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INCENTIVES TO MERGE IN ASYMMETRIC MIXED OLIGOPOLY

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This paper analyzes mergers incentives in an asymmetric mixed oligopoly consisting of two identical private firms and one public firm. It is shown that when there is a technological gap between the public and private firms, both of them will want to merge when the public firm is inefficient and the percentage of the shares owned by private owners after the merger is relatively high. When all firms have identical technology, public and private firms will want to merge when the percentage of the shares owned by private owner in the merged entity is relatively low (Artz et al. 2009). Yet, we show that when the technological gap is high enough, the merger between the public firm and one private firm often includes complete privatization.

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1 Introduction

In recent years, many public firms have acquired equity stake in private firms. For example, in the European automobile industry, the German public firm Volkswagen acquired the Spanish firm SEAT. Recently (July 2008), the french energy firm GDF merged with SUEZ and becomes a group "GDF-SUEZ".

The study of mixed oligopolies, where welfare-maximizing public firms interact with profit-maximizing private firms, has become increasingly popular in recent years (For pioneering works, see Merrill and Schneider 1966; Bös 1986,1991). A number of these studies assumes that firms have constant marginal cost and that private firms produce at lower costs (Megginson and Netter 2001; White 2002). This inefficiency of public firms is justified by the informational and institutional aspects of the market (Hsin and Ogawa 2005). On such a market where costs are linear, the public firm has to be less efficient to guarantee positive output for private firms. If there were any fixed costs, the public firm would be unable to cover them with a positive price-cost margin, and would incur losses (De Fraja and Delbono 1987). To avoid these situations of natural monopolies, a variety of researchers have assumed that firms have an identical technology.

The litterature on mergers has extensively analyzed the decision to merge by private firms (Perry and Porter, 1985; Huck et al., 2001), however there exists few studies on mergers activities in mixed oligopoly.Among these, Barcena-Ruiz and Garzon (2003) explore the case in which a public and a private firms merge into a multiproduct firm and show that both firms want to merge when the shareholding ratio of the public firm's owner takes an intermediate value and the substitutability of the goods produced by both public and private firms is sufficiently low. However, Barcena-Ruiz and Garzon (2003) do not consider the more traditional market with many firms producing identical products. To fill this gap, Nakamura and Inoue (2007) and Mendez Naya (2008), show that if a merger improves productivity, the public firm and the private firm will both want to merge when the shareholding ratio of the owner of the public firm takes an intermediate value after the merger, even though there exist only a few private firms in the market. Recently, Artz et al. (2009), analyse the impact of slope of the marginal cost curve in firms decision to merge. When considering a merger between two private firms, they show that the degree of convexity required to earn profit is larger than would be the case in the absense of the public firm. So, the presence of the public firm reduces the set of mergers that will be profitable. Moreover, Artz et al. (2009) show that in mixed triopoly, the merger between a public firm and a private firm can occur when the percentage of the shares owned by the government and the degree of convexity are relatively low.

Our paper contributes to the literature on mergers in mixed oligopoly by assuming that firms have asymmetric technologies and variable marginal cost. In fact, the analysis of mergers incentives in mixed oligopoly has been developed under the assumption that firms have identical technologies. There are no existing studies on the case of asymmetric technologies. To fill this gap, we investigate a mixed tripoly model where firm's technologies are different and we analyse the impact of this in firm's incentives to merge. In our model, public firm may have a more efficient technology than private firms.

We show that public and private firms want to merge when the shareholding ratio of the public sector is relatively low after the merger, and when public firm is less efficient than a private firm. The studies on the decision to merge by public and private firms in a mixed triopoly show that complete privatization through merger will not take place (Nakamura and Inoue 2007; Mendez Naya 2008; Artz et al. 2009). Nevertheless, we show that when the technological gap is high enough, these mergers include complete privatisation. In addition, there is no possibility of merger when the public firm has a technological forwardness.

The paper is organized as follows. Section 2 sets up the model. In section 3, we present the equilibriums of the different scenarios, and conclusions are drawn in section 4.

