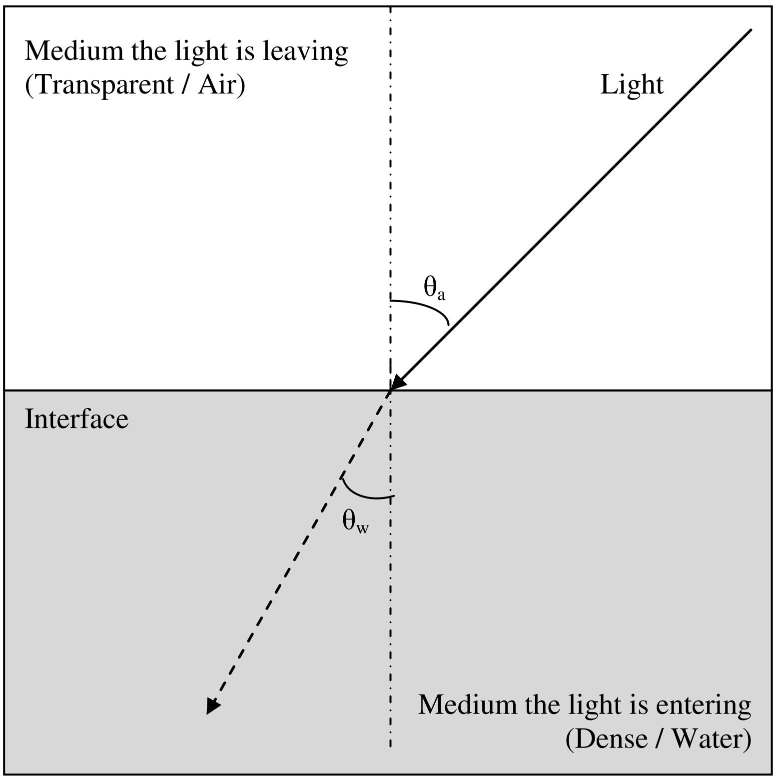
2.1. Discrete case

In geometrical optics, Snell's law of refraction (see Elert 2015, and Jenkins and White 1981) exhibits the relationship between different angles of light as it passes from one transparent medium into another as follows:

$$i_a . \sin(\theta_a) = i_w . \sin(\theta_w), \tag{1}$$

where i_a is the refractive index of the medium a the light is leaving, θ_a is the angle of incidence, i_w is the refractive index of the medium w the light is entering, and θ_w is the angle of refraction. An illustration of refraction (from air to water) is shown in figure 1.

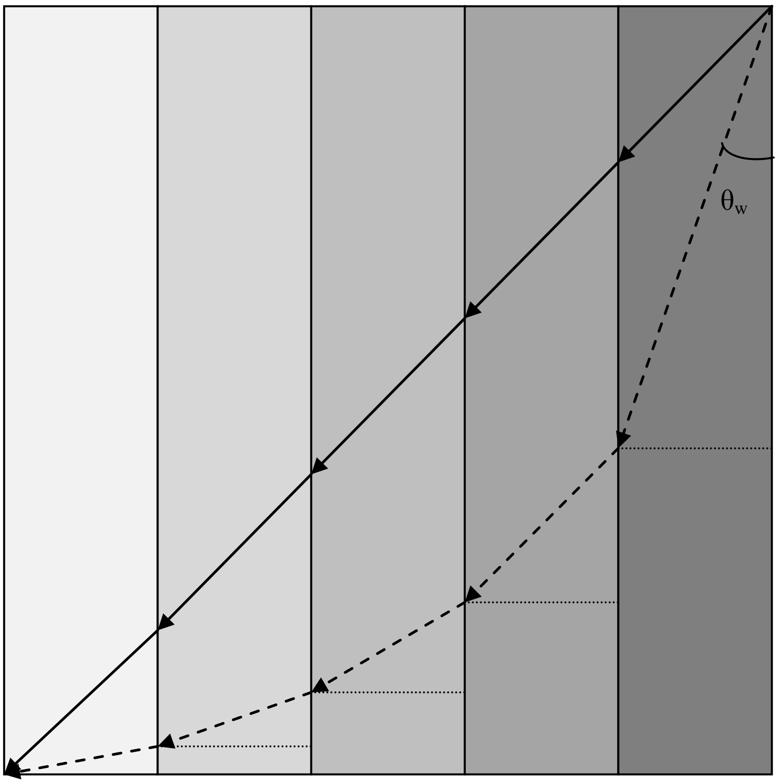
Figure 1. An illustration of refraction (with vertical normal)



[1] It is to be mentioned that Subramanian (2015) is in response to Majumder (2014).

