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Power Attrition of Asymmetric Tail Comovement Test

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Abstract

In the Markov-switching framework, I study the effects of nuisance parameters such as the correlation and transition probability on the power of a recently proposed nonparametric test of asymmetric tail comovement (Li, 2014). As nuisance parameters govern how different states are separated apart and how they interact with each other, it is found that substantial power loss can be incurred when the underlying parameters take certain values. In addition, I show that the power of the test is sensitive to the choice of tail threshold and is adversely affected as one goes into deeper tails.

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1. Introduction

A growing body of empirical research has found asymmetric comovements of portfolio returns, e.g., Ang and Chen (2002) and Hong, Tu, and Zhu (2007). The test statistics used by these authors are mostly based on Pearson's correlation coefficient, the limitations of which are now well understood, i.e., while it is a proper measure when the underlying assets are jointly normally distributed, it does not capture nonlinear dependence that otherwise may be present in the data. Recently Li (2014) proposes a new approach to measure the pairwise comovement between two asset returns. The new framework gives a probabilistic interpretation to the notion of tail comovement and involves the choice of a tail threshold parameter. The test statistic is based on the difference between the probability that the two returns simultaneously move up or down (concordant movements) and the probability that they move up or down in opposite directions (discordant movements). The test does not impose any restrictions on the functional form of the joint distribution of the two returns—which is convenient for practitioners—but, as is shown in Section 2, the price to pay is that the power of the test is sensitive to the model parameters which are not explicitly modeled.

Against this background, the main purpose of this note is to highlight the danger of using model-free test of asymmetric tail comovement in finite samples while power of the test might have drifted away to a very low level. Since the Gaussian Markov-switching model has served as a benchmark for characterizing asset returns in many applications, the performance of the test in this context is of particular interest. Another benefit of using this model is that, by varying the model parameters (mean return, volatility, correlation, transition probability), the regime-switching structure is able to generate a continuum of economically sensible (local) alternatives which can be then used to evaluate the power of the test. In fact, exact moments can be computed for such models (Timmermann, 2000) and it is straightforward although tedious to show that asymmetric tail comovement is an inherent property of Markov-switching models (this seems to be a somewhat overlooked aspect) unless, say, the state-dependent distributions are exactly symmetric about the zero mean, which, from a practical perspective, is hardly possible to find in empirical studies. The same logic applies to other more complicated models with a Markov-switching component.

2. Simulation Results

2.1 Summary of the Test Procedure

Li's test is based on a pair of upside and downside comovement measures:

$$cm^{+}(c) = 2P\big((R_{t+1}^{1} - R_{t}^{1})(R_{t+1}^{2} - R_{t}^{2}) > 0 | R_{t}^{1} > c, R_{t}^{2} > c\big) - 1, \tag{1}$$

and

$$cm^{-}(c) = 2P\big((R_{t+1}^{1} - R_{t}^{1})(R_{t+1}^{2} - R_{t}^{2}) > 0 | R_{t}^{1} < -c, R_{t}^{2} < -c\big) - 1. \tag{2}$$

In Eq. (1) and (2), P is the usual probability measure, R^1 and R^2 are returns of the two assets, and c is some non-negative threshold in percentage points. Accordingly cm^+ measures the strength of upside (right tail) comovement and cm^- measures the strength of downside (left tail) comovement. By construction, the new measure can be viewed as a conditional version of Kendall's τ and is a summary of both linear and nonlinear dependence. For a given threshold c, the null and alternative hypotheses can be stated as

$$\mathrm{H}_0: cm^+(c) = \mathrm{cm}^-(c),$$
$$\mathrm{H}_1: cm^+(c) \neq \mathrm{cm}^-(c).$$

For multiple thresholds, a joint test can be constructed so that $cm^+ = (cm^+(c_1), \dots, cm^+(c_m))$. $cm^+(c)$ and $cm^-(c)$ can be easily estimated nonparametrically using the procedure given in Li (2014):

$$\widehat{cm}^+(c) = \frac{\sum_{t=1}^{n-1} S(R_t^1, R_{t+1}^1, R_t^2, R_{t+1}^2) I(R_t^1 > c, R_t^2 > c)}{\sum_{t=1}^{n-1} I(R_t^1 > c, R_t^2 > c)},$$

$$\widehat{cm}^-(c) = \frac{\sum_{t=1}^{n-1} S(R_t^1, R_{t+1}^1, R_t^2, R_{t+1}^2) I(R_t^1 < -c, R_t^2 < -c)}{\sum_{t=1}^{n-1} I(R_t^1 < -c, R_t^2 < -c)},$$

where $I(R_t^1 < -c, R_t^2 < -c)$ and $I(R_t^1 > c, R_t^2 > c)$ are indicator functions that define the tail events and $S(R_t^1, R_{t+1}^1, R_t^2, R_{t+1}^2) = I[(R_{t+1}^2 - R_t^2)(R_{t+1}^1 - R_t^1) > 0] - I[(R_{t+1}^2 - R_t^2)(R_{t+1}^1 - R_t^1) < 0]$.