2 The model

We consider an industry consisting of three firms with a single homogeneous output. One of the firms is a welfare-maximizing public firm (denoted by index 0), and the other two are profit-maximizing private firms (denoted by indexes 1 et 2). Let q_0 and q_i denote the quantities of the public firm and private firm i, respectively ($i = 1, 2$). The inverse demand function is given by

$$p = 1 - Q.$$

where Q is the total output of the good ($Q = q_0 + \sum_{i=1}^{2} q_i$). The cost function of firm i is represented by the quadratic cost function:

$$C_j = \delta_j q_j^2 \qquad \{j = 0, 1, 2\} \tag{1}$$

where $\delta_j > 0$. This cost function generates linear marginal cost curves with slope $2\delta_j$. This formulation of the cost function allows firms to have different technologies[1]. However, we assume that both private firms have identical technologies ($\delta_1 = \delta_2$) and are therefore symmetric. The profit function of a firm j is given as:

$$\pi_j = pq_j - \delta_j q_j^2 \qquad (j = 0, 1, 2) \tag{2}$$

As it is usually assumed in mixed oligopoly models, each private firm chooses its output level in order to maximize (2). On the other hand, the public firm chooses its output to maximize the social welfare. The social welfare (W) is defined as the sum of consumers' surplus (denoted by CS) and producers' surplus[2]. Therefore, social welfare is given by:

$$W = CS + \sum_{j=0}^{2} \pi_j \tag{3}$$

We suppose that firms have possibility to merge. Furthermore, we assume that mergers which result in a monopoly are prohibited by antitrust laws. Hence, starting with a premerger market structure $M_a = \{0, 1, 2\}$,three mergers are possible, leading to the post-merger market structures

$M_b = \{(1, 2), 0\} \rightarrow$ Merger between two private firms.

$M_c = \{(0, 1), 2\} \rightarrow$ Merger between public firm and private firm 1.

$M_d = \{1, (0, 2)\} \rightarrow$ Merger between public firm and private firm 2.

Since the two private firms are identical, market structures M_c and M_d are symmetric. Without loss of generality, we will not analyze the market structure M_d. When firms i and j merge, the merged firm (ij) retains two plants, one of which is owned by firm (i) and the other by firm (j) before the merger. We assume that the merged entity may allocate its production among its two plants in order to minimize its total production cost. This reflects the underlying advantage of being able to direct output across two plants.In this context, the cost function of the merged firm is expressed as:

[1] This cost function allows us to not restrict our analyse only to the case where private firms are more efficient than the public firm. It also consider the case where the public firm would be most efficient.

[2] We consider here that all firms are domestic and that government assigns equal weight to consumers' surplus and profit of each firm, whether private or public.

$$C_{ij} = \frac{\delta_i \delta_j}{\delta_i + \delta_j} q_{ij}^2 \qquad i = 1,2 \; et \; j = 0,1,2 \; et \; i \neq j \tag{4}$$

where $q_{ij} = q_i + q_j$. The profit of the merged firm is:

$$(\pi_{ij}) = p q_{ij} - \frac{\delta_i \delta_j}{\delta_i + \delta_j} q_{ij}^2 \qquad i = 1,2 \; et \; j = 0,1,2 \; et \; i \neq j \tag{5}$$

To characterize technology differences, we define an indicator of asymmetry representing the difference between the public degree of convexity (δ_0) and private degree of convexity (δ_i). This indicator is denoted θ, where $\theta = \delta_0 - \delta_i$. Without loss of generality, we normalize δ_0 to 1, therefore $\delta_i = 1 - \theta$. Yet $C_0 = q_0^2$ and $C_i = (1-\theta) q_i^2$. We must have $\theta < 1$. Furthermore, θ can be negative when the public firm has a more efficient technology. We speak of "technological forwardness of the public firm" when $\theta < 0$ and "technological backwardness of the public firm" when $\theta > 0$.

We propose a two stages game with the following time. At the first stage, the merger decision is taken and then, given this decision, all firms simultaneously set their output at the second stage. To obtain a subgame perfect equilibrium, the game is solved by backwards induction.

3 Equilibriums

3.1 Pre-merger equilibrium: $M_a = \{0, 1, 2\}$

We consider a mixed triopoly, assuming that the merger has not yet occurred. In this case, the private firm i and the public firm choose respectively q_i and q_0 to maximize profit and social welfare:

$$\pi_{ia} = p_a q_{ia} - (1-\theta)(q_{ia}^w)^2, \qquad W_a = \frac{(Q_a^w)^2}{2} + \sum_{j=0}^{2} \pi_{ja}^w. \qquad (i = 1,2) \tag{6}$$

The first order conditions of the maximization problems give the following Cournot equilibrium:

$$\begin{array}{lll} q_{ia} = \frac{2}{13-6\theta} & q_{0a} = \frac{3-2\theta}{13-6\theta}, & p_a = \frac{6-4\theta}{13-6\theta} \\ \pi_{ia} = \frac{8-4\theta}{(13-6\theta)^2}, & \pi_{0a} = \frac{(2\theta-3)^2}{(13-6\theta)^2}, & W_a = \frac{\left(12\theta^2-68\theta+99\right)}{2(13-6\theta)^2} \end{array} \tag{7}$$

The following corollary are immediate consequences of (7).