We may apply formula (1) to the Lorenz curve framework as demonstrated in figure 2 (with standard concept and notations), where we have five different strata with y_i as the proportion of income or consumption of one particular stratum such that $\sum y_i = 1$ (for i = 1, 2, .., 5 or 1, 2, …, n in general). In that, an ideal condition is the one where light passes diagonally without refraction. As inequality exists, light refracts five times (as we have considered five different strata) while passing from one stratum into another.

Figure 2. Lorenz curve framework with five groups



From figure 2, we may check that there are five different triangles associated with five different strata. Hypotenuses of all the triangles constitute the Lorenz curve. If we assume that light passes from the upward direction (from right to left), the perpendicular of a triangle is 0.20 (i.e., 1/n) and the base is y_i. The hypotenuse of each triangle is:

$$\sqrt{(0.20)^2 + (y_i)^2} \text{ , and} \tag{2}$$

$$\sin(\theta_w) = \frac{0.2}{\sqrt{(0.2)^2 + (y_i)^2}} . \tag{3}$$

The refractive index of the stratum where light enters may be computed with respect to that of the immediate preceding one or relative to that of the ideal condition, where $\theta = 45^0$ with respect to the vertical normal. As the latter seems simple, we compute the index of refraction following the latter. The index of refraction of a particular stratum is [from equation (1)]:

$$i_w = i_a \cdot \frac{\sin(\theta_a)}{\sin(\theta_w)} . \tag{4}$$

As in case of a fully transparent medium and / or in ideal condition the refractive index is 1.00 (by assumption) and the angle of incidence (θ_a) is 45^0,

$$i_w = 1.\frac{\sin(45^0)}{\dfrac{0.20}{\sqrt{(0.20)^2 + (y_i)^2}}}. \tag{5}$$

$$= \frac{\sin(45^0)\sqrt{(0.20)^2 + (y_i)^2}}{0.20}. \tag{6}$$

As y_i and sin (45^0) are known, index of refraction of each stratum can be obtained easily from expression (6). In general, as 0.20 = 1/n and if we denote the hypotenuse in the numerator [as in expression (2)] as h,

$$i_w = n.\sin(45^0).h. \tag{7}$$

When $y_i = 0$, $i_w \approx 0.71$ (as obtained from formula 6). When $y_i = 0.2$ (the ideal condition), $i_w = 1.00$. When $y_i = 1$ (the extreme case), $i_w \approx 3.61$.

If we add all the refractive indices (for all the strata) under the Lorenz curve framework, we get the overall measure of economic inequality for the particular income distribution. As all the hypotenuses constitute the deviated or observed Lorenz curve (say, u), after summation (for i = 1, 2, …, 5 or 1, 2, …, n in general) of all the refractive indices we get:

$$i = n.\sin(45^0).u. \tag{8a}$$

Equivalently, as for the whole triangle under the line of absolute equality, sin (45^0) is nothing but the base of the triangle (perpendicular in true sense with respect to $\theta = 45^0$, whose length is 1.00) divided by the hypotenuse (the diagonal line, whose length is $\sqrt{2}$),

$$i = \frac{n}{\sqrt{2}}.u. \tag{8b}$$

Equivalently, as $\sqrt{2}$ = Lorenz curve in the ideal condition (say, v),

$$i = n.\frac{u}{v}. \tag{8c}$$

One may check that expression in (8c) is equivalent to the measures proposed by Amato (1968) and Kakwani (1980).