Under certain assumptions, Theorem 2 of Li (2014) shows that

$$n(cm^+ - cm^-)\widehat{\Omega}(cm^+ - cm^-) \xrightarrow{d} \chi_m^2, \tag{3}$$

where $\widehat{\Omega}$ is a consistent estimate of the variance-covariance matrix based on the Bartlett-kernel, and n and m are, respectively, the number of observations and thresholds. Standard assumptions (e.g., symmetry and relative convergence rate of the nonstochastic bandwidth) on the kernel function used to estimate $\widehat{\Omega}$ are imposed, so that many commonly used kernels, such as the Bartlett, Parzen, and quadratic-spectral kernels are included. Details are omitted in the interest of brevity. In the next subsection, I focus on the two state single threshold case and choose m to be one. Under the null the test statistic converges to $\chi^2(1)$.

2.2 Power Attrition

Here I illustrate how the power of Li's test varies with model parameters in a regime-switching setup. I adopt the same parameterization (DGP 5 in Li (2014)) as in the bivariate Gaussian Markov-switching model of Ang and Bekaert (2002):

$$R_t = \mu_{S_t} + \Sigma_{S_t}^{1/2} \varepsilon_t. \tag{4}$$

In Eq. (4), the transition probability matrix is $\Gamma = [0.9818, 0.0182; 0.1454, 0.8546]$, $\mu_1 = (1.283, 1.304)$, $\mu_2 = (-1.288, -0.692)$; the elements of $\Sigma_{S_t}^{1/2}$ are $\rho_1 = 0.4455$, $\rho_2 = 0.6097$, $\sigma_1 = (3.7689, 5.2194)$, $\sigma_2 = (7.0376, 13.7177)$. ε_t is i.i.d. normal. Subscript 1 and 2 denote the low volatility and high volatility state. Power is computed based on 1000 simulated samples and the initial state distribution is assumed to be $(0.5, 0.5)$ the latter of which is not made clear in Li (2014). The HAC estimators used to estimate the variance (as in Eq. (3)) are the kernel-HAC, VARHAC, prewhitened HAC and KVB-HAC and I use the Bartlett kernel throughout. Results are collected in Tables 1-3. The first row of each subpanel makes use of the benchmark specification of the transition probability matrix and mean returns given above. Each table corresponds to one of the three thresholds ($c = 0, 0.5, 1$).

Table 1

Power attrition of Li's test, tail threshold c=0, H0: symmetric tail dependence.

	corr = [0.3, 0.7]				*corr* = [0.4455, 0.6097]			
	VHAC	pre-HAC	ker-HAC	KVB	VHAC	pre-HAC	ker-HAC	KVB
T=250	0.259	0.277	0.262	0.167	0.187	0.198	0.18	0.142
prob =0.1	0.124	0.131	0.12	0.082	0.094	0.111	0.101	0.086
prob =0.2	0.144	0.158	0.139	0.128	0.113	0.115	0.108	0.08
prob =0.3	0.159	0.173	0.167	0.124	0.106	0.117	0.111	0.075
μ_{low}=-1.0	0.227	0.246	0.238	0.166	0.176	0.191	0.178	0.142
μ_{low}=-0.5	0.218	0.24	0.226	0.156	0.19	0.192	0.179	0.128
μ_{low}=0.0	0.231	0.244	0.228	0.166	0.172	0.189	0.174	0.127
T=500	0.406	0.412	0.393	0.287	0.339	0.347	0.345	0.247
prob =0.1	0.211	0.213	0.206	0.144	0.126	0.127	0.119	0.097
prob =0.2	0.203	0.213	0.203	0.169	0.158	0.163	0.158	0.123
prob =0.3	0.26	0.264	0.251	0.193	0.154	0.165	0.156	0.119
μ_{low}=-1.0	0.382	0.396	0.386	0.27	0.314	0.328	0.316	0.24
μ_{low}=-0.5	0.408	0.418	0.41	0.294	0.343	0.352	0.341	0.258
μ_{low}=0.0	0.385	0.395	0.389	0.303	0.345	0.358	0.353	0.282
T=1000	0.677	0.682	0.674	0.486	0.561	0.566	0.559	0.386
prob =0.1	0.363	0.369	0.354	0.266	0.21	0.215	0.208	0.161
prob =0.2	0.43	0.439	0.429	0.327	0.237	0.245	0.235	0.183
prob =0.3	0.444	0.452	0.442	0.336	0.305	0.317	0.302	0.225
μ_{low}=-1.0	0.683	0.679	0.669	0.491	0.557	0.561	0.547	0.39
μ_{low}=-0.5	0.674	0.679	0.676	0.485	0.572	0.588	0.569	0.43
μ_{low}=0.0	0.695	0.695	0.683	0.519	0.564	0.578	0.577	0.395

