Economics Bulletin

Volume 35, Issue 3

Bootstrap-based Selection for Instrumental Variables Model

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Abstract

This paper develops bootstrap-based method for addressing the "many instruments" problem in the context of instrumental variable estimation. We propose a plug-in restricted-efficient-residual-based (plug-in RE) bootstrap for choosing optimal number of instruments used for two-stage least squares (TSLS) and limited information maximum likelihood (LIML) estimator. In Monte Carlo experiments, we find that the instrument choice based on our plug-in RE bootstrap generally yields an improvement in finite sample performance.

Citation: Wenjie Wang and Qingfeng Liu, (2015) "Bootstrap-based Selection for Instrumental Variables Model", *Economics Bulletin*, Volume 35, Issue 3, pages 1886-1896
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Submitted: June 14, 2015. **Published:** September 07, 2015.

1 Introduction

There have been a considerable interest in the application and properties of instrumental variables estimators. In practice, empirical researchers can often choose among a large number of instrumental variables. It is now well known that in finite samples, instrumental variables estimators behave poorly when there are many instrumental variables (see, e.g., Kunitomo (1980) [10], Morimune (1983)[11] and Bekker (1994)[2], etc.). In particular, the two stages least squares (TSLS) estimator, which is most widely used in practice, has a bias that is proportional to the number of instruments used in the regression. This motivates the use of instruments selection methods.

In the literature of instruments selection, Donald and Newey (2001)[6] proposed minimizing the asymptotic mean square error (MSE) as a criterion for choosing optimal number of instruments. Their approximate MSE is like that of Nagar (1959)[12], being the MSE of leading terms in an expansion of the estimators of interest. However, the formula of the approximate MSE is quite complicated, and thus could be difficult for empirical researchers to apply. Moreover, one potential drawback of this method is that the neglected terms in the expansion may be nonetheless important in finite sample, especially when the sample size is not large enough for the asymptotic theory to provide a good approximation.

As an alternative to the higher-order expansion of Donald and Newey (2001)[6], this paper proposes a bootstrap-based method to approximate the asymptotic MSE of instrumental variable estimators, and use it as a criterion for instruments selection. The widely used nonparametric i.i.d. bootstrap can be straightforwardly applied to instruments selection. However, such bootstrap procedure may not be suitable for the "many instruments" situation since the instruments are involved in the resample scheme. When the dimension of the instruments is large the nonparametric i.i.d. bootstrap can behave poorly, as demonstrated in our simulation results.

In this paper, we propose a plug-in restricted-efficient-residual-based (RE) bootstrap procedure by extending the idea of Davidson and MacKinnon (2008[3], 2010[4], 2014[5]). Their RE bootstrap is proposed for hypothesis testing and constructing confidence sets, and it uses the value of the structural parameter under the null hypothesis when generating the bootstrap pseudo-data. However, the RE bootstrap cannot be directly applied to instrument selection since the null value of the parameter is not available. To solve this problem, we use similar efficient reduced-form estimator as Davidson and MacKinnon (2008, 2010, 2014) but with some plugged-in preliminary instrumental variable estimator.

A motivating example for the empirical application of many instruments is Angrist and Krueger ([1])'s study of the returns to schooling. Hansen et al. ([9]) show that using 180 instruments help to obtain more precise estimation result than using the original 3 instruments. In such case with large number of instruments, we expect our plug-in RE bootstrap to perform better than the nonparametric i.i.d. bootstrap.

We conducted some Monte Carlo simulations to investigate the finite sample performance of the plug-in RE bootstrap. For comparison, two kinds of i.i.d. bootstraps and a standard residual-based bootstrap were investigated as well. Our Monte Carlo experiments show that for the TSLS estimator, the residual based bootstraps such as the plug-in RE bootstrap typically outperform the nonparametric bootstrap. The plug-in RE bootstrap tends to perform best when the number of instruments is quite large and/or when the degree of endogeneity is high. On the other hand, for the LIML estimator, all the bootstrap procedures have similar performance. In general, estimators using bootstrap-selected instruments always outperform the estimator without instrument selection, confirming the usefulness of bootstrap-based methods for instrument selection.

The organization of the rest of the paper is as follows. In Section 2, we describe the instrument variable model and the TSLS and LIML estimation methods. In Section 3, we propose the plug-in RE bootstrap-based instrument selection method. In Section 4, we report the results of the Monte Carlo experiments. We conclude in Section 5. The Appendix contains all the details of the procedures of the other three bootstraps.