The length of v in (8c) is: $\sqrt{1^2 + 1^2} = \sqrt{2}$. For n=5 and in the extreme case, when all resources are given to one group or individual, (in figure 2) the u takes an upward turn from point (0, 0.8). So, the maximum length of u (for n = 5)[2] is: $0.8 + \sqrt{(0.20)^2 + (1)^2}$. As, in the ideal case v = u, for n=5, from equation (8c),

$$i_{min} = 5.00; \tag{9}$$

and in the extreme case, from equation (8c),

$$i_{max} = 5.\frac{0.8 + \sqrt{(0.20)^2 + (1)^2}}{\sqrt{2}} \approx 6.43. \tag{10}$$

If we want results in a normalised 0-100 scale, the overall measure of economic inequality (which may be termed as Optical Inequality Index or OI index) may be defined as:

$$I_o = 100.\frac{i - i_{min}}{i_{max} - i_{min}}. \tag{11}$$

Using formula (6) or (7) and formulae (8a) or (8b) or (8c) and (11) we will be able to compute (in discrete case) refractive and OI indices respectively for the data set under consideration or for the cases where number of groups or individuals is sufficiently large.

[2] The maximum length is 2 when n is sufficiently large.

2.2. Continuous case

Snell's law of the form 'i sin (θ) = constant', as demonstrated above, is useful in studies when a ray of light passes through different media with refractive index being piece-wise constant for each of the medium. In continuous case, there are infinite numbers of infinitesimally narrow groups or stratum with continuously varying refractive index throughout the unit square. In such a case, the refractive index is to be computed using a differential form of Snell's law (simply by differentiation of the above expression), as shown below.

$$i.\sin(\theta) = \text{const.} \tag{12}$$

Differentiating the above,

$$i.\cos(\theta) + \sin(\theta).\frac{di}{d\theta} = 0, \tag{13}$$

or, $$\frac{\cos(\theta)}{\sin(\theta)} = -\frac{1}{i}.\frac{di}{d\theta}, \tag{14}$$

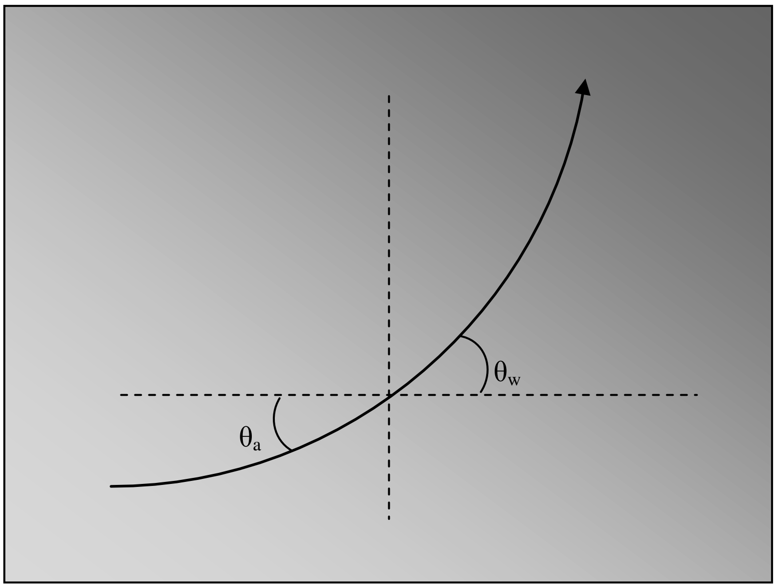
or, $$\cot(\theta)\,d\theta = -\frac{di}{i}. \tag{15}$$

Expression (15) shows the differential form of Snell's law when refraction is considered with respect to the vertical normal (Arovas 2008, and Tatum 2014).

Before I proceed further, I change the angular description to reap some mathematical advantages[3], as shown in figure 3. It illustrates the case of refraction with respect to horizontal normal where, as per sign convention the angles are of opposite signs. With these, the Snell's law takes the following form (Tatum 1999, and Blackstock 2000)[4, 5]:

$$i.\cos(\theta) = \text{const.} \tag{16}$$

Figure 3. An illustration of refraction (with horizontal normal)



Differentiating the expression (16),

$$\tan(\theta)\,d\theta = \frac{di}{i}. \tag{17}$$

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[3] Such as, to express the refractive index in terms of the slope of the tangent line to Lorenz curve.