Note: μ_{low} sets the low return-high volatility state mean return to -1.0, -0.5 and 0.0. *prob* sets the off-diagonal entries of the transition probability matrix to 0.1, 0.2 and 0.3. The test examines whether comovement in the upper tail is the same as comovement in the lower tail and is given in Li (2014). When one-sided hypothesis is tested, i.e., right tail comovement is higher than left tail comovement, power is higher by about 0.09-0.15 in all cases.

Four observations can be made. First, the power of the test decreases in all cases as the threshold gets larger. It is highest when $c = 0$ which is not really a "tail" event. Second, power also depends on the degree to which the correlation coefficients are distinct from each other. In general the more separated apart they are, the higher the power. Third, for reasonable choices of the mean return of the low return-high volatility state ($\mu_{\text{low}} = -1, -0.5, 0$), power tends to be least affected especially when the sample size grows larger (≥ 1000)—this is good news. Finally, the transition probability matrix has the most salient influence on the performance of the test ($prob$ = 0.1, 0.2, 0.3). Since this matrix determines the average time the process spends in each state—or duration—the larger the off-diagonal probabilities, the more "mixed-up" the states are. In an unreported exercise, power is lowest when all entries of the two-by-two transition probability matrix take the value of 0.5 (unrealistic though). It is tempting to conjecture that each one of these effects can be gauged independently of the others but they are actually highly interconnected whenever the DGP has a regime-switching structure.

To fix ideas, suppose the low volatility state has a lower correlation and positive mean return and the conditioning tail threshold is zero. If the low correlation state is more persistent than the high volatility (with higher correlation) state, then, *ceteris paribus*, this will necessarily make upper tail dependence less significant simply because the low volatility-low correlation state has a positive mean which makes cm^{+} smaller. Increasing the threshold or assuming less persistent states only blurs the difference between cm^{+} and cm^{-}. The latter tendency is reinforced when the correlation differential across the two states gets smaller. Moreover, state-dependent mean returns also matter. Imagine the low correlation state has a mean return of 1% compared to -0.5% in the high correlation state, then conditional on a positive threshold return, the test will place more weight on the low correlation state. In general, a nonparametric tail comovement measure is inevitably influenced by the magnitude of correlations, how often the process hits the high correlation state, and how the mean return of each state compares with the tail threshold.

Table 2

Power attrition of Li's test, tail threshold c=0.5, H_0: symmetric tail dependence.