Corollary 1

(a) For every value of $\theta < (>) \frac{1}{2}$, the private firm i has a lower (higher) output than the public firm.

(b) For every value of $\theta < (>) 0.13$, the public firm's profit is higher (lower) than the private firm's.

In contrast to the mixed oligopoly literature, the public firm's output is not always higher than that of private firms *(a)*. Here, private firms can produce more than the public firm when the technology difference is very high ($\theta > \frac{1}{2}$). According to *(b)*, the public firm that seeks to maximize welfare may have higher profit than does a private profit-maximizing firm. Given that the public firm takes into account consumer surplus, it produces more than any private firm when the technology difference is low. In this context, private firms have a smaller potential market and restrict their output (relative to the case where all firms are private). Thus, this lead to a larger market share and higher profit for the public firm, unless θ is very great. This result extends that of Kamaga and Nakamura (2007) for the specific case where all firms have the same technology ($\theta = 0$).

Consumer surplus and social welfare strictly increase with parameter θ, while price decreases as θ increases. This means that price is much lower if the public firm has a "technological backwardness" ($\theta > 0$). The intuition behind of this surprising result is quite straightforward. When parameter θ increases, the public firm decreases her output and private firms increase their output. Nevertheless, the decrease of public output is less than the increase of any private firm. Thus, the total output increases and price decreases.

The profit of two private firms increases with parameter θ while the profit of the public firm decreases with this parameter.

3.2 Post merger equilibrium

- $M_b = \{(1,2), 0\}$

We consider now the case where the two private firms decide to merge into a new private firm denoted 12. The objectives of firm 12 and public firm are given by:

$$(\pi_{12})_b = p_b(q_{12b}) - \frac{1-\theta}{2}(q_{12b})^2 \qquad W_b = \frac{(Q_b)^2}{2} + (\pi_{12})_b + \pi_{0b} \tag{8}$$

Thus we get the following equilibrium:
$q_{12b} = \frac{2}{8-3\theta}, \quad q_{0b} = \frac{2-\theta}{8-3\theta}, \quad p_b = \frac{4-2\theta}{8-3\theta}$
$(\pi_{12})_b = \frac{6-2\theta}{(8-3\theta)^2}, \quad \pi_{0b} = \frac{(\theta-2)^2}{(8-3\theta)^2}, \quad W_b = \frac{3\theta^2-20\theta+36}{2(8-3\theta)^2}$
The output of the merged firm is smaller than that of its pre-merger constituent parts. In response to the merged firm strategy (the merged firm reduces her output), the public firm increases her output. This happens because the reduction in output by the merged firm increases the importance of consumer surplus to the public firm causing it to further increases output. In addition, the total output decreases and the price increases[3]. However, the merger reduces the total welfare. Furthermore, when firms have identical technology ($\theta = 0$), the public firm produces the same output as the merged firm.

- $M_c = \{(0,1), 2\}$

Finally, we consider the case where a private firm and a public firm decide to merge. Following Matsumura (1998), we assume that when the public and the private firms decide to merge, the merging firm (01), is partially owned by private and public owners. If we denote α, the private owner's shareholding proportion in the merged firm, the objective function of the merged firm is given by:

$$V_{01} = (1-\alpha)W_c + \alpha(\pi_{01}) \tag{9}$$

where $\alpha \in [0,1]$. In this case, the objective function of the outsider is:

$$\pi_{2c} = p_c q_{2c} - (1-\theta)(q_{2c})^2 \tag{10}$$

Thus, we get the following equilibrium:
$q_{01c} = \frac{(2\theta-3)}{(6\theta-4\alpha+2\theta\alpha-7)}, \quad q_{2c} = \frac{2\alpha-2\theta-\theta\alpha+2}{(\theta-2)(6\theta-4\alpha+2\theta\alpha-7)}, \quad p_c = \frac{(2\theta-3)(2\theta-2\alpha+\theta\alpha-2)}{(\theta-2)(6\theta-4\alpha+2\theta\alpha-7)}$
$(\pi_{01})_c = \frac{(2\theta-3)^2(\theta-2\alpha+\theta\alpha-1)}{(\theta-2)(6\theta-4\alpha+2\theta\alpha-7)^2}, \quad \pi_{2c} = \frac{-(2\theta-2\alpha+\theta\alpha-2)^2}{(\theta-2)(6\theta-4\alpha+2\theta\alpha-7)^2}$
$W_c = \frac{8\theta^4\alpha+12\theta^4-2\theta^3\alpha^2-68\theta^3\alpha-92\theta^3+13\theta^2\alpha^2+212\theta^2\alpha+251\theta^2-28\theta\alpha^2-284\theta\alpha-286\theta+20\alpha^2+136\alpha+116}{2(\theta-2)^2(6\theta-4\alpha+2\theta\alpha-7)^2}$

Proposition 1 *When public and private firms merge:*

[3] All this changes can be obtained by differentiating the equilibrium values of M_b and M_a. For example, $p_b - p_a = 3\frac{-29\theta+6\theta^2+36}{(6\theta-13)(3\theta-8)} > 0 \ \ \forall\theta < 1$

- *The output of the merged firm decreases (increases) relative to its pre-merger constituent firms as* $\alpha > (<) \frac{2}{(\theta-2)(2\theta-5)}$

- *The output of the outsider decreases (increases) relative to its pre-merger as*

$$\alpha < (>) \frac{2}{(\theta-2)(2\theta-5)}$$

Proof (see appendix 1)

This proposition shows that there exists a critical value of α such that the output of merging firms is equal of its pre-merger constituent firms. For low values of private owner's shareholding, the output of the merged firms decreases relative to its pre-merger constituent firms.

The output of the merged firm and that of the outsider (private firm) are strategic substitutes. Thus, when the merged firm decreases output, the private firm increases output and when the merged firm increases output, the private firm decreases output. Moreover, the total market output and the price are respectively decreasing and increasing functions of α.

3.3 The decision by firms to merge

When public and private firms decide to merge, there are two kinds of merger incentive.

Private incentives to merge: The owners of the private firm (i) will want to merge if the profit that they obtain in the merged entity, $\alpha_i(\pi_{ij})_k$, is greater than the profit obtained by the private firm in the mixed triopoly,π_i.

$\alpha_i(\pi_{ij})_k > \pi_i$ with $i = 1, 2$ and $j = 0, 1, 2$ and $i \neq j$. $\quad k = b, c, d$.

Public incentives to merge: The public firm (0) will want to merge if the welfare after the merger, is greater than that in mixed triopoly.

$W_k > W_0$ with $k = c, d$.

Proposition 2

2.a) *The private owners want to merge with the public firm, only if, after the merger, private owners owns a high enough percentage of the mixed firm. The merger is always profitable for private firm regardless* θ *when* $\alpha \in \left[\frac{2}{3}, 1\right]$.

2.b) *The public firm want to merge with a private firm, only if, after the merger, government owns a high enough percentage of the mixed firm. The merger always increases welfare regardless* α *when* $\theta \in]0.89, 1[$.

Proof (see appendix 2)

Proposition *2.a* is illustrated in Fig.1. Fig.1 shows how the decision to merge by private owners depends on parameters α and θ. α^π is the value of parameter α such that[4] $\Delta\pi_c = 0$. When the point (θ, α) is above the curve representing α^π, the private firm will prefer to merge. And when this point is under the point, it will prefer not to merge.

[4] $\Delta\pi_c = \alpha(\pi_{01})_c - (\pi_{1a})$.

