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## Identifying defectors in a population with short-run players.

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## Abstract

This paper considers a repeated Prisoners' Dilemma game to explore how an information mechanism that labels defectors can help sustain cooperation in societies that include short-run players. We provide sufficient conditions under which there exists equilibria that sustain cooperation for different information technologies that identify defectors. We also analyze imperfect labeling mechanisms.

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## 1. Introduction

The world of the twenty-first century is replete with information on our individual identities and past transactions, which are readily available for the sole purpose of encouraging transactions between otherwise anonymous strangers. It seems that promoting trust to facilitate transactions is the rationale behind making this information publicly available. However, even in Internet markets where information is free and accessible, fraud is common[1] and, therefore, it is not clear how the available information affects the transactions among strangers.

It is well known that community enforcement can sustain cooperation even when agents only count on their own experience to make decisions. Social norms may sustain cooperative outcomes when transactions among members are infrequent even in the absence of information. A key feature of such norms is the threat of sanctions by future partners to deter dishonest behavior. If, however, the transactions of some agents in the society are not only infrequent but also unique then there is no reason to expect cooperation from those members. The disruption created by such agents undermines the ability of the remaining long-run players to cooperate.[2] In these cases, the availability of further information about the rest of players in the society, beyond the pieces derived from our own experience, is crucial.

In this paper, we explore the role of labeling mechanisms in a population with both long-run and short-run players. In particular, we introduce an information technology that intuitively captures the platforms for ratings of the participants in an Internet market. There is a mechanism that attaches labels to the players who "misbehave." We study the plausibility of cooperation among unlabeled players even when they are unable to distinguish between long-run and short-run players. To simplify the analysis, we consider a repeated Prisoner's Dilemma game with random matching.

Cooperation can be sustained even with very limited information when a large population of players is randomly matched.[3]

We begin by considering an information technology that punishes players for their actions independently of the transaction in which they are engaged. This labeling mechanism is unappealing because any defection generates a contagious process that destroys cooperation in the whole society. Following Kandori (1992), we restrict attention to a very straightforward behaviour strategy[4] and show that the presence of short-run players prevents cooperation in

[1]Bolton, Katok, Ockenfels (2004) refer to a research group GartnerG2 report concluding that fraud is 12 times higher in internet transactions.

[2]As shown, e.g., by Moscoso Boedo (2010).

[3]Examples of such results include Kandori (1992), Okuno-Fujiwara and Postlewaite (1995), Ellison (1994), Harrington (1995), Ahn and Suominen (2001), Möller (2005) and Takahashi (2010) . In particular, Kandori (1992) proves a Folk Theorem with a labeling mechanism that allows to vary punishment lengths. Okuno-Fujiwara and Postlewaite (1995) also assume that players possess observable labels and that this information enables cooperation. Ellison (1994) allows for a public randomization device and Takahashi (2010) shows that cooperation can be sustained by mixed strategies using only first order information. All these papers analyze homogeneous populations. Ghosh and Ray (1996) consider a model with heterogeneous agents, but they depart from the case of uniform random matching. Dellarocas (2003) surveys the effects of on-line feedbak mechanisms and compares them to traditional word-of-mouth networks. Bolton, Katok, and Ockenfels (2004) conducted experiments to analyze the enhancement of trade supported by internet feedback and the importance of information in settings with different cooperation costs. Also, Bolton, Katok, and Ockenfels (2005) find that the ratio of cooperation increases when players get better information regarding their opponents' past matches.

[4]Kandori (1992), Definition 2, p. 72.

equilibrium with this mechanism.

We then proceed by considering a technology that monitors some aspects of the players' previous transactions. We make use of the labeling mechanism proposed by Kandori (1992) and obtain the set of restrictions that need to be satisfied in equilibrium for our model. When non-cooperative players are present, long-run players need to be more patient. Furthermore, since defection occurs in equilibrium, the loss when cheated cannot be too large.

Finally, we allow for the presence of errors in the information technologies. In particular, we explore the implications of two kinds of mistakes: a mechanism that sometimes labels "innocent" players and one that sometimes forgets to label "guilty" players. These two kinds of mistakes impose different requirements in order to sustain cooperation. We provide a rationale for the need of caring more about the first type of error, which is more disruptive of cooperation.

The rest of paper is organized as follows. Section 2 introduces the model. Section 3 provides sufficient conditions that sustain cooperation among unlabeled members of the society for different information technologies. Section 4 considers imperfect labeling mechanisms and Section 5 concludes.