	corr = [0.3, 0.7]				*corr* = [0.4455, 0.6097]			
	VHAC	pre-HAC	ker-HAC	KVB	VHAC	pre-HAC	ker-HAC	KVB
T=250	0.185	0.2	0.193	0.128	0.169	0.176	0.17	0.122
prob=0.1	0.103	0.119	0.108	0.103	0.087	0.098	0.097	0.062
prob=0.2	0.134	0.151	0.14	0.108	0.093	0.101	0.096	0.088
prob=0.3	0.128	0.14	0.129	0.106	0.087	0.102	0.098	0.071
μ_{low}=-1.0	0.186	0.203	0.193	0.142	0.167	0.182	0.16	0.122
μ_{low}=-0.5	0.206	0.232	0.215	0.172	0.188	0.196	0.182	0.137
μ_{low}=0.0	0.196	0.211	0.201	0.145	0.183	0.2	0.184	0.126
T=500	0.385	0.395	0.383	0.259	0.288	0.301	0.291	0.206
prob=0.1	0.213	0.218	0.199	0.153	0.116	0.119	0.115	0.093
prob=0.2	0.19	0.198	0.189	0.139	0.158	0.167	0.159	0.127
prob=0.3	0.22	0.232	0.216	0.153	0.133	0.141	0.124	0.11
μ_{low}=-1.0	0.381	0.386	0.378	0.251	0.286	0.299	0.286	0.186
μ_{low}=-0.5	0.403	0.411	0.401	0.276	0.315	0.336	0.326	0.219
μ_{low}=0.0	0.368	0.379	0.372	0.27	0.336	0.342	0.333	0.251
T=1000	0.642	0.648	0.646	0.453	0.547	0.549	0.531	0.377
prob=0.1	0.317	0.324	0.307	0.233	0.182	0.184	0.178	0.142
prob=0.2	0.359	0.367	0.358	0.256	0.203	0.209	0.198	0.157
prob=0.3	0.385	0.395	0.378	0.29	0.272	0.28	0.264	0.19
μ_{low}=-1.0	0.652	0.659	0.647	0.472	0.561	0.561	0.546	0.392
μ_{low}=-0.5	0.658	0.663	0.654	0.46	0.568	0.572	0.561	0.402
μ_{low}=0.0	0.65	0.652	0.643	0.464	0.585	0.596	0.577	0.38

Note: Same as Table 1.

Table 3

Power attrition of Li's test, tail threshold c=1, H0: symmetric tail dependence.

	corr = [0.3, 0.7]				*corr* = [0.4455, 0.6097]			
	VHAC	pre-HAC	ker-HAC	KVB	VHAC	pre-HAC	ker-HAC	KVB
T=250	0.198	0.208	0.196	0.125	0.147	0.157	0.147	0.113
prob =0.1	0.09	0.1	0.091	0.075	0.057	0.064	0.058	0.046
prob =0.2	0.115	0.121	0.11	0.071	0.074	0.081	0.079	0.058
prob =0.3	0.094	0.106	0.098	0.088	0.097	0.107	0.099	0.082
μ_{low}=-1.0	0.176	0.189	0.172	0.114	0.144	0.156	0.146	0.106
μ_{low}=-0.5	0.164	0.182	0.17	0.121	0.13	0.141	0.134	0.094
μ_{low}=0.0	0.172	0.187	0.178	0.131	0.149	0.163	0.159	0.114
T=500	0.33	0.343	0.33	0.237	0.252	0.259	0.255	0.208
prob =0.1	0.148	0.159	0.148	0.122	0.104	0.108	0.105	0.075
prob =0.2	0.169	0.166	0.156	0.117	0.105	0.106	0.102	0.088
prob =0.3	0.195	0.204	0.186	0.132	0.119	0.126	0.122	0.087
μ_{low}=-1.0	0.336	0.339	0.331	0.224	0.268	0.27	0.26	0.19
μ_{low}=-0.5	0.331	0.347	0.323	0.217	0.286	0.292	0.278	0.205
μ_{low}=0.0	0.335	0.342	0.338	0.227	0.268	0.285	0.271	0.176
T=1000	0.594	0.597	0.586	0.421	0.484	0.481	0.466	0.338
prob =0.1	0.259	0.269	0.252	0.189	0.115	0.115	0.104	0.104
prob =0.2	0.3	0.304	0.288	0.234	0.175	0.184	0.177	0.123
prob =0.3	0.345	0.353	0.339	0.245	0.197	0.205	0.196	0.159
μ_{low}=-1.0	0.591	0.591	0.59	0.382	0.465	0.476	0.469	0.314
μ_{low}=-0.5	0.598	0.598	0.589	0.429	0.522	0.526	0.51	0.345
μ_{low}=0.0	0.62	0.627	0.62	0.448	0.509	0.521	0.513	0.364

Note: Same as Table 1.

2.3 Empirical Illustration

To see how the power of the test depends on the choice of tail threshold in a real world example, I conduct a small scale empirical exercise using daily adjusted returns from the CRSP database. In particular, I apply Li's test of asymmetric tail comovement to the four largest components of DJIA 30 and four market indices. The market indices are chosen for the financially highly integrated economies with a significant weight in the world index since the late 1990s, namely, S&P 500 (U.S.), FTSE 100 (U.K.), DAX 30 (Germany) and Nikkei 225 (Japan); see Lane and Milesi-Ferretti (2003). From the previous subsection, for mildly large samples the difference between ker-HAC and prewhitened HAC vanishes, so I only report p-values based on the standard ker-HAC.

