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Domestic creditors as last lenders in debt crises: a simple model with multiple equilibria

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Abstract

It is widely acknowledged that the ratios of public debt over GDP reached historically high levels in the Euro area during the recent sovereign debt crisis. More unnoticed however is the simultaneous increase in the share of government debt held by residents that has started in late 2008 in most fragile economies of the area. This paper develops a simple theoretical framework, in which multiple equilibria arise, to explain why exogenous increases in the debt level may cause this share to increase, due to distinct expected returns on domestic sovereign debt for domestic and foreign creditors in times of turmoil.

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1. Introduction

At the time that most Eurozone economies experienced a deep increase in their sovereign debt to GDP ratios as a consequence of the 2007-2009 global financial crisis, the share of public debt held by the resident sector simultaneously rose in the fragile economies of the area, as shown by Figure 1. Thus, the debt to GDP ratio increased from 41% to 90% between 2006 and 2013 in Spain and from 104% to 129% in Italy, whereas the proportion of domestic holdings rose from 49% to 70% in Spain and from 49% to 65% in Italy.

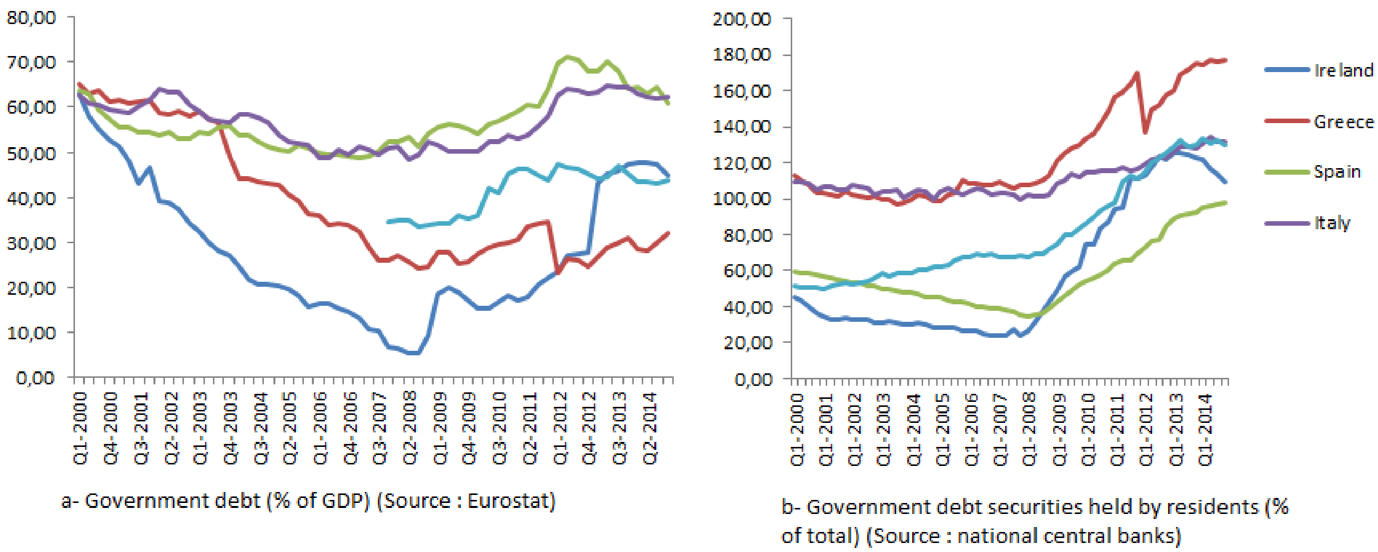

Figure 1: Joint increase in public debt and the share of debt held by the resident sector.

Several recent papers emphasized that the proportion of domestic public debt holdings increased in peripheral countries in the context of the Eurozone debt crisis (Merler and Pisani-Ferry (2012), Van Riet (2014), Acharya et al. (2014)), with domestic creditors apparently playing the role of lenders of last resort. Empirical literature showed that this increase was a direct consequence of the stress period on the Eurozone sovereign debt markets (Brutti and Sauré (2016), Battistini et al. (2014)). This period was characterized by higher debt to GDP ratios and more pessimistic expectations regarding the fiscal sustainability of the Eurozone peripheral sovereigns.