[4] Both the authors derived differential form of Snell's law in the field of physical acoustics, where acoustic weave or ray of sound, as in case of light, obeys Snell's law of geometrical optics.

[5] One should take care that figures 5.1 and 5.2 in Arovas (2008) correspond to equation 16 and the derivation presented by him corresponds to the equation 12 as shown above (in the present paper).

As i and θ are continuous functions of the coordinate x, expression (17) may be rewritten as follows:

$$\tan(\theta).\frac{d\theta}{dx}=\frac{1}{i}\frac{di}{dx}\ . \tag{18}$$

If we express the path as y = y (x),

$$\tan(\theta)=y'\text{ , and} \tag{19}$$

$$\theta'=\frac{d}{dx}\tan^{-1}y', \tag{20}$$

$$=\frac{y''}{1+y'^2}\ . \tag{21}$$

Replacing the results of (19) and (21) in (18), we have:

$$y'.\frac{y''}{1+y'^2}=\frac{i'}{i}\ , \tag{22}$$

$$\text{or,}\ \ i'=y'.\frac{y''}{1+y'^2}.i\ . \tag{23}$$

As the quantities in the right-hand side (with the first-order derivative being the slope of the tangent line to the Lorenz curve and i being the initial refractive index) are known, i' or change in the refractive index due to the tiniest change in proportion of population (measured along x axis) can be known.

In continuous case, the overall measure of inequality (OI index), which is based on the length of the Lorenz curve, can be computed simply by replacing the summation used in case of equation (8a) by an integral.

Further, in continuous case, there is a point on the Lorenz curve where the slope of the tangent line is equal to that of the diagonal one. This is the point of inflection, as it divides the population into two groups with a refractive index of less than 1.00 in the left and more than 1.00 in the right. This concept may be used to derive a line of inequality in accordance with that of poverty.

3. Refractive index for each stratum

I utilise data on distribution of income or consumption from the WDI 2014 for 148 countries (as per completeness of information) and compute refractive index for each stratum using formula (6). Results of some countries (selected arbitrarily) are displayed in table 1[6].

It is learnt that in the ideal condition refractive index is equal to 1.00 [as discussed in relation to formula (7)]. So, an index value of 1.00 is desirable for each of the strata. Deviation from 1.00 is undesirable. Any value less than 1.00 is strictly undesirable. Standard literature in optics maintains that an index value of less than 1.00 does not represent a physically possible system (Nave 2012). Further, in case of light, a refractive index value of less than 1.00 represents an ‘anomalous refraction’ (Feynman 2011). However, the condition, which does not represent a physically possible system or which is considered ‘anomalous’ in physical science, appears to be true and too common for many of us in reality. For example, in table 1, we see that 60-80 % common mass in each country are subject to such an ‘anomalous refraction’ and presumably a miserable condition of economic inequality too.

Refractive index with a value of more than 1.00 indicates higher concentration of wealth or income [with the upper limit being 3.61 in the extreme case, as discussed in relation to formula (7)]. However, the refractive index of the highest 20 % group in Namibia in 2010 is

[6] Results of all the 148 countries are presented in table 5 in Majumder (2014).

2.44, which is close to that of Diamond (2.42)[7]. Similarly, the richest group in South Africa, in the same year, commands far more luxury as its refractive index (2.57) is seen to exceed that of Diamond. It is to be noted that in both the countries, 80 % of total population live in an 'anomalous' and miserable conditions of inequality.