In Table 4, (marginally) significant pairs are in boldface and a positive (negative) test statistic corresponds to larger upper (lower) tail dependence. First note that lower tail dependence is not always stronger and many are insignificant. (For the DJIA sample, the sign can be even reversed when one goes from $c = 0$ to $c = 1$.) The result also depends on which tail threshold is being used and hence needs to be evaluated case-by-case. For the large caps, because of their low overall volatility, returns tend to be more concentrated about a small positive mean which limits the number of effective observations exceeding the threshold.

Table 4

Pairwise state-free tail dependence test, H_0: symmetric tail dependence.

	DJIA Top 4 (January 1, 2007-December 31, 2010, 1007 obs.)				U.S.-U.K.-Ger-Jp (1 Jan. 1998-31 Dec. 2012, 3483 obs.)		
				tail threshold c=0			
	CVX	MMM	BA		FTSE	DAX	NKY
IBM	-1.650 (**0.099**)	-1.328 (0.184)	0.819 (0.413)	S&P	0.852 (0.394)	0.917 (0.359)	1.199 (0.231)
CVX	0	-2.126 (**0.033**)	-1.731 (**0.084**)	FTSE	0	-0.507 (0.612)	1.115 (0.265)
MMM		0	-1.617 (0.106)	DAX		0	2.508 (**0.012**)
BA			0	NKY			0
				tail threshold c=1			
	CVX	MMM	BA		FTSE	DAX	NKY
IBM	1.285 (0.199)	0.257 (0.797)	1.339 (0.181)	S&P	1.740 (**0.082**)	0.745 (0.456)	1.749 (**0.080**)
CVX	0	-0.086 (0.931)	1.180 (0.238)	FTSE	0	-0.462 (0.644)	1.899 (**0.058**)
MMM		0	1.202 (0.229)	DAX		0	0.241 (0.809)
BA			0	NKY			0

Note: p-values are in parenthesis. Standard errors are based on the Newey-West HAC estimator. The test statistic is squared and compared to a $\chi^2(1)$ distribution.

Since the test is conservative, when it does reject, it is strong evidence that asymmetric tail comovement is present. When the null is not rejected, a natural question to ask is how to avoid overinterpreting the test result. Based on the above findings, one possible solution is to fit the data first according to some benchmark model and evaluate the estimated parameters or conduct a simple bootstrap experiment. Another solution is to rephrase the hypothesis testing question and view the p-value as a continuous reflection of the strength of comovement asymmetry rather than a yes/no answer.

3. Concluding Remarks

In asset allocation and risk management, it is often argued that models matter most when they are least useful partly because in market downturns correlation ceases to be an adequate summary of comovement and tend to be misleading for investors to make their decisions. Against this background, quite a few authors have made contributions in this field by extending the classical correlation-based measure of comovement to more comprehensive ones such as Li (2014). While the use of more elaborate measures and tests is of central interest to the financial industry and academia alike, it is shown in this note that the performance of the model-free approach does have the drawback of being sensitive to underlying parameters. The interaction between tails of the marginal distribution across different states is intimately related to factors such as the correlation differential, tail threshold and mixture probabilities. In view of this, one should always bear a grain of caution in mind when interpreting the test result.

References

Andrews, D. W. K. (1991). "Heteroskedasticity and autocorrelation consistent covariance matrix estimation." *Econometrica,* 59(3), 817-858.

Ang, A., and Bekaert, G. (2002). "International asset allocation with regime shifts." *The Review of Financial Studies,* 15(4), 1137-1187.

Ang, A., and Chen. (2002). "Asymmetric correlations of equity portfolios." *Journal of Financial Economics,* 63(3), 443-494.

Hong, Y., Tu, J., and Zhu, G. (2007). "Asymmetries in stock returns: statistical tests and economic evaluation." *The Review of Financial Studies,* 20(5), 1547-1581.

Lane, P. R., and Milesi-Ferretti, G. M. (2003). "International financial integration." *IMF Staff Papers,* 50, 82-113.

Li, F. (2014). "Identifying asymmetric comovements of international stock market returns." *Journal of Financial Econometrics,* 12(3), 507-543.

Timmermann, A. (2000). "Moments of Markov-switching models." *Journal of Econometrics,* 96(1), 75-111.

White, H. (1984). Asymptotic Theory for Econometricians. San Diego: Academic Press.