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A mechanism to pick the deserving winner

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Abstract

A group of individuals is choosing an individual (the winner) among themselves, when the identity of the deserving winner is a common knowledge among individuals. A simple mechanism of voting by veto is proposed as an alternative to the mechanism studied by Amorós (2011). Like Amorós' (2011), the suggested mechanism implements the socially desirable outcome (the deserving winner is chosen) in subgame perfect equilibria.

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1. Introduction

Consider a group of individuals who must choose a winner among themselves, when the identity of the deserving winner, w , is common knowledge among all individuals. The social optimal outcome, prescribed by the social choice function, is that the deserving winner wins. However, each individual i is selfish in the sense that, i prefers most of all to be the winner. But at the same time, each individual i is impartial towards the rest, i.e. if i is not chosen as the winner, i prefers w to be chosen.

Amorós (2011) studies the same problem of a jury choosing one winner from a set of agents, each juror favoring one different agent, or, in particular case, where the jury is made up of all agents. To reach the socially desirable outcome Amorós (2011) proposes a mechanism à la Maskin (1999) that implements the social choice function in subgame perfect equilibria. For each extensive form mechanism and each state of the world, a subgame perfect equilibrium induces a Nash equilibrium in every subgame (see Moore and Repullo 1998, Abreu and Sen 1990). In spite of the criticism received by being unnatural (see Jackson 1992), these mechanisms can be applied to specific problems, as done in Amorós (2011), who provides a simple and “natural” extensive form mechanism.

The present paper replicates Amorós’ (2011) result by suggesting an alternative mechanism. The proposed mechanism can be considered as a reversal of the one by Amorós (2011). The individuals, instead of voting for the most preferred candidate, are given the possibility to veto a candidate. A mechanism of voting by veto (hereinafter, veto mechanism) also implements the desired social choice function in subgame perfect equilibria. Moreover, the proposed veto mechanism works for three individuals, improving upon Amorós’(2011), whose mechanism needs at least four individuals to work and fails with three individuals.

The rest of the paper is organized as follows. Section 2 provides the model. Section 3 describes the veto mechanism. Section 4 analyses the case of the implementation of the socially optimal rule in subgame perfect equilibria with $n = 3$ individuals. Section 5 presents the general implementability result of the veto mechanism. Section 6 concludes the paper.

2. The model

Let $N = \{1, 2, \dots, n\}$ be a set of $n \geq 3$ individuals who must choose one individual (the winner) among them. All individuals know who deserves to win: the “deserving winner”. The socially optimal outcome is that the deserving winner wins. However, each individual $i \in N$ is selfish: i always wants to be the winner. But at the same time, if i is not chosen as the winner, i prefers the deserving winner w to be chosen.

There is a fixed individual $w \in N$, interpreted as the deserving winner. The individuals have preferences defined over N , i.e. transitive and complete binary relations on N . A preference of individual i can be considered as i ’s ranking of all individuals in the group, including himself, from most to least preferred individual, with i being first in his preference profile and w being second. Let R_i denote i ’s preference and P_i denote the strict part of R_i .

Definition. A preference R_i of individual $i \in N$ is admissible if:

- (i) for each $w \in N$ and each $j \in N$ such that $j \neq i$, $i P_i j$, and

(ii) for each $w \in N$ and each $j \in N$ such that $j \neq w$ and $j \neq i$, $w P_i j$.

Let Θ_i designate the set of admissible preferences for individual i . A social choice function with the deserving winner w is a function $f_w: \prod_{i \in N} \Theta_i \rightarrow N$ that, for every admissible preference profile, selects the deserving winner w , i.e. for all $R \in \prod_{i \in N} \Theta_i$, $f_w(R) = w$.