Table 1. Refractive Index of each stratum and OI index for some selected countries in 2014

Country	Year of survey	Refractive index corresponding to each stratum					OI index
		Lowest 20%	Second 20%	Third 20%	Fourth 20%	Highest 20%	
China	2010	0.73	0.79	0.88	1.08	1.81	20
India	2010	0.77	0.82	0.90	1.02	1.66	12.7
Italy	2010	0.74	0.82	0.93	1.08	1.64	14.9
Namibia	2010	0.72	0.74	0.78	0.91	2.44	39.9
Slovenia	2011	0.79	0.88	0.95	1.07	1.41	7.2
South Africa	2011	0.71	0.72	0.76	0.91	2.57	46.9
Sweden	2005	0.79	0.86	0.95	1.07	1.45	7.9

Source: Self-elaboration

Among other countries, the refractive index of the richest 20 % group in China (1.81) is close to that of Ruby (1.76)[8] or Sapphire (1.76)[9]. The said index values of India (1.66) and Italy (1.64) are higher than that of Topaz (1.62)[10], and those of Slovenia (1.41) and Sweden (1.45) are close to that of Opal (1.45)[11]. Interpretation of results of economic inequality with the refractive indices of precious gemstones is simply ornamental and has no special scientific meaning. However, from this classification, one may beautifully relate extent of concentration of wealth among rich people with the gemstones in order of their hierarchy.

4. Optical Inequality index and Gini coefficient

Optical Inequality index (OI index) is computed using formula (8a) or (8b) or (8c) and (11). It is nothing but the summation of all the refractive indices of the five different income groups or strata expressed in a 0-100 point normalised scale. Index values are displayed in the final column of table 1. Interpretation of the OI index is similar to that of Gini coefficient.

Table 2. The Summary and goodness of fit statistics of Quadratic models

	Statistic	Value	Standard error	F or t*	Sig.
Model I (n=7)	Adjusted R square	1.000	0.184	22123.973	0.000
	Constant	0.268	0.892	0.301	0.779
	Gini coefficient	-0.009	0.047	-0.183	0.864
	Gini coefficient square	0.013	0.001	23.577	0.000
Model II (n=148)	Adjusted R square	1.000	0.150	235621.836	0.000
	Constant	1.499	0.222	6.738	0.000
	Gini coefficient	-0.070	0.012	-6.009	0.000
	Gini coefficient square	0.014	0.000	96.970	0.000

* F for adjusted R square, t for the constant and the coefficients

Source: Self-elaboration

Gini coefficient and OI index are perfectly correlated by quadratic equation as shown in table 2 and in figures 4 and 5. As, OI index is obtained from the grouped data on distribution

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7 There are several texts, which display refractive indices of common gemstones, such as Elert (2015), Jenkins and White (1981), and Reed (2015).

8 *Ibid*. 7.

9 *Ibid*. 8.

10 *Ibid*. 9.

11 *Ibid*. 10.

of income or consumption, the relationship is explored after computing Gini coefficient from the same data following the standard measure under the mean difference approach[12, 13].

Figure 4. Gini coefficient vs. OI index (Model I, n =7)