## 2. The Model

There is a population $\mathcal{M} := \{1, 2, \ldots, M\}$ of players indexed by $i$, where $M \geq 4$ is an even number. A subset $\mathcal{S} := \{i_1, \ldots, i_S\} \subset \mathcal{M}$, with $S < M$, of the players are *short-run players*. In each period $t \in T := \{0, 1, 2, \ldots\}$, players are randomly matched into pairs. The *matching rule of the infinitely repeated game* is a function $\mu : \mathcal{M} \times T \to \mathcal{M}$, where $\mu(i,t)$ indicates the player to whom $i$ is matched at $t$. For each $i \in \mathcal{M}$ and each $t \in T$, the matching rule is assumed to satisfy: (a) $\mu(i,t) \neq i$, i.e., no player can be matched to himself, and (b) $\mu(\mu(i,t),t) = i$, i.e., matchings are pairwise consistent. In each period $t$, the $S$ short-run players leave the repeated game and are replaced by new $S$ short-run players who are matched to play in $t+1$. Players can observe only the transactions in which they are personally engaged and, in particular, they cannot observe their opponents' identities. Furthermore, a player has no information about how other players have been matched, neither about the actions chosen by any other pair of players. All *long-run players* $i \in \mathcal{M} \setminus \mathcal{S}$ have (common) discount factor $\delta \in (0,1)$ and their payoffs in the infinitely repeated random matching game are the normalized sum of the discounted payoffs from the stage-games.

After being matched, each pair of players play the following Prisoner's Dilemma (PD) stage-game:

| | $C$ | $D$ |
|---|---|---|
| $C$ | $1, 1$ | $-l, 1+g$ |
| $D$ | $1+g, -l$ | $0, 0$ |

Figure 1

where $l > 0$ and $g > 0$ indicate, respectively, the loss when cheated and the gain from defection. Thus, each player $i \in \mathcal{M}$ chooses in each $t \in T$ an *action* $a_{it} \in A := \{C, D\}$ and receives a payoff according to the PD stage-game payoff function $u_i : A^M \to \mathbb{R}$, whose payoffs, for any matching $(i, \mu(i,t)) \in \mathcal{M} \times \mathcal{M}$, are specified by the matrix in Figure 1 above. Let $a_t := (a_{it})_{i \in \mathcal{M}} \in A^M$ denote an *action profile in period $t$*.

A *matching profile in period t* is an $M$-dimension tuple of pairs of matched players $\sigma_t := (i, \mu(i,t))_{i \in \mathcal{M}} \in (\mathcal{M} \times \mathcal{M})^M$. In this model, we consider that each player $i \in \mathcal{M}$ receives a *label* $b_{it} \in B := \{U, L\}$ in each period $t \in T$ according to a (common) exogenous trustworthy *labeling mechanism* $\beta : B \times A \to B$. Here, $U$ indicates "unlabeled" while $L$ indicates "labeled." Let $b_t := (b_{it})_{i \in \mathcal{M}}$ denote a *profile of labels* in period $t$. The labeling specified by the mechanism depends only on the players' current actions and on their previous period labels. This mechanism cannot ex-ante assess whether a player is either long-run or short-run. We assume that each player is informed both about his own and his opponent's label before choosing his action. In this paper, we will consider two kind of labeling mechanisms: one that monitors players' actions and another that monitors some characteristics of their previous transactions. In addition, we will consider as well the case in which the labeling mechanism works imperfectly, either mistakenly not penalizing "bad behavior" or penalizing "good behavior."

Short-run players enter the game for just one period and they have a discount factor of zero. Since they do not care about the future, they trivially choose always the myopic best response of the PD stage-game, $D$, for each $i \in \mathcal{S}$ and each $t \in T$. Therefore, we take this myopic best response as given and do not formalize neither private histories nor strategies for the short-run players. In contrast, long-run players are crucially concerned about the future and, as mentioned earlier, maximize the expected lifetime utility given their (common) discount factor $\delta$. Thus, we need specify carefully their private histories and strategies.

A *private history for a long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ up to period $t$* is a sequence of his own actions, of actions chosen by his opponents, of his own labels, and of labels assigned to his opponents

$$h_{it} := ((a_{i0}, a_{\mu(i,0)0}, b_{i1}, b_{\mu(i,1)1}), \ldots, (a_{i(t-1)}, a_{\mu(i,t-1)(t-1)}, b_{it}, b_{\mu(i,t)t})) \in \mathcal{H}_{it} := (A \times A \times B \times B)^t.$$

Let $h_{-it} := (h_{jt})_{j \in \mathcal{M} \setminus (\mathcal{S} \cup \{i\})}$ be a *profile of private histories of the long-run players other than player $i$*. A *history up to period $t$* is a sequence of matching profiles, action profiles, and label profiles

$$h_t := ((\sigma_0, a_0, b_1), \ldots, (\sigma_{t-1}, a_{t-1}, b_t)) \in \mathcal{H}_t := ((\mathcal{M} \times \mathcal{M})^M \times A^M \times B^B)^t.$$

A *(pure) behavior strategy* for a long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ is a function $\alpha_i : \mathcal{H}_i \to A$ that specifies an action $\alpha_i(h_{it}) = a_{it}$ for each history $h_{it} \in \mathcal{H}_i := \bigcup_{t=0}^{\infty} \mathcal{H}_{it}$. Let $\alpha := (\alpha_i)_{i \in \mathcal{M} \setminus \mathcal{S}}$ be a *(pure) strategy profile for the long-run players* and let $\alpha_{-i}(h_{-it}) := (\alpha_j(h_{jt}))_{j \in \mathcal{M} \setminus (\mathcal{S} \cup \{i\})}$ be a *strategy profile for the long-run players other than player i after the profile of private histories $h_{-it}$*.[5]