2 Instrumental Variable Model and Estimation Methods

We consider the following instrumental variable model

$$y_i = X_i'\beta + \varepsilon_i \tag{1}$$

$$X_i = f(z_i) + v_i \tag{2}$$

where y_i is a scalar outcome variable, X_i is a $l \times 1$ vector of endogenous variables, z_i is a vector of exogenous variables, ε_i and v_i are unobserved random variables with second moments which do not depend on z_i, and $f(z_i) = E[X_i|z_i]$. As has been pointed out by Donald and Newey (2001)[6], this set up is general enough to allow for many types of instrumental variables. The vector z might be a few continuous variables, in which case $f(z)$ allows for unknown functional form. Also, z might be many dummy variables, for which $f(x)$ represents a fully saturated model. Because X_i is endogenous, instrumental variables are needed for the purpose of estimating β. The set of instrumental variables has the form $Z_k \equiv (\psi_1(z_i), ..., \psi_k(z_i))$, where k is the number of instruments, and ψ_k's are functions of z_i such that $Z_{k,i}$ is a $k \times 1$ vector of instruments. Let $y = (y_1, ..., y_n)'$and define X, ε, f, and v similarly.

The two commonly used estimators for the parameter of interest β are the two stages least squares (TSLS) estimator and the limited information likelihood (LIML) estimator. The TSLS estimator using Z_k, $\hat{\beta}_{TSLS}(k)$, is defined as

$$\hat{\beta}_{TSLS}(k) = \left(X'P_kX\right)^{-1} X'P_k y$$

where $P_k = Z_k(Z_k'Z_k)^{-1}Z_k'$, and the limited information maximum likelihood (LIML) estimator, $\hat{\beta}_{LIML}(k)$, is defined as

$$\hat{\beta}_{LIML}(k) = \left(X'P_kX - \hat{\Lambda}(k)X'X\right)^{-1} \left(X'P_k y - \hat{\Lambda}(k)X'y\right)$$

where $\hat{\Lambda}(k) = min_\beta \frac{(y-X\beta)'P_k(y-X\beta)}{(y-X\beta)'(y-X\beta)}$.

3 Bootstrap-based Selection of Instrumental Variables

In this section, we propose the plug-in RE bootstrap-based instruments selection method. Instead of using analytical formulas, the bootstrap is used to approximate the MSE of the TSLS and LIML estimators, and then the optimal number of instruments is chosen by minimizing the bootstrap-based approximate MSE.

Suppose we have estimated the equation (1) and equation (2), and have yielded the residuals for both equations. Then we can generate bootstrap pseudo-data with these residuals and the observed sample. Particularly, when we use the least square to estimate equation (2), we call such procedure the standard residual-based bootstrap. The details of the algorithm of this bootstrap is described in Appendix. However, as pointed out by Davidson and MacKinnon (2008, 2010, 2014), using the least square estimator is not an efficient way to estimate the reduced-form equation (2). To improve the performance of bootstrap procedures, when possible it is desirable to use a more efficient estimator of the reduced-form equation, instead of using the least square estimator. In addition, for their purpose of testing the hypothesis $H_0 : \beta = \beta_0$, one may also generate the bootstrap pseudo-data imposing the null hypothesis. Indeed, their RE bootstrap is implemented using a null-restricted efficient reduced-form estimator

$$\tilde{\pi}(\beta_0) = \left(Z'Z\right)^{-1} Z' \left(X - \varepsilon(\beta_0)\frac{\varepsilon(\beta_0)'M_k X}{\varepsilon(\beta_0)'M_k\varepsilon(\beta_0)}\right)$$

where $\varepsilon(\beta_0) = y - X\beta_0$. Then residuals of the reduced form equation can be obtained by computing $\tilde{v}(\beta_0) = X - Z\tilde{\pi}(\beta_0)$ and the bootstrap disturbances are obtained by drawing from the empirical distribution function of $(\varepsilon_i(\beta_0), \tilde{v}_i(\beta_0))_{i=1}^N$.