Nevertheless, theoretical literature that strives to provide a credible rationale to understand the causal relation between debt stress and the increase in the share of domestic holdings has remained scarce. This makes it crucial to provide micro-foundations that explain the variations in the share of debt held by the resident or the foreign sector. Understanding the transmission mechanism between sovereign risk and the proportion of domestic creditors is indeed likely to have important consequences for default risk because the identity of creditors has been shown to impact the government's decision (Mengus (2014), Gennaioli et al. (2014), Broner et al. (2014)).

To contribute to this recent strand of literature, we extend Gros (2012) simple model of sovereign debt crisis by distinguishing between domestic and foreign creditors and by modeling the government's decision as a function of the share of debt held at home. Multiple equilibria, in which the share of debt held by residents is the endogenous aggregate outcome that results from micro-founded creditors' decisions, arise in the model. The model then allows the investigation of the consequences of an exogenous increase in the debt level on the domestic share of debt and on the government's decision to default. The model also provides some preliminary insights on the policy implications of a rising proportion of domestic creditors during debt turmoil.

The mechanism of the model relies on two key assumptions that are backed by empirical and theoretical literature. The first assumption is that the expected implicit return on domestic sovereign debt is higher for domestic creditors than for foreign creditors in times of sovereign debt turmoil. Indeed, as argued by Battistini et al. (2014), when the expected probability of default differs from zero, domestic creditors have more incentives than foreign ones to invest in domestic sovereign bonds due to a combination of factors. First, they explain this finding by financial repression episodes in which national governments attempt to oblige domestic banks to buy domestic public debt. Financial repression makes it indirectly more profitable to invest in domestic bonds for residents because it spares the implicit cost of not complying with government pressure. Second, this finding can relate to carry-trades in under-capitalized banks (mainly located in fragile economies), which make risky debt more attractive for those banks. Third, this finding can be justified by the possibility of an exit from the Eurozone that would make payoffs on domestic sovereign bonds less attractive for foreign investors due to tran- saction costs. Finally, this finding can be explained by ex-post bailouts for domestic creditors (Van Riet (2014), Mengus (2014), Brutti and Sauré (2016)), which make default less costly for them. Therefore, although the expected nominal return (that is, the nominal interest rate) remains similar for domestic and foreign creditors in times of debt crisis, the implicit expected return differs due to the additional expected benefits for domestic creditors and the additional expected costs for foreign creditors.

The second key assumption is that an increase in the share of debt held at home decreases the government's incentives to default, all else being equal. According to the literature, a higher share of debt held at home decreases the economic and political cost of repaying debt. When domestic creditors hold a higher share of domestic public debt relative to foreigners, economic inefficiencies are relatively less costly because the outcome of taxation is mainly redistributed to domestic creditors. Perhaps more importantly, a higher share of debt held domestically also makes domestic bondholders less reluctant to pay taxes that are directed at funding the reimbursement of sovereign debt because it enables them to be paid back on the bonds they hold (Gros (2012)). Conversely, a higher domestic share of debt raises the cost of default for the domestic economy for a given amount of debt because the volume of debt held domestically increases (Kremer and Mehta (2000), Mengus (2014), Gennaioli et al. (2014)).

This paper's objective is to contribute to the literature on sovereign debt repatriation in times of crisis by developing a simple stylized framework. This paper provides a broader micro-founded explanation to the debt repatriation phenomenon, interpreting it as the result of diverging implicit returns for domestic and foreign investors due to a combination of factors. In addition, the model includes the distinct effects of an increase in the domestic share of debt on the government's incentives to default, which were identified separately in the literature. Before identifying the cases in which a marginal increase in the debt causes the share of debt held at home to rise, we first present the model and show that multiple equilibria arise.