Following the related literature, the matching rule $\mu$ is assumed to be uniform and independent across periods $t \in T$. Therefore,[6] $\mathcal{P}_\alpha(\mu(i,t) = j \mid h_{it}) = \frac{1}{M-1}$, for each $i, j \in \mathcal{M}$, such that $i \neq j$, and each $h_{it} \in \mathcal{H}_i$. Also, the probability that a long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ faces a short-run player, given a private history $h_{it}$, is $\rho := \mathcal{P}_\alpha(\mu(i,t) \in \mathcal{S} \mid h_{it}) = \frac{S}{M-1}$. Further, by letting $K_{h_t}(\alpha) := |\{i \in \mathcal{M} \setminus \mathcal{S} : \alpha_i(h_{it}) = D \;\; \forall t \in T\}|$ be the number of long-runs

[5] Without ambiguity, we will refer to this profile as the strategy profile of the repeated game since, in fact, short runs do not make any strategic choice.

[6] Formally, a *state of the world* in this setting is an infinite history $\omega := ((\sigma_0, a_0, b_1), (\sigma_1, a_1, b_2), \ldots) \in \Omega := ((\mathcal{M} \times \mathcal{M})^M \times A^M \times B^B)^\infty$. Then, our probability space of reference is $(\Omega, \mathcal{B}_\Omega, \mathcal{P}_\alpha)$, where $\mathcal{B}_\Omega$ is the Borel $\sigma$-algebra on $\Omega$ and $\mathcal{P}_\alpha$ is the probability measure induced by the matching rule $\mu$ and the strategy profile $\alpha$.

who defect, up to history $h_t$, associated to the strategy profile $\alpha$, we obtain the probability $\kappa_{h_t}(\alpha) := \mathcal{P}_\alpha(\alpha_{\mu(i,t)}(h_{\mu(i,t)t}) = D \mid h_{it}) = \frac{S+K_{h_t}(\alpha)}{M-1}$ that a non-defector faces a defector, up to history $h_t$, according to $\alpha$.

We need some extra notation to specify the equilibrium concept that we use. The *continuation strategy induced by history $h_{it}$* for a long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ is the strategy specified as $\alpha_i|_{h_{it}}(h_{i\tau}) = \alpha_i(h_{it}, h_{i\tau})$ where $h_{it}, h_{i\tau}$ is the concatenation of the history $h_{it}$ followed by the history $h_{i\tau}$. Also, we write $\alpha_{-i}|_{h_{-it}} := (\alpha_j|_{h_{jt}})_{j \in \mathcal{M} \setminus (\mathcal{S} \cup \{i\})}$ to indicate a *profile of continuation strategies the long-run players other than player $i$*. For $i \in \mathcal{M} \setminus \mathcal{S}$, let $U_i(\alpha \mid \beta, h_{it})$ be *player $i$'s expected payoff* for the strategy profile $\alpha$, conditioned on the mechanism $\beta$ and on the private history $h_{it}$. We have

$$U_i(\alpha \mid \beta, h_{it}) = (1-\delta) \sum_{\tau=0}^{\infty} \delta^\tau \mathcal{P}_\alpha(h_\tau \mid \beta, h_{i(t+\tau)}) u_i\big(\alpha_i(h_{i(t+\tau)}), \alpha_{-i}(h_{-i(t+\tau)})\big). \tag{1}$$

Our definition of equilibrium is standard in the literature (see, e.g., Mailath and Samuelson, 2006, Definition 12.2.3, p. 395) and it extends the original concept of sequential equilibrium by Kreps and Wilson (1982) to the current setting of infinitely repeated games with random matching. In our model, a strategy profile is a sequential equilibrium if, after each private history, each long-run player best responds to the behavior of the other players, given his beliefs over histories which are consistent with his own private history. A technical remark is perhaps in order here. Notice that, in our setting, the beliefs of each long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ over histories, conditioned on any mechanism $\beta$ and on any private history $h_{it}$, and for any strategy profile $\alpha$, $\mathcal{P}_\alpha(h_t \mid \beta, h_{it})$, are always determined according to Bayes' rule. Thus, so are his beliefs over the continuation play of the other players. As a consequence, such posterior beliefs trivially satisfy the consistency requirement (with respect to any strategy profile) of sequential equilibrium.[7]

**Definition 1.** A strategy profile $\alpha^*$ is a *(pure-strategy) sequential equilibrium* of the repeated game under the labeling mechanism $\beta$ if for each long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ and each private history $h_{it} \in H_i$, we have:

$$U_i(\alpha_i^*, \alpha_{-i}^*|_{h_{-it}} \mid \beta, h_{it}) \geq U_i(\alpha_i, \alpha_{-i}^*|_{h_{-it}} \mid \beta, h_{it}) \qquad \text{for each strategy} \quad \alpha_i.$$

## 3. Perfect Mechanisms with Short-Runs

This section considers two plausible labeling mechanisms: (a) one that monitors own actions and assigns labels to any non-cooperative player, and (b) another that monitors both own actions and (immediately) previous transactions by labeling only those players who either defect against an unlabeled opponent or who fail to switch from $D$ to $C$ when required. We interpret switching from $D$ to $C$ as a sort of "repentance."