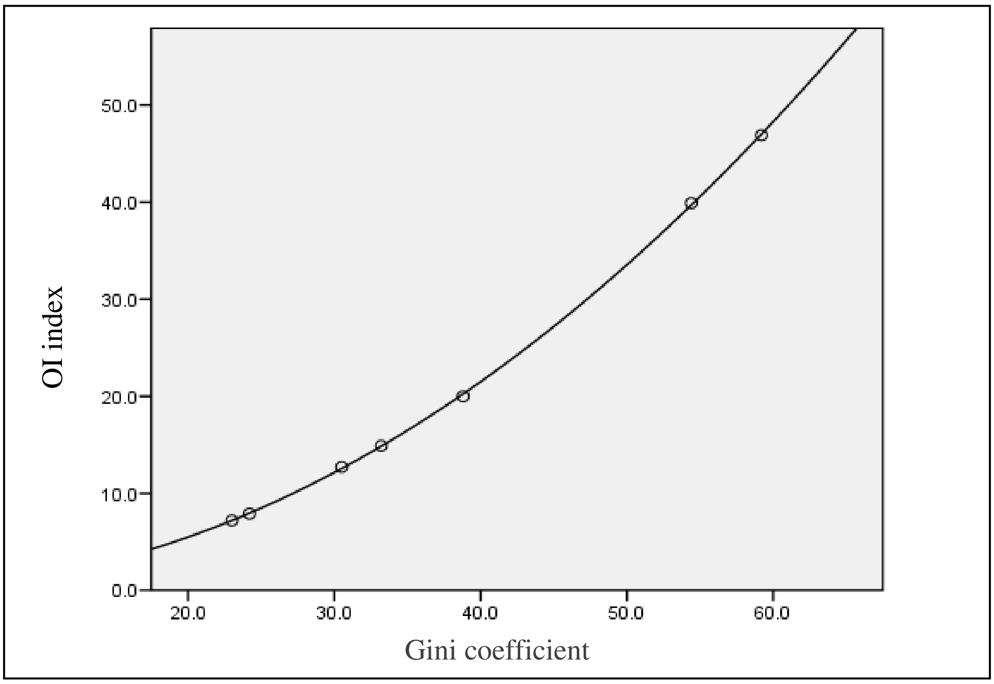
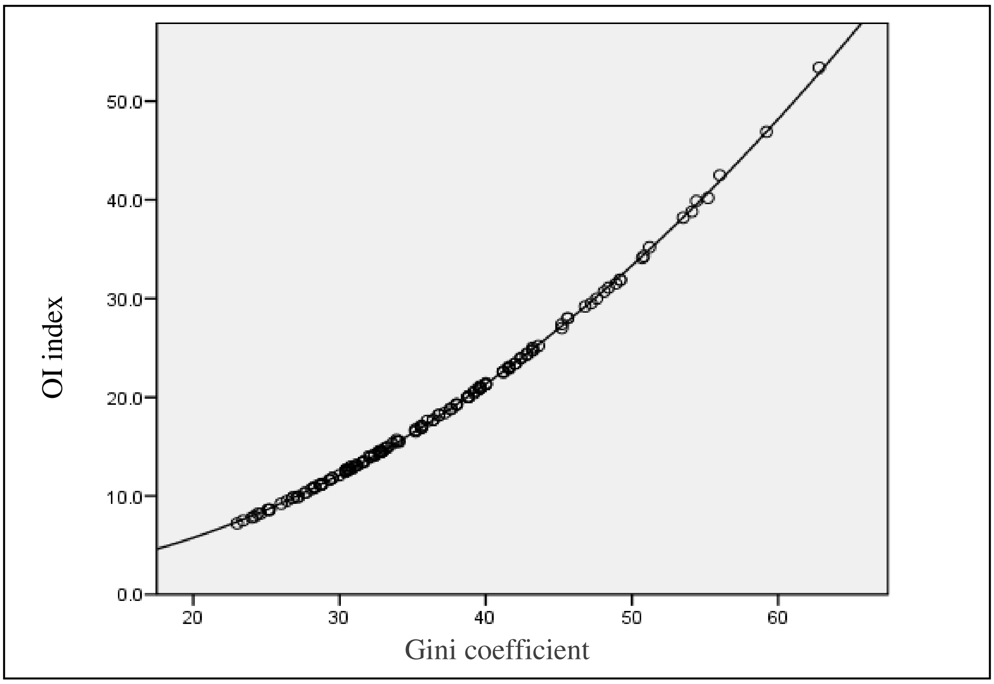


Figure 5. Gini coefficient vs. OI index (Model II, n = 148)



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[12] Say, G_3 in Anand (1983) after multiplying it by 100.

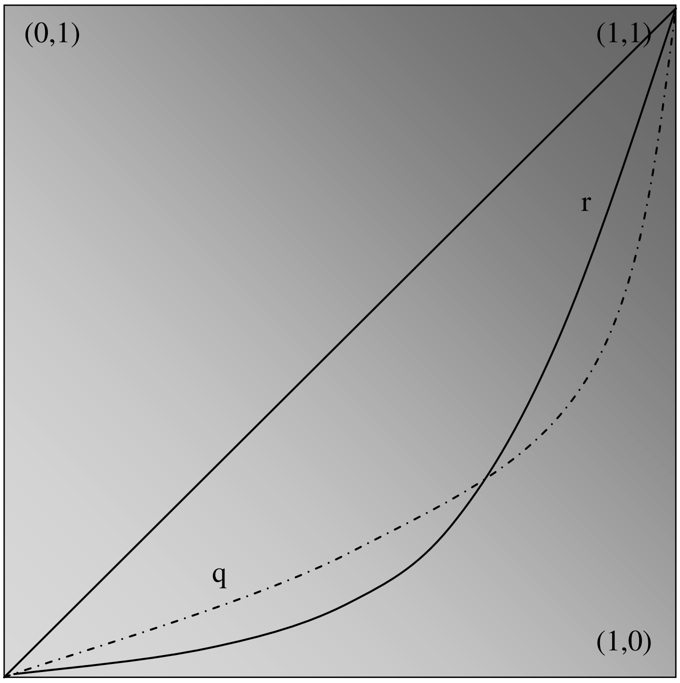
[13] Gini coefficient is computed from grouped data and hence it shows lower values than those based on micro data.