However, for our current purpose of approximating the asymptotic MSE of TSLS or LIML, it is infeasible to impose the null hypothesis since we do not know the true value of β. Thus, instead of using β_0 we suggest to replace β_0 in $\tilde{\pi}(\beta_0)$ with a preliminary consistent estimator of the structural parameter (say, $\tilde{\beta}$) for our instrument selection procedure. The TSLS/LIML estimator computed with all the available instruments or some preliminarily chosen instruments can be used as $\tilde{\beta}$. Specifically, the algorithm of our plug-in RE bootstrap based selection procedure is as follows:

Algorithm of the plug-in RE bootstrap based selection procedure

1. Obtain the residuals $\left(\tilde{\varepsilon}_i, \tilde{v}_i(\tilde{\beta})\right)_{i=1}^N$ from $\tilde{\varepsilon}_i = y - X\tilde{\beta}$ and $\tilde{v}_i(\tilde{\beta}) = X_i - Z_i'\tilde{\pi}(\tilde{\beta})$, where $\tilde{\pi}(\tilde{\beta}) = \left(\tilde{Z}'\tilde{Z}\right)^{-1}\tilde{Z}'\left(X - \tilde{\varepsilon}\frac{\tilde{\varepsilon}'\tilde{M}X}{\tilde{\varepsilon}'\tilde{M}\tilde{\varepsilon}}\right)$, $\tilde{\varepsilon} = y - X\tilde{\beta}$, $\tilde{M} = I - \tilde{Z}\left(\tilde{Z}'\tilde{Z}\right)^{-1}\tilde{Z}'$, $\tilde{Z}$ is some preliminary choice of instruments and $\tilde{\beta}$ is some preliminary estimator of β.

2. Draw the bootstrap disturbances $(\varepsilon_i^*, v_i^*)_{i=1}^N$ from the empirical distribution function of (de-meaned) $\left(\tilde{\varepsilon}_i, \tilde{v}_i(\tilde{\beta})\right)_{i=1}^N$.

3. For each number of instrument k, generate the bootstrap DGP:

$$y_i^*(k) = X_i^*(k)\cdot\hat{\beta}(k) + \varepsilon_i^*,\ X_i^*(k) = Z_{k,i}'\tilde{\pi}(k,\tilde{\beta}) + v_i^*,$$

where $\tilde{\pi}(k,\tilde{\beta}) = (Z_k'Z_k)^{-1} Z_k' \left(X - \tilde{\varepsilon}\frac{\tilde{\varepsilon}' M_k X}{\tilde{\varepsilon}' M_k \tilde{\varepsilon}}\right)$, and $\hat{\beta}(k)$ corresponds to $\hat{\beta}_{TSLS}(k)$ or $\hat{\beta}_{LIML}(k)$. Compute the bootstrap analogues of the TSLS/LIML estimator using $\left(y_i^*(k), X_i^*(k), Z_{k,i}\right)_{i=1}^N$.

4. Repeat Steps 2-3 B times. Note that the same choice of $\tilde{Z}$ and $\tilde{\beta}$ need to be used for each replication. Compute the bootstrap-based approximate MSE

$$BMSE(k) = B^{-1} \sum_{b=1}^{B} \left(\hat{\beta}_b^*(k) - \hat{\beta}(k)\right)^2,$$

where $\hat{\beta}_b^*(k)$ denotes the bootstrap analogue of the TSLS/LIML estimator derived from the b-th time of replication, then choose k that minimizes $BMSE(k)$.

4 Monte Carlo Simulations

In this section, we use similar Monte Carlo design as Donald and Newey (2001)[6]. Our data generating process is the following model: $y_i = X_i\beta + \varepsilon_i$ and $X_i = Z_i'\pi + v_i$ for $i = 1, ..., N$, where Y_i is a scalar, β is a scalar parameter of interest. $Z_i \sim N(0, I_K)$, and (ε_i, v_i) is i.i.d. jointly normal with variance 1 and covariance c. The integer K is the total number of instruments considered in each experiment. We fix the true value of β at 0.1 and examine how well the various bootstrap-based instruments selection methods perform relative to existing methods. In this framework, each experiment is indexed by the vector of specifications: (N, K, c, π). We set $N = 100$.The largest number of instruments is set to 10 and 30. The degree of endogeneity is controlled by the covariance c and set to $c = 0.1, 0.5, 0.9$. The number of Monte Carlo experiments is set to 1000 and the number of bootstrap replications is set to 399 in all experiments.