An extensive form mechanism, denoted by $\Gamma(M, g)$, consists of a set of (pure) strategies profiles of all individuals, $M = \prod_{i \in N} M_i$, and the order, in which the individuals choose their strategies. An outcome function $g: M \rightarrow N$ associates an individual $g(m)$ with each profile m of messages and the order of the sequential game. For every profile $R \in \prod_{i \in N} \Theta_i$, the pair (Γ, R) constitutes an extensive form game. It is a game of perfect information, as each individual, when playing his pure strategy, knows the previous history of the game and acts according to this history. A subgame perfect equilibrium (SPE) of a perfect information game is a strategy profile that induces an equilibrium in every subgame of the game. The social choice function f_w is subgame perfect equilibria implementable if there exists a sequential mechanism Γ such that the set of SPE outcomes of the game (Γ, R) has one element: $f_w(R)$, that is, w .

3. The mechanism

Veto mechanism. Given an arbitrary linear ordering $(1, 2, \dots, n)$ of the $n \geq 3$ individuals, each individual from 1 to $n - 1$ announces an individual to veto from those not having been vetoed before. Once individual $n - 1$ has made his announcement, there only remains one individual, v . Let z be the first individual in the ordering $(1, 2, \dots, n - 1)$ that does not veto himself (i.e. the first individual that vetoes an individual different from himself), if such an individual exists. If no such z exists or if $z \neq v$, then the outcome of the mechanism is that v is chosen as a winner; if $z = v$, then the outcome of the mechanism is determined by letting n choose the winner between v and the individual v^* vetoed by v : n chooses the most preferred individual, if there is one, and any of the two, if n is indifferent between v and v^* .

4. The three individual case

This section analyses the mechanism when there are $n = 3$ individuals considering the different positions that w can occupy in the linear order. This analysis will demonstrate that all SPE paths lead to the election of the deserving winner w as the final outcome.

Lemma 1. *For $n = 3$ the veto mechanism implements the social choice function f_w in subgame perfect equilibria.*

Proof. Suppose that the linear order is $(1, 2, 3)$. It will be demonstrated that f_w is implementable in subgame perfect equilibria by means of the veto mechanism. The mechanism starts with individual 1 announcing his veto. Individual 1 has three options: to veto 1, to veto 2 or to veto 3. Each option leads to a different path. The proof depends on the position that the deserving winner w occupies.

Case 1: $w = 1$.

Path 1: 1 vetoes 1. Then 2 vetoes either 2 or 3. If 2 vetoes 2, then no individual vetoes himself, so $v = 3$ is chosen as the winner. If 2 vetoes 3, then $z = v = 2$ and, consequently, $n = 3$

chooses the winner between $v = 2$ and $v^* = 3$. As $3 P_3 2$, 3 chooses himself as the winner. Therefore, no matter whether 2 vetoes 2 or 3, 3 is the winner.

Path 2: 1 vetoes 2. Then 2 vetoes either 1 or 3. If 2 vetoes 1, then $z = 1$, $v = 3$, and as $z \neq v$, $v = 3$ becomes the winner. If 2 vetoes 3, then $z = v = 1$, and $n = 3$ chooses the winner between $v = 1$ and $v^* = 2$. Since $1 P_3 2$, 3 will choose $1 = w$ as the winner. At 2's node, given that $1 P_2 3$, the best option for 2 is to veto 3, so that $1 = w$ is chosen as the winner.

Path 3: 1 vetoes 3. Then 2 vetoes either 1 or 2. If 2 vetoes 1, $z = 1$ and $v = 2$. Since $z \neq v$, $v = 2$ is chosen as the winner. If 2 vetoes 2, $z = v = 1$. Therefore, $n = 3$ picks the winner between $v = 1$ and $v^* = 3$. Since $3 P_3 1$, 3 will choose $v^* = 3$ as the winner. As a result, at 2's node the best option for 2 is to veto 1.

Given the outcomes of paths 1 - 3, at 1's node the best option for 1 is to veto 2, as it is the only strategy that leads to the best outcome for 1: $1 = w$ is chosen as the winner. This proves that, when $w = 1$, all subgame perfect equilibria lead to the outcome when the deserving winner wins.

Case 2: $w = 2$.