2. A simple model of debt crisis with multiple equilibria

2.1. The setting

The timing of the model is as follows: first, two groups of investors – a group of domestic creditors and a group of foreign creditors with logarithmic utility – encounter an optimal portfolio-consumption problem in a 2-period setup. Creditors decide what to consume in period 1 (the variable C_1) and what share ω of their savings $W_0 - C_1$ (with W_0 the initial wealth) to allocate to one-period maturity sovereign bonds, for which an exogenous quantity d is sold on the sovereign bonds market. The remaining share of post-consumption wealth $1-\omega$ is allocated to a risk-free asset, which provides an exogenous payoff (assumed to be zero for simplicity). Both groups are two infinite continuums of identical atomistic agents over the interval [0,1]. Being a negligible part of an infinite continuum, each individual investor takes the aggregate decision as given. The optimal individual decision is given by the solution to the following maximization program (the subscript j refers to domestic investors when equal to D and to foreign ones when equal to F):

$$\max_{C_{1,j},\omega_j} \ln(C_{1,j}) + \beta E_1[\ln(C_{2,j})],$$

$$C_{1,j} + \frac{E_1[C_{2,j}]}{\omega_j E_1[R_j] + (1-\omega_j)} \leq W_0,$$
$$0 \leq \omega_j \leq 1.$$

$E_1[R_j]$ is the expected gross return on domestic sovereign bonds for investors of group j. The key assumption of the model is that this return differs for domestic and foreign investors as soon as the probability of default is non-null.
For a given i^* and a given expected probability of default $E[p]$:

$$E_1[R_D] = (1 - E[p])(1 + f(i^*)),$$

with $f(i^*)$ the implicit payoff from i^* for domestic investors and $f'(i^*) > 0$, as the implicit return increases in the nominal return. Similarly, for foreign investors:

$$E_1[R_F] = (1 - E[p])(1 + g(i^*)),$$

with $g'(i^*) > 0$.[1] Due to the additional benefits for domestic creditors and the additional costs for foreign creditors that arise when the probability of default is distinct from 0: $f(i^*) = g(i^*) = i^*$ if $p = 0$ and $f(i^*) > i^* > g(i^*)$ if $p \neq 0$.
The difference between payoff functions f and g for a given i^* is increasing in the probability of default p because a higher probability of default deepens the differences in the

[1] Both f and g are assumed to be continuous functions in i^*.

domestic and foreign implicit returns on domestic sovereign bonds.
Second, a random macroeconomic shock r affects the government's fiscal sustainability after investors make their decision. r can take two values $r_1 > 0$ and $r_2 < 0$ with respective probabilities p_1 and $1 - p_1$. This shock generates ex-ante uncertainty on the government's decision. Then, market clearing on the domestic sovereign bonds market imposes that total demand Q_T equals the exogenous amount of debt d.
Third, the government observes the aggregate outcome of the investors' decision (i^*, H^*), with i^* the equilibrium nominal interest rate paid on sovereign bonds and H^* the share of public debt held by domestic creditors in equilibrium, and the realization of the shock r (either r_1 or r_2). The government then decides between repaying its debt $d(1 + i^*)$ (which is augmented by the service of the debt in the second period) or defaulting on part of this debt. The government makes its optimal decision by comparing C_R the cost function of full debt repayment and C_D the cost function of default. When $C_R < C_D$, the government repays its full debt $d(1 + i^*)$, and the creditors are repaid on their investment in domestic sovereign bonds. When $C_D < C_R$, the government does not repay part of its debt, and the creditors loose part of the amount they invested in domestic sovereign bonds.
Both the government's cost functions are assumed to be quadratic functions of the debt. This assumption is motivated by the fact that it is intuitively appealing that a marginal increase in the debt has a higher impact on the cost of debt repayment and on the cost of default, the higher the level of debt. It is also motivated by the desire to maintain things as simply as possible. In addition, the shape of the cost functions implies that the loss associated with debt repayment increases more with the debt than the loss associated with default, which ensures that, *ceteris paribus*, the probability of default increases in the debt, in accordance with intuition.[2]
The cost of full debt repayment C_R writes:

$$C_R(H^*, i^*, d, r) = u(H^*)d^2(1 + i^*)^2 + r, \tag{1}$$

The cost of debt repayment is related to the cost of taxation for the government. Raising taxes to repay the debt is costly for the government: it generates economic inefficiencies and, in a political economics perspective, decreases the support from domestic creditors, who are also voters, for the government. $u(H^*) > 0$ represents a variable cost of taxation coefficient, depending on the level of debt held domestically H^*. $u'(H^*) < 0$, as a higher proportion of domestic debt holdings decreases the cost of taxation.[3]