First, I estimate a model with the seven countries included in table 1 and then I repeat the exercise with the data of 148 countries as listed in table 5 in Majumder (2014). It is found, in both the models, that 100 % variability in the OI index is explained by the Gini coefficient with identical adjusted R square value of 1.00. The precise relationships as estimated in the models are shown in figures 4 and 5 respectively as above.

5. Properties of Optical Inequality index

As discussed previously (in the introductory section), OI index is a ‘left-wing’ or pro transfer sensitive inequality measure. It is also shown that the OI index is related to Gini coefficient by a quadratic equation. One may confirm that it possess all the desirable properties as the Gini coefficient does, which is transfer-neutral. Although, Gini coefficient satisfies the Pigou-Dalton condition, it is not differentially sensitive to transfers at either the lower or the upper end of an income distribution.

Figure 6. Lorenz curves with different skewness



I cite one simple numerical example to clarify the issue of sensitivity of OI index[14]. Consider the following distributions with five income groups: p = (7, 13, 20, 27, 33), q = (10, 10, 20, 27, 33) and r = (7, 13, 20, 30, 30). It can be seen that q has been derived from p by a downward transfer of 3 income units to the lowest 20 % from the second 20 %; and r has been derived from p by an identical transfer of 3 income units to the fourth 20 % from the highest 20 %. One may check that the areas enclosed by the Lorenz curves represented by q and r with the diagonal of the unit square are the same (and hence, Gini coefficients for the two are the same), although q is skewed towards (0,0) - ‘bulges at the top’; and r towards (1,1) - ‘bulges at the bottom’. Figure 6 represents such ideas more clearly. An inequality measure (say, Z), which satisfies the Pigou-Dalton transfer axiom, will be transfer-neutral if Z(p) > Z(q) = Z(r); and Z will be pro transfer-sensitive[15] if Z(p) > Z(r) > Z(q). For the numerical example under review, and given equations (8a) or (8b) or (8c) and (11) (for OI

[14] In accordance with Subramanian (2015).

[15] The third case is of anti transfer-sensitivity, which requires Z(p) > Z(q) > Z(r); a ‘right-wing’ inequality measure satisfies this condition.

index, I_o) and any standard measure for Gini coefficient (G)[16], it can be verified that G(p) [= 26.4] > G(q) = G(r) [= 25.2]: the Gini coefficient is transfer-neutral; and $I_o(p)$ [= 10.1] > $I_o(r)$ [= 9.9] > $I_o(q)$ [= 9.3]: OI index is pro transfer-sensitive.

Further, as the OI index is additive, it is confirmed that each component under different strata maintains the spirit of the Pigou-Dalton condition. For example, for the stratum where the index value is more than 1.00, in response to any positive transfer to it, index value increases indicating further escalation of inequality. On the other hand, for the stratum where the index value is less than 1.00, in response to any outward transfer from it, index value decreases aggravating the 'anomalous'condition further and vice-versa.

Finally, I present a crude exercise to get an idea about how change in one refractive index (holding others constant) brings change in the OI index. The exercise is done by estimating Cobb-Douglas type function (for n = 148)[17], results of which are presented in table 3.

It is prominent from the results that major diminution in inequality may come from the positive changes at the lower end.