The goal here is to understand whether each of the mechanisms enables the sustainability of cooperation among permanent members of a society by identifying defectors. With no

[7] A common (and quite convenient) feature of this class of models is that all information sets are achieved with strictly positive probability under any strategy profile so that we do not need to worry about how beliefs are determined in information sets not achieved under the proposed strategy profile. This simplifies greatly the analysis of sequential equilibrium in this and in other related settings.

labeling mechanism, the presence of short-run players prevents cooperation among long-run players (see, e.g., Moscoso Boedo, 2010). Hence, we are introducing informational mechanisms that, for instance, capture intuitively the possibility of obtaining negative reports about non-cooperative participants in an Internet market.

All players, long-runs and short-runs, enter the game unlabeled. Then, under either of the two labeling mechanisms $\beta$ mentioned above, a label identifies a long-run player who has defected. However, the strategy chosen by an unlabeled player remains uncertain to his opponent. Such an unlabeled player could be either a cooperative long-run player or a short-run player. Thus, if a long-run player cooperates with unlabeled opponents, then he needs to be willing to accept some periods of loss. In particular, notice that the payoff to a long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ who chooses $C$ in some period $t$ when his opponent chooses $D$ is bounded from above and it cannot exceed

$$\mathcal{P}_\alpha(\mu(i,t) \in \mathcal{S} \mid \beta, h_{it})[-l] + \mathcal{P}_\alpha(\mu(i,t) \in \mathcal{M} \setminus \mathcal{S} \mid \beta, h_{it})[1] = \rho[-l] + (1-\rho)[1].$$

This bound is obtained by considering that the mechanism $\beta$ labels short-run players and by supposing that all long-run players choose $C$. Then, the payoff to a long-run cooperative player is individually rational only if $l \leq \frac{1-\rho}{\rho}$. Thus, an increase in the proportion of short-runs ($\rho$) places tighter restrictions on the parameter $l$. Obviously, if the society approximates to one where all players are short-runs ($\rho \to 1$), then there is no positive $l$ that can sustain cooperation.

### *3.1. (Unforgiving) Labels for any Non-Cooperative Behavior*

We consider first a mechanism $\beta^u$ which assigns a label forever to those who play $D$ at some $t \in T$. Formally, for each long-run player $i$,

$$b_{it} = \beta^u(b_{i(t-1)}, a_{i(t-1)}) = \begin{cases} U & \text{if } b_{i(t-1)} = U \text{ and } a_{i(t-1)} = C, \\ L & \text{otherwise.} \end{cases}$$

Labels capture here the idea of "permanent negative reports." Any player who does not cooperate gets a label forever so that a label indicates past defection.[8] Importantly enough, notice that all short-run players get a label but this is irrelevant for their opponents' choice because short-runs leave the game immediately. Since short-run players do not have any past interactions, we have, by construction, $b_{it} = U$ for each $i \in \mathcal{S}$.

Now, let us consider, for each long-run player $i \in \mathcal{M} \setminus \mathcal{S}$, the following "unforgiving strategy" $\alpha_i^u$: *"Cooperate always with unlabeled players, defect against labeled players and, once you are labeled, defect from then onwards."*[9] Formally,

$$\alpha_i^u(h_{it}) = \left\{ \begin{array}{ll} C & \text{if } b_{\mu(i,t)t} = U, \\ D & \text{if } b_{\mu(i,t)t} = L, \\ D & \text{if } b_{i\tau} = L \text{ for some } \tau \leq t. \end{array} \right\}$$

Then, $\alpha^u$ constitutes a sequential equilibrium of the repeated game if the following three conditions hold for each long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ after each private history $h_{it}$:

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[8]In this sense, the label provides a type of first order information which is not as detailed as in Takahashi (2010).

[9]Takahashi (2010) calls this strategy "pairwise grim-trigger " and shows that, for a continuum population with homogeneous players, this is not always an equilibrium strategy.

1. If $i$ is unlabeled at $t$, then playing $C$ at $t$ when matched with an unlabeled opponent must be preferred to playing $D$;

2. If $i$ is unlabeled at $t$, then playing $D$ at $t$ when matched with a labeled opponent must be preferred to paying $C$;

3. If $i$ has been labeled at some $\tau \leq t$, then playing $D$ at $t$ must be preferred to playing $C$.

To verify Condition 1 above, note that

$$\mathcal{P}_{\alpha^u}\left(\alpha^u_{\mu(i,t)t}(h_{\mu(i,t)t}) = D \;\middle|\; \beta^u, b_{\mu(i,t)t} = U, h_{it}\right) = \mathcal{P}_{\alpha^u}\left(\mu(i,t) \in \mathcal{S} \mid \beta^u, h_{it}\right) = \rho$$

In words, conditional on observing $b_{\mu(i,t)t} = U$, an unlabeled long-run player $i$ must assign probability $\rho$ to the event that his opponent plays $D$. This is so because, conditional on observing that the opponent is unlabeled, he must conclude that such opponent is either long-run (who will then play $C$, according to the recommendation of $\beta^u$) or short-run (who will be unlabeled, since he has not had any previous interactions, and will play $D$). In this case, note that only short-run opponents will play $D$. Then, by using the expression in (1), the equilibrium condition of Definition 1 requires that

$$\begin{gathered}\sum_{\tau=0}^{\infty} \delta^\tau \Big[\rho[-l] + (1-\rho)[1]\Big] \geq \Big[\rho[0] + (1-\rho)[1+g]\Big] \\ \Leftrightarrow \delta \geq \frac{\rho l + (1-\rho)g}{(1-\rho)(1+g)}.\end{gathered} \tag{2}$$

That is, the long-run player must prefer to face a defector, with probability $\rho$, and continue to be unlabeled versus defecting and getting a label from then onwards. Intuitively, long-run players need to be patient enough for being willing to cooperate in order to ignore some periods of losses, which are caused by the presence of short-run players.