The cost of default C_D is:

$$C_D(H^*, i^*, d, h, L) = u(H^*)((1 - h)d)^2(1 + i^*)^2 + v(H^*)(hd)^2(1 + i^*)^2 + L, \tag{2}$$

where $0 \leq h \leq 1$ represents the share of debt which is defaulted on. Its optimal value is derived below. Therefore, the cost of debt repayment still must be paid for the portion of

[2] The specification of the cost functions relies on Gros (2012) but is modified in order to take into account the role of the share of debt held by domestic creditors.

[3] The specification of the cost of debt repayment accounts for the stochastic dimension of the government's decision, by incorporating a random macroeconomic shock. When $r = r_2 < 0$, the cost of debt repayment decreases. Thus, the shock positively affects the government's fiscal sustainability. It what follows, we refer to this shock as the 'positive shock'. On the other hand, when $r = r_1 > 0$, the shock negatively affects the government's fiscal sustainability. In what follows, we refer to this shock as the 'negative shock'.

debt that is not defaulted on, as explained by the term $u(H^*)((1-h)d)^2$.
In addition, the government encounters a cost of default for the portion of debt that is defaulted on. $L > 0$ is a lump-sum cost of default, which is independent of the amount of debt defaulted on and which can be interpreted as the cost of reputation; it occurs whenever the government decides to default. $v(H^*) > 0$ is a variable cost of default coefficient, depending on the share of debt held domestically H^*, such that $v'(H^*) > 0$ because the cost of default for the domestic economy increases in the domestic share of debt. The optimal share of debt defaulted on, h^*, results from the minimization of the cost function in case of default:

$$\min_h u(H^*)((1-h)d)^2 + v(H^*)(hd)^2(1+i^*)^2 + L \Leftrightarrow h^* = \frac{u(H^*)}{u(H^*)+v(H^*)}. \quad (3)$$

Interestingly, the optimal share of debt that is defaulted on depends on the size of the variable cost of taxation coefficient relative to the sum of the variable cost of default coefficient and the variable cost of taxation coefficient. The share of debt which is defaulted on thus decreases with the cost of default coefficient $v(H^*)$, whereas it increases in the cost of taxation coefficient $u(H^*)$, in accordance with intuition. For a given H^*, if the cost of taxation coefficient is higher than the cost of default coefficient, more than half the debt is defaulted on in case of default.

Finally, when substituting h by the above expression (its optimal value), the cost function C_D writes:

$$C_D(H^*, i^*, d, L) = \frac{v(H^*)u(H^*)}{u(H^*)+v(H^*)} d^2(1+i^*)^2 + L. \quad (4)$$

Both cost functions depend on the aggregate outcome of investors' individual decisions: the equilibrium interest rate on debt i^* and the equilibrium share of debt held domestically H^*. Therefore, for fixed values of the debt d and of the model's parameters, the optimal government's decision varies with creditors' decisions, which depend on the expected probability of default. This finding leads to multiple equilibria, as described below.

2.2. The model's multiple equilibria

Three distinct cases in the government's decision may arise. In the first case, $C_R < C_D$ (meaning that it is costlier for the government to default than to fully repay the debt) in both possible states, that is, even when the negative shock r_1 hits. In this case, the probability of default is equal to 0. In the second case, $C_D < C_R$ (meaning that it is costlier for the government to fully repay the debt) in both possible states, that is, even when the positive shock r_2 occurs. The probability of default is 1. In the third case, the government's decision depends on the realization of the shock. When $r = r_2$, $C_R < C_D$, it is thus optimal for the government to repay the debt. When $r = r_1$, $C_D < C_R$: it is optimal for the government to default on part of the debt. The probability of default is p_1.
The government's best response to creditors' aggregate decision is anticipated by both continuum of investors when making their decision in period 1. Consequently, ex-ante, creditors can coordinate on three distinct expected probabilities of default : 0, 1 and p_1,

which leads to three distinct equilibria.