Path 1: 1 vetoes 1. Then 2 vetoes either 2 or 3. If 2 vetoes 2, then no individual vetoes himself, so $v = 3$ is chosen as the winner. If 2 vetoes 3, then $z = v = 2$, and so $n = 3$ picks the winner between $v = 2$ and $v^* = 3$. As $3 P_3 2$, 3 chooses himself as the winner. Therefore, no matter if 2 vetoes 2 or 3, 3 is the winner.

Path 2: 1 vetoes 2. 2 vetoes either 1 or 3. If 2 vetoes 1, then $z = 2$, $v = 3$, and since $z \neq v$, $v = 3$ becomes the winner. If 2 vetoes 3, $z = v = 1$, then $n = 3$ chooses the winner between $v = 1$ and $v^* = 2$. Since $2 P_3 1$, 3 picks $2 = w$ as the winner. At 2's node, since $2 P_2 3$, the best choice for 2 is to veto 3.

Path 3: 1 vetoes 3. 2 vetoes either 1 or 2. If 2 vetoes 1, then $z = 1$, $v = 2$, and since $z \neq v$, $v = 2$ is chosen as the winner. If 2 vetoes 2, $z = v = 1$, and since $z = v$, $n = 3$ picks the winner between $v = 1$ and $v^* = 3$. As $3 P_3 1$, 3 will choose $v^* = 3$ as the winner. At 2's node, as $2 P_2 3$, the best choice for 2 is to veto 1. Therefore, $2 = w$ is chosen as the winner.

Given the outcomes of paths 1 - 3, at 1's node the best choice for 1 is to veto 2 or 3, as these paths both result in the best outcome for 1: $w = 2$ is chosen as the winner. Thus, it has been demonstrated that, when $w = 2$, all subgame perfect equilibria lead to the outcome that the deserving winner wins.

Case 3: $w = 3$.

Path 1: 1 vetoes 1. 2 vetoes either 2 or 3. If 2 vetoes 2, then no individual vetoes himself, so $v = 3$ is chosen as the winner. If 2 vetoes 3, $z = v = 2$, and, consequently, $n = 3$ chooses the winner between $v = 2$ and $v^* = 3$. As $3 P_3 2$, 3 chooses himself as the winner. Therefore, no matter whether 2 vetoes 2 or 3, $3 = w$ is the winner.

Path 2: 1 vetoes 2. 2 vetoes either 1 or 3. If 2 vetoes 1, $z = 1$ and $v = 3$, and since $z \neq v$, $v = 3$ becomes the winner. If 2 vetoes 3, as $v = z = 1$, at 3's node, $n = 3$ chooses the winner between

$v = 1$ and $v^* = 2$. Here consider two subcases: (i) if $1 P_3 2$, 3 will choose 1 as the winner; (ii) if $2 P_3 1$, 3 will choose 2 as the winner.

At 2's node, in subcase (i), if $1 P_3 2$, given that $3 P_2 1$ when $w = 3$, the best choice of 2 is to veto 1, so that the outcome is $w = 3$. In subcase (ii), if $2 P_3 1$, the best option for 2 is to veto 3, as 2 prefers himself to be the winner.

Path 3: 1 vetoes 3. 2 vetoes either 1 or 2. If 2 vetoes 1, $z = 1$, $v = 2$, and since $z \neq v$, $v = 2$ is chosen as the winner. If 2 vetoes 2, $z = v = 1$, and, therefore, $n = 3$ picks the winner between $v = 1$ and $v^* = 3$. Given that $3 P_3 1$, 3 will choose $v^* = 3$ as the winner. At 2's node, since $2 P_2 3$, the best option for 2 is to veto 1.