As for Condition 2, notice that it is satisfied if the a present loss from facing a defector does not exceed the future discounted gains from staying unlabeled in a community with defectors. Since

$$\mathcal{P}_{\alpha^u}\left(\alpha_{\mu(i,t)}(h_{\mu(i,t)t}) = D \mid \beta, b_{\mu(i,t)t} = L, h_{it}\right) = \kappa_{h_t}(\alpha^u),$$

we have that, using the expression in (1), the required condition is

$$\begin{gathered}\Big[\kappa_{h_{(t+\tau)}}(\alpha^u)[0] + (1-\kappa_{h_{(t+\tau)}}(\alpha^u))[1+g]\Big] \geq \sum_{\tau=0}^{\infty} \delta^\tau \Big[\kappa_{h_{(t+\tau)}}(\alpha^u)[-l] + (1-\kappa_{h_{(t+\tau)}}(\alpha^u))[1]\Big] \\ \Leftrightarrow \delta \leq \frac{\kappa_{h_{(t+\tau)}}(\alpha^u) l + (1-\kappa_{h_{(t+\tau)}}(\alpha^u))g}{(1-\kappa_{h_{(t+\tau)}}(\alpha^u))(1+g)} \quad \text{for any } h_{(t+\tau)} \in \bigcup_{\tau=0}^{\infty} \mathcal{H}_{t+\tau} \text{ and } t \in T.\end{gathered}$$

Finally, notice that

$$\mathcal{P}_{\alpha^u}\left(\Big\{i \in \mathcal{M} \setminus (\mathcal{S} \cup \{i\})\Big\} \bigcap \Big\{\alpha^u_i(h_{i\tau}) = D\Big\} \;\Big|\; \beta^u, b_{it} = L, h_{\mu(i,t)t}\right) = 1 \qquad \text{for some} \quad \tau \leq t-1.$$

Thus, after observing $b_{it} = L$, any player $\mu(i,t)$ who is matched with $i$ at $t$ must assign probability one to the event that $i$ is a long-run player who has played $D$ at some previous period.

Therefore, Condition 3 is trivially satisfied for player $i$ for any $l > 0$ since, given the beliefs above, player $\mu(i,t)$ will play $D$ following the recommendation of his strategy $\alpha^u_{\mu(i,t)}$.

In short, we have obtained the following set of sufficient conditions for $\delta$ under which cooperation can be sustained:

$$\frac{\rho l + (1-\rho)g}{(1-\rho)(1+g)} \leq \delta \leq \frac{\kappa_{h_t}(\alpha^u)l + (1-\kappa_{h_t}(\alpha^u))g}{(1-\kappa_{h_t}(\alpha^u))(1+g)} \qquad \text{for any} \quad h_t \in \bigcup_{\tau=0}^{\infty} \mathcal{H}_\tau.$$

Finally, notice that the constraints above on the values of $\delta$ are well-defined since, for any strategy profile $\alpha^u$ and any history $h_t \in \bigcup_{\tau=0}^{\infty} \mathcal{H}_\tau$, we have $\rho \leq \kappa_{h_t}(\alpha^u)$. In addition, to ensure that

$$\frac{\rho l + (1-\rho)g}{(1-\rho)(1+g)} < 1,$$

so that the requirement above may hold for meaningful values of $\delta$, we need $\rho < 1/(1+l)$. As indicated earlier, the later inequality is satisfied when a long-run player who chooses to cooperate is (strictly) individually rational for any strategy profile.

### *3.2. Labels for Defection against a Cooperative Player*

In this subsection, we provide sufficient conditions under which, regardless of his own label, a long-run player always cooperates against an unlabeled opponent and defects against a labeled one. Notice that this strategy encompasses the idea of "repentance" since the player is recommended to cooperate against an unlabeled opponent even if the latter defects. To sustain this behavior, we consider a mechanism $\beta^r$ which assigns a label to each long-run player who does not follow the strategy above mentioned. More precisely, for each long-run player $i \in \mathcal{M} \setminus \mathcal{S}$, we propose the following "repentance strategy" $\alpha^r_i$: *"Regardless of your own label, cooperate always with unlabeled players and defect against labeled players."* Formally,

$$\alpha^r_i(h_{it}) = \left\{ \begin{array}{ll} C & \text{if } b_{\mu(i,t)t} = U, \\ D & \text{if } b_{\mu(i,t)t} = L. \end{array} \right\}$$

Then, the labeling mechanism that we propose is

$$b_{it} = \beta^r(b_{i(t-1)}, a_{i(t-1)}) = \left\{ \begin{array}{ll} U & \text{if } a_{i(t-1)} = \alpha^r_i(h_{i(t-1)}), \\ L & \qquad \text{otherwise.} \end{array} \right.$$

In contrast with the technology of the previous subsection, now the labeling mechanism monitors not only a player's own actions but also some characteristics of his, and of his opponent, past transactions as the label depends on his opponent's label. An intuitive interpretation of this kind of behavior is that, in many interactions, only cooperative players are likely to take the time and make to effort to fill in a complaint.