The optimistic equilibrium In case (1) in which agents expect the probability of default to be 0, the expected implicit return on debt is similar for domestic and foreign creditors and equal to the nominal return (and $f(i^*) = g(i^*) = i^*$). Creditors' optimal decision (characterized by optimal consumption in period 1 C_1 and the optimal share of post-consumption wealth that is allocated in domestic sovereign bonds ω_j^*) is given by:

$$\{C_1^* = \frac{1}{1+\beta} W_0, 0 \leq \omega_D^{*1} \leq 1, 0 \leq \omega_F^{*1} \leq 1\}.$$

Because both domestic and foreign implicit return are equal when the expected probability of default is 0, domestic and foreign demands for sovereign bonds are equal ($\omega_D^{*1} = \omega_F^{*1}$). This yields the equilibrium share of debt held domestically: $H^{*1} = \frac{1}{2}$. Indeed, the share of public debt held by domestic investors in equilibrium is, by definition, the ratio of the debt held at home over total debt holdings (those of both domestic and foreign creditors):

$$H^* = \frac{\omega_D^*(W_0 - C_1^*)}{(\omega_D^* + \omega_F^*)(W_0 - C_1^*)}.$$

Thus, in this case, this yields $H^{*1} = \frac{1}{2}$, meaning that domestic creditors hold half of the domestic public debt stock in equilibrium.[4]

Beliefs are then validated if and only if the probability of default is indeed null for this set of individual decisions, meaning that $C_R < C_D$ regardless of the realization of the shock, that is, even for the negative shock r_1. If this is not the case, because creditors are rational (in the sense that the probability distribution they rely on coincides with the actual ex-ante probability distribution), this set of decisions is never chosen in equilibrium. Whether this set of decisions is optimal depends notably on the level of the debt d. Such an optimistic equilibrium can arise solely provided that the debt level is not excessive. The interval of values of d for which the optimistic equilibrium is a rational equilibrium is the interval of values of d for which the probability of default is indeed null when agents expect it to be and act accordingly (which yields $H^* = \frac{1}{2}$ and $i^* = 0$). This interval is:

$0 \leq d \leq \frac{\sqrt{(L-r_1)(u(\frac{1}{2})+v(\frac{1}{2}))}}{u(\frac{1}{2})}$ if $L \geq r_1$, and it is empty if $L < r_1$.[5]

[4] As the probability of default is expected to be null, agents are indifferent between investing in sovereign bonds or in the risk-free asset. Market clears if and only if $d \leq 2\omega_D^*(W_0 - C_1)$ and it clears in $i^{*1} = 0$ (as the risk-free rate is assumed to be zero).

[5] The interval is obtained by isolating d in the inequality $C_R - C_D < 0$ in which r is substituted by the negative shock r_1 and the aggregate outcome of agents' individual decisions H^* and i^* are substituted by $\frac{1}{2}$ and 0 in equations (1) and (4). If $L < r_1$, it is never optimal for the government not to default when the negative shock r_1 hits because the lump-sum cost of default is too low relative to the shock that negatively affects the government's fiscal sustainability.

The pessimistic equilibrium In case (2) in which agents expect the probability of default to be 1, their optimal decision is given by:

$$\{C_1^* = \frac{1}{1+\beta}W_0, \omega_D^{*2} = 0, \omega_F^{*2} = 0\}.$$

Indeed, in this case, given that a default is expected to occur in all states, there is no demand for domestic sovereign bonds in period 1 (thus, the share of savings invested in bonds is null $\omega_D^{*2} = \omega_F^{*2} = 0$) and the market does not clear. Such pessimistic beliefs are validated for all values of d. Indeed, when there is no demand for domestic sovereign bonds, the government is pushed into a roll-over crisis in period 1, regardless of the value of $d > 0$. Therefore, the share of debt held domestically is null in the pessimistic equilibrium: $H^{*2} = 0$.