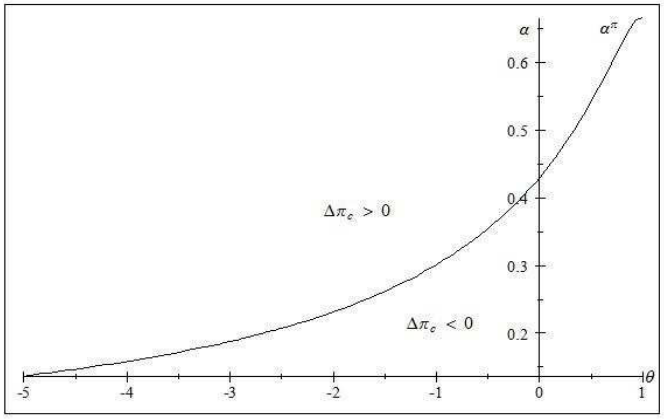

Fig.1: Private firm incentives to merge

Proposition *2.a* shows that private owners want to merge with the public firm if, after the merger, the shareholders of the private firm own a high enough percentage of the shares in the mixed merged firm ($\alpha > \alpha^{\pi}$). α^{π} is an increasing [5] function of θ. Therefore, when $\theta > 0$, the private firm would prefer not to merge for low values of α ($\alpha < 0.43$). Let us note that $\lim_{\theta} \rightarrow 1\alpha^{\pi} = \frac{2}{3}$. This result implies that irrespective of θ, the shareholders of the private firm will always want the merger when they will have more than $\frac{2}{3}$ percentage of the shares in the merging firm.

Proposition *2.b* is illustrated in Fig.2. Fig.2 shows how the decision to merge by public firm depends on parameters α et θ. α^{w} is the value of parameter α such that[6] $\Delta W_c = 0$. When the piont (θ, α) is under the curve representing α^{w}, the public firm will prefer to merge. And when this point is above the point, it will prefer not to merge.

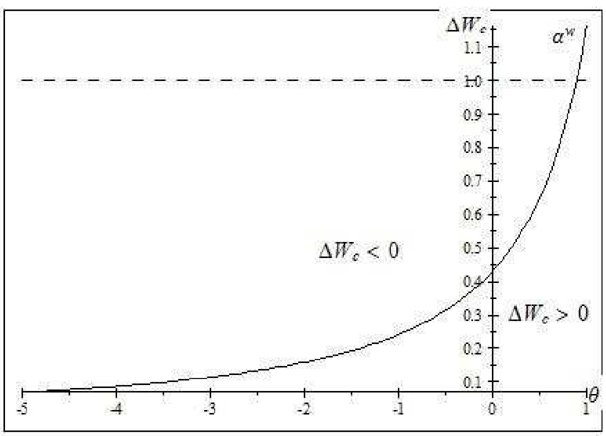

Fig.2: Public firm incentives to merge

Proposition *2.b* shows that even if the merged entity behaves like a private firm by maximizing profit, the public firm could have an incentive to merge. Indeed, when $\alpha = 1$, the merger improves the welfare if the technology gap is high ($\theta > 0.89$). By considering a mixed triopoly and considering that firms have identical technology, Nakamura and Inoue (2007) and Artz et al. (2009) show that when $\alpha = 1$, the public firm have no incentive to merge with one private firm. We show in this paper that if costs are asymmetric, the public firm could have an incentive to merge.

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[5] $\frac{d}{d\theta}(\alpha^{\pi}) > 0$

[6] $\Delta W_c = W_c - W_a$

The following lemma compares α^w and α^π.

Lemma 1 $\alpha^w > \alpha^\pi \ \ \forall\theta \geq 0$; There exists $\widetilde{\theta} \in \left]-\infty; 0\right[$ such that $\alpha^w \leq \alpha^\pi$ for $\theta \leq \widetilde{\theta}$.

According to Lemma 1, when private firms technology is at least identical to that of public firm ($\theta \geq 0$), α^w is greater than α^π. On the other side, when Parameter θ is low than the critical value $\widetilde{\theta}$, the reverse happens ($\alpha^w \leq \alpha^\pi$). The approximative value of $\widetilde{\theta}$ (value of θ such that $\alpha^w = \alpha^\pi$) is -0.00038.

Taking into account propositions 6, 7 and lemma 1, we obtain the following proposition:

Proposition 3 *The private firm and the public firm will merge when $\theta > \widetilde{\theta}$ and $\alpha^\pi < \alpha < \alpha^w$.*