We consider the specification of the vector π as used in Donald and Newey (2001)[6]. Let R_f^2 denote the theoretical R^2 of the first-stage regression in equation (2). The kth element of π is given by

$$\pi_k = a(K)\left(1 - \frac{k}{K+1}\right)^4,$$

where $a(K)$ is chosen to satisfy $\pi'\pi = R_f^2 / \left(1 - R_f^2\right)$. We use $R_f^2 = 0.1$. The strength of the instruments decreases gradually with k, this case is most relevant for empirical applications where there is always some information about which instruments are important. And we choose the optimal number of instruments to be used for estimation based on the plug-in RE bootstrap-based methods. For comparison, we also perform simulations for two kind of nonparametric i.i.d. bootstrap-based methods, the pairwise bootstrap and Freedman (1984)'[7]s bootstrap, and the standard residual-based bootstrap-based method. The details of the algorithms of the two nonparametric i.i.d. bootstrap-based methods are described in the Appendix.

Tables 1 - 4 report the result of the experiments. Summary statistics are computed for TSLS and LIML estimators. For each estimator, we compute the median bias (BIAS) and the median absolute deviation (MAD). We use these "robust" measures of central tendency and dispersion because of concerns about the existence of moments of of the LIML estimator. "TSLS-all" and "LIML-all" denote the estimators computed using the largest set of instruments. "Pair", "Freedman", "Standard", and "plug-in RE" denotes the four bootstrap algorithms presented in the previous section and the Appendix.

For the case of TSLS estimator (Tables 1 - 2), the most commonly used instrumental variable estimator, the pairwise bootstrap and the Freedman's nonparametric bootstrap typically have similar performance as TSLS-all. Freedman's procedure performs slightly better than the pairwise bootstrap when the endogeinity is high (c=0.9). Both bootstrap procedures tend to select a large number of instruments. This results in relatively bad performance of these two procedures since the bias of the TSLS estimator increases considerably when the number of instruments becomes large. Among the two residual-based bootstrap, the performance of the standard one is close to that of the nonparametric bootstraps. This is somewhat expected since the reduced form equation is not estimated efficiently in the standard residual-based bootstrap. On the other hand, the plug-in RE bootstrap turns out to perform well relative to other methods. Especially in the case with high endogeneity, both BIAS and MAD are considerably reduced when the plug-in RE is used. Tables 3 - 4 report the results for the LIML estimator. Contrary to the case of the TSLS estimator, all the bootstrap procedures have similar performance with nonparametric bootstraps performing relatively better when the endogeneity is low and the plug-in RE bootstraps better when the endogeneiry is high. When K=30, the bootstrap-based methods almost always outperform LIML-all, indicating that the bootstrap procedures succeed to select only relatively important instruments. Again, the plug-in RE bootstrap tend to have best performance among all the bootstraps when K=30.

5 Conclusion

In this paper, we propose plug-in RE bootstrap-based method for instrument selection and we investigate the effectiveness of bootstrap based criteria by Monte Carlo experiments. Our Monte Carlo experiments show that for the TSLS estimator, the residual based bootstraps such as the plug-in RE procedure typically outperform the nonparametric bootstrap procedures. On the other hand, for the LIML estimator, all the bootstrap procedures have similar performance. Furthermore, estimators using bootstrap-selected instruments almost always outperform the estimator without instrument selection, confirming the usefulness of bootstrap-based methods for instrument selection. Overall, the results are encouraging and should stimulate further research on bootstrap based selection methods, e.g., for general nonlinear cases where estimators such as the Generalized Method of Moments (GMM) and Generalized Empirical Likelihood (GEL) are used.

6 Appendix

First, we introduce one of the nonparametric i.i.d. bootstraps, the pairwise bootstrap, in which each bootstrap sample is drawn from the empirical distribution function of the data.

For the pairwise bootstrap, the ith row of each bootstrap sample is simply one of the row of the matrix $(y : X : Z)$, chosen at random with probability 1/n. The instruments selection procedure based on the pairwise bootstrap is as follows:

Algorithm of the pairwise bootstrap

1. For each number of instruments k , draw the bootstrap sample $W^*_{k,i} = \left(y^*_i, X^*_i, Z^*_{k,i}\right)_{i=1}^N$ from the empirical distribution function of the data, $\hat{F}_k(w) = N^{-1}\sum_{i=1}^N I\{W_{k,i} \leq w\}$ where $W_{k,i} = \left(y_i, X_i, Z_{k,i}\right)_{i=1}^N$.
2. Compute the TSLS and LIML estimators using the bootstrap samples for each k. For example, the bootstrap analogue of the TSLS estimator reads
$$\hat{\beta}^*_{TSLS}(k) = \left(X^{*'} P^*_k X^*\right)^{-1} X^{*'} P^*_k y^*,$$
where $P^*_k = Z^*_k (Z^{*'}_k Z^*_k)^{-1} Z^{*'}_k$.
3. Repeat Steps 1-2 B times. Compute the bootstrap-based approximate MSE
$$BMSE(k) = B^{-1} \sum_{b=1}^{B} \left(\hat{\beta}^*_b(k) - \hat{\beta}(k)\right)^2,$$
where $\hat{\beta}^*_b(k)$ denotes the TSLS/LIML estimator derived from the b-th time of replication, then choose k that minimizes $BMSE(k)$.

Hahn (1996)[8] shows that under the standard textbook asymptotics (in which the number of instruments k is assumed to be fixed), the pairwise bootstrap is asymptotically valid in the sense that the asymptotic distribution of the bootstrap analogue is the same as that of the original estimator.

However, the pairwise bootstrap can be problematic since under this procedure, the bootstrap moment condition $E^*\left[Z^*_{k,i}\left(y^*_i - X^{*'}_i \hat{\beta}(k)\right)\right] = 0$ does not hold in general; E^*denotes the expectation induced by the pairwise bootstrap. This is because under current bootstrap scheme
$$E^*\left[Z^*_{k,i}\left(y^*_i - X^{*'}_i \hat{\beta}(k)\right)\right] = N^{-1}\sum_{i=1}^{N} Z_{k,i}\left(y - X^{'}_i \hat{\beta}(k)\right)$$
and there is no value of $\hat{\beta}$ that satisfies the condition when the model is over-identified. It is thus important to also consider the procedure where the moment conditions even hold in the bootstrap world.

The second nonparametric i.i.d. bootstrap is based on Freedman's method which is a modification of the pairwise bootstrap. In this procedure, the residual of the structural form equation (1) is made orthogonal to the instruments, so that each bootstrap sample can be drawn from a distribution that satisfies the moment conditions. More precisely, our instruments selection procedure based on Freedman's bootstrap is as follows:

Algorithm of the Freedman's bootstrap

1. Let $\tilde{\varepsilon}_K = M_K \left(y - X\hat{\beta}(K) \right)$, where $\hat{\beta}(K)$ is the TSLS or the LIML estimator based on the largest set of instruments Z_K, and $M_K = I - Z_K \left(Z_K' Z_K \right)^{-1} Z_K'$.

2. For each number of instruments k, draw the bootstrap sample $\left(X_i^*, Z_{k,i}^*, \varepsilon_i^* \right)_{i=1}^N$ from the empirical distribution of $\left(X_i, Z_{k,i}, \tilde{\varepsilon}_{i,K} \right)_{i=1}^N$. Generate bootstrap pseudo-data $y_i^*(k)$ using the structural form equation (1):

$$y_i^*(k) = X_i^{*'} \hat{\beta}(k) + \varepsilon_i^*$$

   Compute the instrumental variable estimators using the bootstrap sample $\left(X_i^*, Z_{k,i}^*, y_i^*(k) \right)$.

3. Repeat Steps 2 B times. Compute $BMSE(k)$, the bootstrap-based approximate MSE, then choose the number of instruments k that minimizes $BMSE(k)$.

Note that for different choices of k, the bootstrap moment condition always holds under current procedure since $E^* \left[Z_{k,i}^* \left(y_i^* - X_i^{*'} \hat{\beta}(k) \right) \right] = \frac{1}{N} \left(Z_k' M_K (y - X\hat{\beta}(K)) \right) = 0$, by the fact that M_K is constructed in Step 1 from the largest set of instruments.