Then, $\alpha^r$ constitutes a sequential equilibrium of the repeated game if the following two conditions hold for each long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ after each private history $h_{it}$:

1. Regadless of whether $i$ is labeled or not, playing $C$ at $t$ when matched with an unlabeled opponent must be preferred to playing $D$;

2. Regardless of whether $i$ is labeled or not, playing $D$ at $t$ when matched with a labeled opponent must be preferred to paying $C$.

For $\tau \geq 1$, let $x_{i\tau} := \mathcal{P}_{\alpha^r}\left(\beta_{\mu(i,\tau)\tau} = U \,\middle|\, \beta^r, h_{i\tau}\right)$ denote the unconditional probability that a long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ is matched at $\tau$ with an unlabeled opponent under the proposed labeling mechanism and strategy profile. Notice that $\rho \leq x_{i\tau} \leq 1$ since, under mechanism $\beta^r$ and strategy profile $\alpha^r$, not only short-runs are unlabeled. Long-runs who cooperate against unlabeled opponents and defect against labeled opponents at $\tau - 1$ are also unlabeled at $\tau$.

To verify Condition 1, we have to consider separately the cases where a long-run is unlabeled and where he is labeled. First, if a long-run agent $i \in \mathcal{M} \setminus \mathcal{S}$ is unlabeled ($b_{it} = U$), then the sequential rationality requirement imposed in Definition 1 that gives him the incentives to choose $C$ at the current period is[10]

$$\begin{aligned}
&\rho[-l] + (1-\rho)[1] + \delta\Big[x_{i(t+1)}\big(\rho[-l] + (1-\rho)[1]\big) + (1 - x_{i(t+1)})\big(0[0] + 1[1+g]\big)\Big] \geq \\
&\qquad \rho[0] + (1-\rho)[1+g] + \delta\Big[x_{i(t+1)}\big(1[-l] + 0[1]\big) + (1 - x_{i(t+1)})\big(1[0] + 0[1+g]\big)\Big] \\
&\qquad \Leftrightarrow \delta \geq \frac{\rho l + (1-\rho)g}{x_{i(t+1)}(1-\rho)(1+l) + (1 - x_{i(t+1)})(1+g)} =: \delta_{1t} \quad \forall t \in T.
\end{aligned} \tag{3}$$

In words, if the long-run player $i$ cooperates against an unlabeled opponent and defects against a labeled one, then he obtains, in the current period, the expected payoffs derived from choosing $C$ in the PD stage-game, and from meeting either a short-run (with probability $\rho$) or a long-run (with probability $1 - \rho$). In the next period, he will obtain these very same payoffs when he meets an unlabeled opponent, with probability $x_{i(t+1)}$. However, he will meet a labeled (long-run) opponent with probability $1 - x_{i(t+1)}$ and, in this case, his strategy will recommend him to play $D$. This will give him payoff $1 + g$ since such a long-run opponent observes that $i$ is unlabeled and thus must play $C$. On the other hand, if the long-run player $i$ defects in the current period then he obtains at $t$ the expected payoffs derived from choosing $D$ in the PD stage-game, and from meeting either a short-run (with probability $\rho$) or a long-run (with probability $1 - \rho$). As a consequence, he gets a label so that in the next period all opponents will defect against him. Thus, by switching back to the proposed strategy at $t + 1$, player $i$ will get the expected payoff either of $-l$, when he faces an unlabeled opponent, or of zero, when he faces a labeled one.

Analogously, if the long-run player is labeled ($b_{it} = L$), then the required sequential optimality condition, which, in intuitive terms, recommends the long-run players to cooperate at the current period "in repentance," is now

$$\begin{aligned}
&1[-l] + 0[1] + \delta\Big[x_{i(t+1)}\big(\rho[-l] + (1-\rho)[1]\big) + (1 - x_{i(t+1)})[1+g]\Big] \geq \\
&\qquad 1[0] + 0[1+g] + \delta x_{i(t+1)}[-l] \\
&\qquad \Leftrightarrow \delta \geq \frac{l}{x_{i(t+1)}(1-\rho)(1+l) + (1 - x_{i(t+1)})(1+g)} =: \delta_{2t} \quad \forall t \in T.
\end{aligned} \tag{4}$$

[10]Note that, under the usual stationarity conditions, both sides of the required inequality include the common term $\sum_{\tau=2}^{\infty} \delta^{\tau}\left[x_{i(t+\tau)}\left[\rho(-l) + (1-\rho)(1) + (1 - x_{i(t+\tau)})(1+g)\right]\right]$, which describes the expected payoff to the long-run player from period $t + 2$ onwards. For simplicity, we do not include it in the required condition as it trivially cancels out.