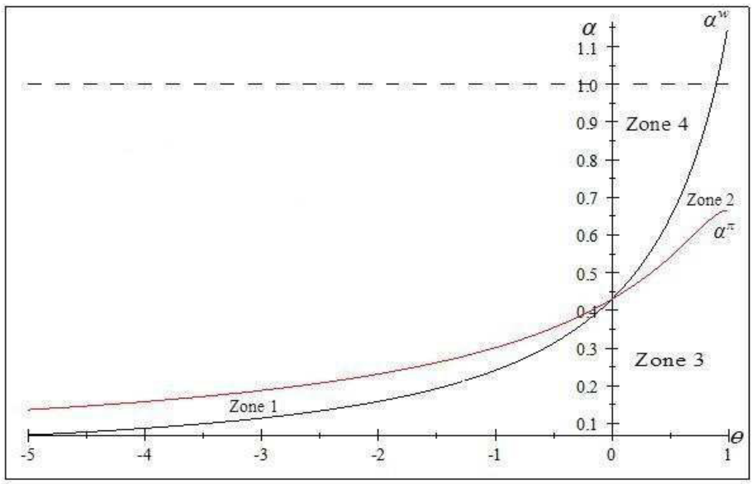

Fig.3: Public and private firms incentive to merge
Zone 1- Neither the public nor the private firm want to merge
Zone 2- Both, public and private firms want to merge
Zone 3- Only the public firm wants to merge
Zone 4- Only the private firm wants to merge

From this proposition, it can be concluded that public and private firms merge when the shareholders of the private firm own a high enough percentage of the shares in the mixed merged firm and $\theta \in \left]\widetilde{\theta}; 1\right[$. It should be emphasized that the interval in which this merger can occur when public firm has "technological forwardness" ($\left[\widetilde{\theta}; 0\right[$) is very narrow, with $0 - \widetilde{\theta} \approx 0.00038$. In other words, there is practically no possibility of merger when the public firm has "technological forwardness".

4 Conclusion

This paper explores the incentives for mergers in an asymmetric mixed oligopoly consisting of a single public firm and two symmetric private firms. When considering the merger of a public firm with a private one, we show that both firms decide to merge if $\theta > \widetilde{\theta}$ and the shareholding ratio of private firm is $\alpha \in (\alpha^\pi, \alpha^w)$. This happens because when the public firm is less efficient than a private firm, the gains in efficiency resulting from the merger are greater than the deadweight loss that results from market power after the merger. Moreover, when $\theta \geq 0.9$, both firms want to merge even if the merged firm is owned only by private sector. This result is fairly remarkable in that it expands the result obtained by Artz et al. (2009). They show that in mixed triopoly where firms have identical technology, the merger can occur when α is relatively low. Yet, we show that when the technological gap is high enough, the merger between the public firm and one private firm often includes complete privatization.

Two interesting extensions of our model remain. One is to explore the model where private firms are owned by foreign shareholders. This situation would have an impact on the firm's decision to merge and equilibrium outcomes, since social welfare may not include the profits of the foreign firm [7]. The other extension is to endogenize the decision to merge. Assuming a model with two symmetric private firms and one inefficient public firm with linear cost, Kamijo and Nakamura (2009) show that the only stable market structure contains a merged public-private firm. Kamaga and Nakamura (2007) obtained similar results by considering that firms have identical technologies with increasing marginal cost. The introduction of asymmetry across the firm's technologies is important for further research of endogenous mergers in mixed oligopoly.

APPENDIX

Appendix 1:

Change in output for merged firm: $q_{01c} - (q_{0a} + q_{1a}) = 2\frac{10\alpha - 9\theta\alpha + 2\theta^2\alpha - 2}{(13-6\theta)(6\theta - 4\alpha + 2\theta\alpha - 7)}$

Setting this expression equal to zero, yields to: $\alpha = \frac{2}{(\theta-2)(2\theta-5)}$. If $\alpha > \frac{2}{(\theta-2)(2\theta-5)}$ $\rightarrow$ The output of the merged firm decreases. If $\alpha < \frac{2}{(\theta-2)(2\theta-5)}$ $\rightarrow$ The output of the merged firm increases.

Change in output for outsider: $q_{2c} - q_{2b} = -\frac{10\alpha - 9\theta\alpha + 2\theta^2\alpha - 2}{(\theta-2)(6\theta-13)(6\theta - 4\alpha + 2\theta\alpha - 7)}$

Setting this expression equal to zero, yields to: $\alpha = \frac{2}{(\theta-2)(2\theta-5)}$. If $\alpha < \frac{2}{(\theta-2)(2\theta-5)}$ $\rightarrow$ The output of the outsider decreases. If $\alpha > \frac{2}{(\theta-2)(2\theta-5)}$ $\rightarrow$ The output of the outsider increases.