The semi-optimistic equilibrium In case (3) (the intermediate case) in which agents expect the probability of default to be $0 < p_1 < 1$, their optimal decision is given by:

$$\{C_1^* = \frac{1}{1+\beta}W_0, \omega_D^{*3} = \frac{f(i^{*3}) - p_1(1 + f(i^{*3}))}{f(i^{*3})}, \omega_F^{*3} = \frac{g(i^{*3}) - p_1(1 + g(i^{*3}))}{g(i^{*3})}\}.$$

The optimal share of post-consumption wealth ω_j^{*3} allocated to domestic sovereign bonds results from the first order conditions of the representative creditor's maximization program. Once again, individual decisions determine the aggregate outcomes: the equilibrium share of debt held domestically H^* and the market interest rate i^*.
Thus, in this semi-optimistic equilibrium, the share of debt held by domestic creditors is higher than that held by foreign creditors, as $H^{*3} = \frac{\omega_D^{*3}}{\omega_D^{*3}+\omega_F^{*3}} > \frac{1}{2}$, due to the fact that domestic demand for domestic sovereign bonds is higher than foreign demand ($\omega_D^{*3} > \omega_F^{*3}$).[6]
Therefore, the share of debt held domestically in the semi-optimistic equilibrium is higher than the share of debt held domestically in the optimistic equilibrium.

Regarding the market interest rate, the market clearing condition that is imposed on the domestic sovereign bonds market is such that total demand Q_T equals the exogenous level of debt d: $i^{*3} = Q_T^{-1}(d)$.[7]

Beliefs are validated if and only if the cost of default indeed exceeds the cost of debt repayment when the positive shock hits (that is, $C_R < C_D$ when $r = r_2$) and the cost of default is smaller when the negative shock hits (meaning that $C_D < C_R$ when $r = r_1$) for such a set of aggregate outcomes i^{*3} and H^{*3}.

[6]Indeed, as $p_1 \neq 0$, we have $f(i^{*3}) > g(i^{*3}) > 0$ as stated above. Thus, $\omega_D^{*3} - \omega_F^{*3} = \frac{p_1(f(i^{*3})-g(i^{*3}))}{f(i^{*3})g(i^{*3})} > 0$.

[7]Indeed, $\frac{\partial \omega_D^3}{\partial i^{*3}} = \frac{p_1 f'(i^{*3})}{f(i^{*3})^2} > 0$ and $\frac{\partial \omega_F^3}{\partial i^{*3}} = \frac{p_1 g'(i^{*3})}{g(i^{*3})^2} > 0$. Therefore, total demand for domestic sovereign bonds is strictly increasing in i^{*3} and thus bijective (as f and g are assumed to be continuous on $\mathbf{R}^+$ with images in $\mathbf{R}^+$). Consequently, there is only one value of i^{*3} such that market clears.

Therefore, the semi-optimistic equilibrium is rational solely for an intermediate range of the debt level d, that is for all d such that:

$$\frac{\sqrt{(L-r_1)(u(H^{*3})+v(H^{*3}))}}{u(H^{*3})(1+i^{*3})} \leq d \leq \frac{\sqrt{(L-r_2)(u(H^{*3})+v(H^{*3}))}}{u(H^{*3})(1+i^{*3})} \text{ if } L \geq r_1,$$

and such that: $0 \leq d \leq \frac{\sqrt{(L-r_2)(u(H^{*3})+v(H^{*3}))}}{u(H^{*3})(1+i^{*3})}$ if $L < r_1$.

Indeed, when d is excessive, default becomes optimal, regardless of the creditors' beliefs, although they coordinate on semi-optimistic beliefs; then equilibrium (3) is not a rational equilibrium.
Finally, three distinct equilibria arise in the set-up: an optimistic equilibrium in which domestic debt is equally held by domestic and foreign creditors, a pessimistic equilibrium in which a roll-over crisis occurs, and a semi-optimistic equilibrium in which domestic creditors hold a larger share of domestic sovereign debt relative to foreign creditors.

They may all be rational equilibria for the same level of debt d, depending on whether agents coordinate rationally on the optimistic equilibrium, the semi-optimistic one or the pessimistic one. Appendix A distinguishes the values of d for which several equilibria may arise (that is, for which ranges for the debt level d identified above for each equilibrium coincide), depending on creditors' expectations.
We now assess the impact of a marginal increase in the public debt to GDP ratio from d to $d' = d + \varepsilon$ (with $\varepsilon = \triangle d$) on the equilibrium, to show how debt turmoil that is characterized by higher levels of sovereign debt may be associated with increases in the share of debt held by domestic creditors and with changes in the default probability.