Again, long-run players need to be patient enough for being willing to cooperate in order to ignore some periods of losses, which are caused by the presence of defectors in the society.

As for the lower bounds on the discount factor $\delta$ specified in equations (3) and (4), let $\delta_k := \sup_{t \in T} \delta_{kt}$ for $k \in \{1,2\}$, and let $x_i \in [\rho, 1]$ be the number such that $x_i(1-\rho)(1+l) + (1-x_i)g = \inf_{t \in T} \{x_{i(t+1)}(1-\rho)(1+l) + (1-x_{i(t+1)})g\}$. Then, note that

$$\delta_1 - \delta_2 = \frac{(1-\rho)(g-l)}{x_i(1-\rho)(1+l) + (1-x_i)(1+g)}.$$

Therefore, it trivially follows from the expression above that if $g \leq l$, the requirement in condition (4) implies the one in condition (3) whereas if $g > l$, the requirement in condition (3) implies the one in condition (4).

Finally, note that Condition 2 is trivially satisfied since

$$\mathcal{P}_{\alpha^r}\left(\mu(i,t) \in \mathcal{M} \setminus (\mathcal{S} \cup \{i\}) \,\middle|\, \beta^r, b_{\mu(i,t)t} = L, h_{it}\right) = 1$$

and, therefore, the strategy $\alpha_i^r$ implies that $i$ enjoys a present gain by defecting and a future stream of gains because he is unlabeled.

To summarize, the sufficient condition under which the proposed strategy profile $\alpha^r$, together with the labeling mechanism $\beta^r$, constitutes a sequential equilibrium of the infinitely repeated game is:

$$\delta \geq \frac{\max\left\{\rho l + (1-\rho)g,\, l\right\}}{x_i(1-\rho)(1+l) + (1-x_i)(1+g)}$$

for each long-run player $i \in \mathcal{M} \setminus \mathcal{S}$, where $x_i \in [\rho, 1]$.

## 4. Imperfect Mechanisms with Short-Run Players

In this section, we comment briefly on the implications of having imperfect mechanisms that monitor transactions and that identify deviators for only one period. To this end, we consider the "repetance" strategy profile $\alpha^r$ explored in Subsection 3.2 and allow for two types of errors: (a) error of type I (I), consisting of not labeling a player (with probability $\varepsilon > 0$) who, according to the mechanism, should be labeled and (b) error of type II (II), consisting of labeling a player (with probability $\varepsilon > 0$) who, according to the mechanism, should not be labeled.

For the type of error $k \in \{\text{I}, \text{II}\}$ and for $\tau \geq 1$, let $x_{i\tau}^k = \mathcal{P}_{\alpha^r}\left(\beta_{\mu(i,\tau)\tau} = U \,\middle|\, \beta_k^r, h_{i\tau}\right)$ be the unconditional probability that a long-run player $i \in \mathcal{M} \setminus \mathcal{S}$ is matched at $\tau$ with an unlabeled opponent under the proposed strategy profile and the labeling mechanism $\beta_k^r$ with error of type $k$. Also, for any $k \in \{\text{I}, \text{II}\}$, let $x_i^k$ be the probability such that $x_i^k(1-\rho)(1+l) + (1-x_i^k)g = \inf_{t \in T}\left\{x_{i(t+1)}^k(1-\rho)(1+l) + (1-x_{i(t+1)}^k)g\right\}$. Note that, given the nature of the errors of type I and II described above, we have $x_i^{\text{II}} < x_i < x_i^{\text{I}}$.

A mechanism that incurs the error of type I can be regarded as "extra-forgiving," since it implies that a long-run player who does not follow the cooperative strategy is sometimes not penalized. From the analysis of when Condition 1 of the Subsection 3.2 is satisfied, it follows that the required sequential rationality conditions derived from the expressions in (3) and (4) are, respectively:

$$\rho[-l] + (1-\rho)[1] + \delta\Big[x_i^{\mathrm{I}}\Big(\rho[-l] + (1-\rho)[1]\Big) + (1-x_i^{\mathrm{I}})[1+g]\Big] \geq$$
$$(1-\rho)[1+g] + \delta\Big[(1-\varepsilon)x_i^{\mathrm{I}}[-l] + \varepsilon\Big[x_i^{\mathrm{I}}\Big(\rho[-l] + (1-\rho)[1]\Big) + (1-x_i^{\mathrm{I}})[1+g]\Big]\Big]$$

and

$$-l+\delta\Big[x_i^{\mathrm{I}}\Big(\rho[-l] + (1-\rho)[1]\Big) + (1-x_i^{\mathrm{I}})[1+g]\Big] \geq$$
$$\delta\Big[(1-\varepsilon)x_i^{\mathrm{I}}[-l] + \varepsilon x_i^{\mathrm{II}}\Big(\rho[-l] + (1-\rho)[1]\Big) + (1-x_i^{\mathrm{I}})[1+g]\Big].$$

Notice that the two sufficient conditions above are satisfied if

$$\delta \geq \frac{\max\Big\{\rho l + (1-\rho)g,\, l\Big\}}{(1-\varepsilon)\Big[x_i^{\mathrm{I}}(1-\rho)(1+l) + (1-x_i^{\mathrm{I}})(1+g)\Big]} =: \delta^{\mathrm{I}}.$$

Thus, allowing this type of error in the mechanism only requires players to be more patient than before. The previous restriction on the discount factor is now the one applied to a discount factor modified by the probability $(1-\varepsilon)$ of not making a mistake.