Table 3. The Summary and goodness of fit statistics of the Cobb-Douglas type function

Statistic	Value	Standard error	F or t*	Sig.
Adjusted R square	0.999	0.016	20360.171	0.000
Constant	0.202	0.029	6.858	0.000
A	-3.545	0.520	-6.816	0.000
B	-1.494	0.431	-3.470	0.001
C	0.146	0.367	0.398	0.691
D	0.649	0.303	2.138	0.034
E	2.185	0.430	5.080	0.000

* F for adjusted R square, t for the constant and the coefficients

A, B, C, etc. denote natural logarithms of Refractive Indices (I_w) of various strata from the lower end.

Source: Self-elaboration

6. Conclusion

The inherent objective of the paper has been to propose an alternative measure of economic inequality under the Lorenz curve framework, which could be far more lively and responsive to our senses as compared to Gini coefficient. Consequently, the overall workability of the proposed index, in parts and together with its sensibility favouring the worse-off ones has been tested and found satisfactory. Further, amalgamation of the principles and propositions of physical and economic sciences together, makes a ground for all of us to envisage about a world without anomalous condition of economic inequality. Being overly simple but contented, the proposed measure of economic inequality based on the index of refraction of light could be a good substitute of the said Gini coefficient and similar ones.

[16] *Ibid.* 12.

[17] As displayed in table 5 in Majumder (2014).

References

Amato, V. (1968) *Metodologia Staistica Strutturale, vol.1*. Cacucci: Bari.

Anand, S. (1983) *Inequality and Poverty in Malaysia: Measurement and Decomposition*, Oxford University Press: New York.

Arovas, D. (2008) "Chapter 5: Calculus of Variations" Classical Mechanics - Lectures and Reading: San Diego. Available at: http://physics.ucsd.edu/students/courses/winter2008/physics110b/LECTURES/CH05_VARIATIONAL.pdf (accessed on 19 March 2015).

Blackstock, D. T. (2000) *Fundamentals of Physical Acoustics*, Wiley-Interscience: New York.

Elert, G. (2015) "Refraction" The Physics Hypertextbook: New York. Available at: http://physics.info/refraction (accessed on 19 March 2015).

Feynman, R. P. (2011) *Feynman Lectures on Physics 1: Mainly Mechanics, Radiation, and Heat*, Basic Books: New York.

Jenkins, A. F. and H. E. White (1981) *Fundamentals of Optics*, McGrawhill International Editions: New Delhi.

Kakwani, N. (1980) *Income Inequality and Poverty: Methods of Estimation and Policy Applications*, Oxford University Press: New York.

Majumder, A. (2014) "An alternative measure of economic inequality in the light of optics" ECINEQ Working Paper 2014-346: Verona. Available at: http://www.ecineq.org/milano/WP/ECINEQ2014-346.pdf (accessed on 19 March 2015).

Mazumdar, K. G. (1983) *A Text Book of Optics*, Ideal Publishers: Calcutta.

Nave, R. C. (2012) "Refraction of Light" HyperPhysics: Atlanta. Available at: http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/refr.html, (accessed on 19 March 2015).

Reed, R. (2009) "Refraction - Snell's Law" Interactagram.com – Diagrams for Interactive Learning: Columbia, Missouri. Available at: http://interactagram.com/physics/optics/refraction (accessed on 19 March 2015).

Subramanian, S. (2015) "More tricks with the Lorenz curve" *Economics Bulletin* **35 (1)**, 580-589.

Subramanian, S. (2010) "Tricks with the Lorenz Curve" *Economics Bulletin* **30 (2)**, 1594-1602.

Tatum, J. B. (2014) "Chapter 1: Reflection and Refraction" Geometric Optics: Victoria. Available at: http://astrowww.phys.uvic.ca/~tatum/goptics/geomop1.pdf (accessed on 19 March 2015).

Tatum, J. B. (1999) "Fireballs: Interpretation of airblast data" *Meteoritics and Planetary Science* **34**, 571-585.

Xu, K. (2004) "How Has the Literature on Gini's Index Evolved in the Past 80 Years?": Halifax. Available at http://myweb.dal.ca/kxu/howgini.pdf (accessed on 05 November 2014).

Yitzhaki, S., and E. Schechtman (2013) "More Than a Dozen Alternative Ways of Spelling Gini" in *The Gini Methodology: A Primer on a Statistical Methodology*, Springer: New York: 11-31.