Given the outcomes of paths 1 - 3, at 1's node the best option for 1 is to veto 1 or 2 in subcase (i) (when $1 P_3 2$), so that the winner is $w = 3$; and to veto 1 in subcase (ii) (when $2 P_3 1$), thus, $w = 3$ is chosen as the winner. Therefore, all SPE outcomes result in the election of w as the winner. ■

5. Main result

Lemma 2. For a given $R \in \Pi_{i \in N} \Theta_i$, let p be a path connecting the root of the game (Γ, R) with one of its outcomes. Let r be a decision node reached by the path such that: (i) the individual i assigned to r is vetoed by some predecessor j along p ; and (ii) individual j is also vetoed by some predecessor along p (therefore, $i \geq 3$). Then no path starting at node r leads to outcome i .

Proof. For i to be reached from r by another path p' (that coincides with p before r) under the conditions of Lemma 2 it is necessary (a) that the non-vetoed individual v along p' is i or (b) that the non-vetoed individual v along p' is the one that has vetoed i . By (i), i has already been vetoed before r is reached, so (a) cannot hold. By (ii), the individual j who has vetoed i has also been vetoed before r is reached, for which reason (b) cannot hold. ■

Proposition. If $n \geq 3$, then the veto mechanism implements the social choice function f_w in subgame perfect equilibria.

Proof. Since Lemma 1 proves the result when $n = 3$, let $n \geq 4$ and assume the result true for all $n' \in \{3, \dots, n - 1\}$.

For a given $R \in \Pi_{i \in N} \Theta_i$, consider the game (Γ, R) induced by the veto mechanism when the deserving winner is a given $w \in \{1, \dots, n\}$ and the preferences of the individuals are the admissible preferences with deserving winner w . It must be shown that w is the only subgame perfect equilibrium of the game.

- Case 1: $w = 1$. Let p be the path that results when, for all $k \in \{1, \dots, n - 1\}$, k vetoes $k + 1$. Observe that, along path p : (i) no one vetoes w and, hence, $v = w$; and (ii) the first individual not vetoing himself is 1, that is, $z = 1$. Since $w = 1$, the outcome is given by the choice of individual n between $w = 1$ and the individual 2 vetoed by 1. Given that $n \geq 4$ prefers w to 2, n chooses w . To sum up, path p leads to outcome w .

By Lemma 2, no individual $k \geq 3$ has an incentive to deviate from p , because by deviating from p no such individual can obtain the only outcome more preferred than w : outcome k . Obviously, being 1 the deserving winner w , 1 has neither an incentive to deviate. Finally, as 2 has been vetoed by 1, the only reason why 2 could deviate from p is that $v = z = 1$, in which case individual n choose from 1 and 2 (the individual vetoed by 1). Yet, being 1 the deserving winner, $n \neq 2$ prefers 1 to 2, on account of which no deviation by 2 from p makes it possible for 2 to obtain a better outcome than w .

The final conclusion is that no individual has an incentive to deviate from p . This makes p lead to a subgame perfect equilibrium outcome (the deserving winner) and no other subgame perfect equilibrium outcome can be different from w .

- Case 2: $w = n$. Let now the path p be the one that results when, for all $k \in \{1, \dots, n - 2\}$, k vetoes $k + 1$ and $n - 1$ vetoes 1. In this case, z (the first individual not vetoing himself along p) is 1, whereas the non-vetoed individual is the deserving winner w . Given that $v \neq z$, the outcome that corresponds to p is $v = w$.

As in case 1, by Lemma 2, no individual $k \geq 3$ has an incentive to deviate from p . As regards $k = 2$, the only reason that could justify a deviation from p by 2 is that some deviation leads to outcome 2, the only outcome more preferred by 2 to w . But even if there existed a subgame perfect equilibrium path starting at 2's node leading to outcome 2, this would not constitute a subgame perfect equilibrium of the whole game because, by vetoing himself, 1 can force the occurrence of outcome w . In fact, when 1 vetoes 1, the subgame that starts at 2's node is the game induced by the veto mechanism when the deserving winner is a given $w \in \{2, \dots, n\}$ and the preferences of the $n - 1$ individuals are the admissible preferences with deserving winner w and, by the induction hypothesis, the only subgame perfect equilibrium outcome of this game is w .