The error of type II describes intuitively a mechanism that is "extra-unconfident" since it penalizes even those players who behave according to the suggested cooperative strategy.

Under this type of error, the restrictions in (3) and (4) lead now to the following sequential rationality requirement for a long-run player $i \in \mathcal{M} \setminus \mathcal{S}$:

$$\rho[-l] + (1-\rho)[1] + \delta\Big[(1-\varepsilon)\Big[x_i^{\mathrm{II}}\Big(\rho[-l] + (1-\rho)[1]\Big) + (1-x_i^{\mathrm{II}})[1+g]\Big] + \varepsilon x_i^{\mathrm{II}}[-l]\Big] \geq$$
$$(1-\rho)[1+g] + \delta\Big[x_i^{\mathrm{II}}[-l]\Big]$$

and

$$-l + \delta\Big[(1-\varepsilon)\Big[x_i^{\mathrm{II}}\Big(\rho[-l] + (1-\rho)[1]\Big) + (1-x_i^{\mathrm{II}})[1+g] + \varepsilon x_i^{\mathrm{II}}[-l]\Big] \geq \delta x_i^{\mathrm{II}}[-l].$$

The two sufficient conditions above are satisfied if

$$\delta \geq \frac{\max\Big\{\rho l + (1-\rho)g,\, l\Big\}}{(1-\varepsilon)\Big[x_i^{\mathrm{II}}(1-\rho)(1+l) + (1-x_i^{\mathrm{II}})(1+g)\Big]} =: \delta^{\mathrm{II}}.$$

Obviously, an interesting question that arises when designing a labeling mechanism is which of the previous two types of mistakes is more relevant to avoid. A plausible approach to this comparison requires us to study the restrictions imposed on the discount rate $\delta$. From the specifications above of the upper bounds $\delta^{\mathrm{I}}$ and $\delta^{\mathrm{II}}$, it follows that

$$\delta^{\mathrm{I}} \geq \delta^{\mathrm{II}} \;\Leftrightarrow\; \Big(x_i^{\mathrm{I}} - x_i^{\mathrm{II}}\Big)\Big[(l-g) - \rho(1+l)\Big] \leq 0.$$

Now, note that $\rho(1+l) > 0$ and recall that a long-run player who cooperates is (strictly) individually rational under any strategy profile if $\rho(1+l) < 1$. Then, since $(x_i^{\mathrm{I}} - x_i^{\mathrm{II}}) > 0$, we have that: (a) if $l - g \leq \rho(1 + l)$, then the error of type I is relatively worst whereas (b) if $l - g > \rho(1 + l)$, then the error of type II is relatively worst. The intuition here is clear. If the payoff from defecting $g$ is relatively large, then an error in the mechanism that lowers the punishment when deviating is less likely to sustain cooperation. On the other hand, if the repentance payment (i.e., the loss $l$ derived from meeting a short-run) is too high, then a mechanism that forces players to repent by making mistakes will make it more difficult to support cooperation.

In practice, Internet feedback mechanisms are often more concerned with avoiding false bad labels than with monitoring the veracity of good ones. In our setting, good labels means not having a label. Then, under the implications of our model, this behavior suggests that the monitoring error of type II is relatively more costly. Thus, the costs when cheated might be larger than the gain from punishment.

## 5. Conclusions

In this paper, we have explored a model of random pairwise interactions in a population of agents who play a PD stage-game and get some information from a labeling mechanism that identifies defectors. We have shown that the inclusion of short-run players makes the sustainability of a cooperative outcome more complex. When only one label provides information, long-run players need to be more patient, and the loss when meeting a short-run has to be relatively low. Furthermore, the presence of short-run players results in the collapse of a cooperative strategy based on a system that monitors players actions instead of transactions.

The setting studied here is built on Kandori (1992)'s model, which provides a cornerstone Folk theorem for a matching game with homogenous agents and a labeling mechanism. The ability of the information mechanism to adjust punishment lengths is essential for Kandori's result. In this paper, we have restricted attention to a mechanism with only one label.

We have obtained that the presence of short-run players leads to more restrictive conditions on discount rates and players' payoffs. Finally, we have studied the effects of having an imperfect labeling mechanism. When "innocent" players are mistakenly labeled the set of parameters that sustain cooperation is further restricted. In many settings, this kind of mistake is more detrimental for cooperation than forgetting to label a "guilty" player.

While this paper analyzed the effects of an informational technology that identifies defectors in an economy with heterogeneous players, some interesting questions are still unanswered. First, a strategy that sustains cooperation for any set of parameters in a population with short-run players and labels for misbehavior remains to be defined. Secondly, studying the implications of having a mechanism with more labels (i.e., longer punishment periods or labels for good and bad behavior) seems very interesting. Finally, in an attempt to explain the functioning of Internet feedback, it would be important to analyze the functioning of an endogenous mechanism that depends on players willingness to provide feedback.

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