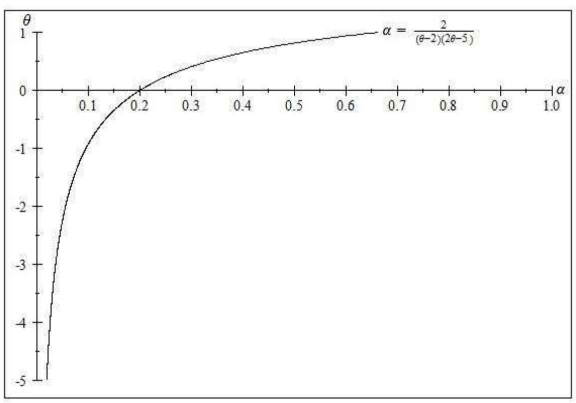

Fig.4: Graphical representation of $\alpha = \frac{2}{(\theta-2)(2\theta-5)}$

Appendix 2:

Proof of proposition 2.a: The owners of the private firm will want to merge if the profit that they obtain in the merged entity $\alpha(\pi_{01})_c$, is greater than the profit obtained by the private firm in the mixed triopoly (π_{1a}). Let $\Delta\pi_c = \alpha(\pi_{01})_c - (\pi_{1a})$

$$\Delta\pi_c = -\frac{\begin{bmatrix} \alpha^2(\theta-2)(-3240\theta + 2776\theta^2 - 1040\theta^3 + 144\theta^4 + 1393) \\ +\alpha(28841\theta - 4480\theta^2 + 3240\theta^3 - 1104\theta^4 + 144\theta^5 - 625) \\ +2116\theta^2 - 21288\theta - 912\theta^3 + 144\theta^4 + 784 \end{bmatrix}}{(2-\theta)(6\theta-13)^2(6\theta - 4\alpha + 2\theta\alpha - 7)^2}$$

If we denote α^π, the value of parameter α such that $\Delta\pi_c^w = 0$:

$$\alpha^\pi = -\frac{1}{2}\frac{\begin{bmatrix} 28841\theta - 4480\theta^2 + 3240\theta^3 - 1104\theta^4 + 144\theta^5 - 625 \\ +(2\theta-3)(6\theta-13)\sqrt{-20\,618\theta + 28\,825\theta^2 - 20\,968\theta^3 + 8360\theta^4 - 1728\theta^5 + 144\theta^6 + 6001} \end{bmatrix}}{(\theta-2)(-3240\theta + 2776\theta^2 - 1040\theta^3 + 144\theta^4 + 1393)}$$

The sign of $\Delta\pi_c$ depends on that of its numerator. Since this numerator is a quadratic and convex function of α and is equal to zero when $\alpha = \alpha^\pi$, therefore $\alpha(\pi_{01})_c > (\pi_{1a})$ if and only if $\alpha > \alpha^\pi$.

[7] Fjell and Pal (1996), Pal and White (1998) et Fjell and Heywood (2002) analyzed a mixed oligopoly that includes foreign firms.

Proof of proposition 2.b: Since the public firm aims to maximize the social welfare, it has an incentive to merge if the welfare gain is positive after the merger. Let $\Delta W_c = W_c - W_a$

$$\Delta W_c = -\frac{\begin{bmatrix} \alpha^2(\theta-2)^2 - (1554\theta + 1184\theta^2 - 392\theta^3 + 48\theta^4 + 739) \\ -4\alpha(\theta-2)(163\theta - 80\theta^2 + 12\theta^3 - 101) \\ +434\theta - 312\theta^2 + 72\theta^3 - 200 \end{bmatrix}}{2(\theta-2)^2(6\theta-13)^2(6\theta-4\alpha+2\theta\alpha-7)^2}$$

If we denote α^w, the value of parameter α such that $\Delta W_c = 0$:

$$\alpha^w = \frac{1}{6}\frac{-\frac{427}{4}\theta+\frac{323}{4}\theta^2-26\theta^3+3\theta^4+\frac{101}{2}+(\theta-\frac{3}{2})(\theta-\frac{13}{6})(\frac{3}{2}\sqrt{2})\sqrt{-692\theta+762\theta^2-415\theta^3+112\theta^4-12\theta^5+248}}{(-\frac{259}{8}\theta+\frac{74}{3}\theta^2-\frac{49}{6}\theta^3+\theta^4+\frac{739}{48})(\theta-2)^2}$$

The sign of this expression depends on that of its numerator. Since this numerator is a quadratic and convex function of α and is equal to zero when $\alpha = \alpha^w$, therefore $W_c > W_a$ if and only if $\alpha < \alpha$.

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