The advantage of using nonparametric bootstrap is that it does not depend on certain model specification and it is robust to heterogeneity in the disturbances. However, the instruments Z is also involved in the resampling scheme. This makes the nonparametric bootstrap potentially unreliable when the dimension of Z becomes large. On the other hand, for our purpose of selecting instruments, we need a bootstrap procedure that is able to deliver good finite sample performance even when k, the number of instruments, is relatively large. This motivates using residual-based bootstrap methods. The standard residual-based bootstrap is one of such procedures:

Algorithm of standard residual-based bootstrap

1. Using $\tilde{\beta}$ and $\tilde{\pi}$, some preliminary estimator of β and the least square estimator of the first stage reduced-form, obtain the residuals $(\tilde{\varepsilon}_i, \tilde{v}_i)_{i=1}^N$, i.e.,

$$\tilde{\varepsilon}_i = y_i - X_i' \tilde{\beta}, \; \tilde{v}_i = X_i - \tilde{Z}_i' \tilde{\pi}$$

   where $\tilde{\pi} = \left(\tilde{Z}' \tilde{Z} \right)^{-1} \tilde{Z}' X$, and $\tilde{Z}$ is certain preliminary choice of instruments. The TSLS or LIML estimator $\tilde{\beta}$ is calculated using $\tilde{Z}$. Then, de-mean the residuals $(\tilde{\varepsilon}_i, \tilde{v}_i)_{i=1}^N$, to obtain $(\bar{\varepsilon}_i, \bar{v}_i)_{i=1}^N$. For the choice of $\tilde{Z}$, one may use the largest set of instruments as in Algorithm 2 or using the number of instruments selected by some goodness of fit criterion for estimation of the first-stage reduced form.

2. Draw the bootstrap disturbances $(\varepsilon_i^*, v_i^*)_{i=1}^N$ from the empirical distribution function of the de-meaned residuals.

3. For each number of instruments k, generate the bootstrap DGP using equations (1) and (2) with the bootstrap disturbance obtained in Step 2

$$y_i^*(k) = X_i^*(k)\hat{\beta}(k) + \varepsilon_i^*,\ X_i^*(k) = Z_{i,k}'\hat{\pi}(k) + v_i^*,$$

where $\hat{\pi}(k) = \left(Z_k'Z_k\right)^{-1} Z_k'X$ and obtain $(y_i^*(k), X_i^*(k))_{i=1}^N$. Then, compute the estimate of interest using the bootstrap sample.

4. Repeat Steps 2 - 3 B times. Compute $BMSE(k)$, then choose the number of instruments that minimize $BMSE(k)$.

Note that in Step 3, the instrument Z_k is always kept fixed without being re-sampled.

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Table 1. Monte Carlo Results: TSLS, K=10

	TSLS-all	Pair	Freedman	Standard	plug-in RE
			c=0.1		
BIAS	0.055	0.058	0.055	0.050	0.052
MAD	0.165	0.157	0.163	0.169	0.176
			c=0.5		
BIAS	0.226	0.226	0.217	0.226	0.216
MAD	0.238	0.237	0.237	0.239	0.244
			c=0.9		
BIAS	0.410	0.400	0.349	0.369	0.264
MAD	0.410	0.400	0.352	0.370	0.288

Table 2. Monte Carlo Results: TSLS, K=30

	TSLS-all	Pair	Freedman	Standard	plug-in RE
			c=0.1		
BIAS	0.074	0.080	0.076	0.074	0.074
MAD	0.122	0.117	0.119	0.122	0.125
			c=0.5		
BIAS	0.364	0.375	0.356	0.360	0.354
MAD	0.364	0.375	0.358	0.361	0.357
			c=0.9		
BIAS	0.651	0.651	0.552	0.622	0.422
MAD	0.651	0.651	0.552	0.622	0.424

Table 3. Monte Carlo Results: LIML, K=10

	LIML-all	Pair	Freedman	Standard	plug-in RE
			c=0.1		
BIAS	0.018	0.039	0.029	0.033	0.027
MAD	0.290	0.237	0.257	0.249	0.269
			c=0.5		
BIAS	0.020	0.048	0.035	0.047	0.010
MAD	0.276	0.225	0.254	0.252	0.267
			c=0.9		
BIAS	0.023	0.085	0.025	0.026	0.007
MAD	0.216	0.214	0.215	0.210	0.204

Table 4. Monte Carlo Results: LIML, K=30

	LIML-all	Pair	Freedman	Standard	plug-in RE
			c=0.1		
BIAS	0.008	0.024	0.035	0.025	0.022
MAD	0.394	0.224	0.303	0.329	0.339
			c=0.5		
BIAS	0.042	0.132	0.112	0.109	0.058
MAD	0.415	0.256	0.351	0.369	0.359
			c=0.9		
BIAS	0.006	0.150	0.034	0.033	0.004
MAD	0.280	0.248	0.284	0.278	0.259