Finally, it rests to be shown that no subgame perfect equilibrium leads to outcome 1 when 1 vetoes $x \neq 1$, for in that case p would lead to the subgame perfect equilibrium outcome w and no other such outcome would exist.

To this end, notice that, as just shown, there is no subgame perfect equilibrium in which 2 is the outcome. Consequently, 2's best prospect is to make w the winner. The claim is that 2 can ensure that w is the winner by vetoing 1 whenever 1 vetoes $x \neq 1$. Observe that the game obtained when, for all $x \neq 1$, 2 chooses to veto 1 is like the game in which individual 1 has been removed and 2 vetoes x . The induction hypothesis ensures that the only subgame perfect equilibrium of this game is the deserving winner w . In view of this, individual 1 cannot do better than trying to get w and this is ensured by path p .

- Case 3: $1 < w < n$. Let now the path p be the one that results when, for all $k \in \{1, \dots, n - 1\} \setminus \{w - 1, w\}$, k vetoes $k + 1$, $w - 1$ vetoes $w + 1$ and w vetoes 1. Similarly to Case 2, z (the first individual not vetoing himself along p) is 1, while the non-vetoed individual v is the deserving winner w . Given that $z \neq v$, the outcome that p results in is $v = w$.

As in all previous cases, by Lemma 2, no individual $k \geq 3$ has an incentive to deviate from p if $k = w$, this follows from the fact that w is his most preferred individual. To complete the proof, first consider individual $k = 2$. If $2 \neq w$, then the only outcome 2 prefers more than w is when 2 becomes the winner. But 2 has been vetoed by 1, so the only possibility for 2 to be chosen as a winner is when 1 is not vetoed by anyone along the path, so that $z = v = 1$, and n picks between 1 and 2. If it happens that n prefers 2 more than 1, n could choose 2. But this outcome would not constitute a subgame perfect equilibrium of the whole game because, by vetoing himself, 1 can force the occurrence of outcome w .

If $2 = w$, then $w + 1$ has been vetoed by $w - 1$, namely 1. If 2 vetoes 1, then notice that this is the same game as, when 1 is removed and 2 vetoes the individual that 1 has vetoed, namely $w + 1$ (subsequent individual in the ordering after $w = 2$). Now we are in Case 1, when w is the first individual in the ordering, vetoing the subsequent individual. As it has been previously proved, this path results in the outcome w .

Finally, what is left to show is that no subgame perfect equilibrium leads to outcome 1 when 1 vetoes $x \neq 1$. In that case p would lead to the subgame perfect equilibrium outcome w and no other such outcome would exist.

As it has just been shown, there is no subgame perfect equilibrium in which 2 is the outcome, unless 2 is the deserving winner himself. Consequently, if $2 \neq w$, the best option for 2 is to make w the winner. As in the previous claim of Case 2, individual 2 can ensure that w is the winner by vetoing 1 whenever 1 vetoes $x \neq 1$. The game obtained when, for all $x \neq 1$, 2 chooses to veto 1 is like the game in which individual 1 has been removed and 2 vetoes x . By the induction hypothesis, the only subgame perfect equilibrium of this game is the deserving winner w . Consequently, individual 1 cannot do better than trying to get the outcome w and this is ensured by path p . ■

6. Conclusion

We have analyzed the problem of choosing a winner among the individuals when the identity of the deserving winner is common knowledge. It has been proved that the proposed veto mechanism implements the socially desirable outcome (that the deserving winner wins) in subgame perfect equilibria. One contribution is that the veto mechanism conceptualizes a counterpart to Amorós' (2011) mechanism: in his mechanism the individuals *choose*, whereas in the veto mechanism the individuals *reject*. In practice, it seems easier to reject a bad (or worse) option, than to pick the best option (or sufficiently good option). The other contribution is that the veto mechanism works when there are at least three individuals, improving upon Amorós' (2011), which requires a minimum of four individuals.

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