2.3. The impact of an increase in sovereign risk on the share of debt held at home and on ex-post default risk

A marginal increase in the debt can affect the initial equilibrium, either because the initial expected probability of default is no longer validated when debt is higher, or because aggregate outcomes H^* and i^* directly depend on the level of debt. Comparing the distinct equilibria of the model in terms of the share of debt held at home allows the identification of the cases in which this share increases following a rise in the debt. The three distinct equilibria determined in the previous section are: $\{p = 0, C_1^*, \omega_D^{*1}, \omega_F^{*1}, H^{*1} = \frac{1}{2}, i^{*1} = 0\}$, $\{p = 1, C_1^*, \omega_D^{*2}, \omega_F^{*2}, H^{*2} = 0, i^{*2} = +\infty\}$, $\{p = p_1, C_1^*, \omega_D^{*3}, \omega_F^{*3}, H^{*3} > \frac{1}{2}, i^{*3} > 0\}$.

One can immediately observe from those results that the share of debt held at home increases when there is a switch from equilibrium (1) to equilibrium (3) following the marginal increase in d. In economic terms, the switch in equilibrium means that the debt level is currently excessively high for full debt repayment to be the government's optimal decision in all states. Default becomes the optimal solution for the government when the negative shock hits; the probability of default then increases from 0 to p_1. In this case, the share of debt held by domestic creditors changes from $H^{*1} = \frac{1}{2}$ to $H^{*3} > \frac{1}{2}$. This increase results from the fact that the higher debt level raises the default risk, which makes the implicit domestic return on domestic debt higher than the implicit foreign return ($f(i^*) > g(i^*)$). Thus, domestic demand is higher than foreign demand ($\omega_D^* > \omega_F^*$).

Whether such a switch occurs depends on the initial conditions (the initial level of debt d and the initial beliefs), because they determine the initial equilibrium (1). Second, starting from equilibrium (1), the likelihood of an increase in the domestic share of debt H^* when the debt rises is impacted by the size of the differences in the implicit returns on domestic sovereign bonds for domestic and foreign creditors ($f(i^*) - g(i^*)$). Third, the switch depends on the impact of the domestic share of debt on the cost of debt repayment C_R and the cost of default C_D (through the variable cost of taxation coefficient $u(H^*)$ and the variable cost of default coefficient $v(H^*)$) and thus, on whether the increase in the domestic share of debt from $H^{*1} = \frac{1}{2}$ to $H^{*3} > \frac{1}{2}$ (which decreases the government's incentives to default as stated above) or the increase in the debt from d to d' and in the interest rate from $i^{*1} = 0$ to $i^{*3} > 0$ (which raises the cost of debt repayment) has more impact on the government's decision. This finding means that the domestic share of debt increases if and only if its stabilizing impact on the government's strategic decision of default is expected to be lower than that of the destabilizing initial increase in public debt. Otherwise, the probability of default would not increase from 0 to p_1, and then the equilibrium would not switch from the optimistic equilibrium to the semi-optimistic equilibrium. Eventually, the switch depends on the impact of random macroeconomic conditions, represented here with two possible states.

Second, the domestic share of debt rises when the new equilibrium following the marginal increase in the debt is equilibrium (3) starting from initial equilibrium (3), in which the probability of default is p_1, and the implicit return on domestic sovereign bonds for domestic creditors increases sufficiently relative to the implicit return for foreign creditors. Indeed, in equilibrium (3), the share of domestic debt holdings H^{*3} depends on the debt level d, because it is defined by $H^{*3} = \frac{\omega_D^{*3}}{\omega_D^{*3} + \omega_F^{*3}}$, where $\omega_D^{*3} = \frac{f(i^*) - p_1(1+f(i^*))}{f(i^*)}$ and $\omega_F^{*3} = \frac{g(i^*) - p_1(1+g(i^*))}{g(i^*)}$, with i^{*3} a function of the debt d: $i^{*3} = Q_T^{-1}(d)$, as shown above.
Whether H^{*3} is an increasing function of the debt level then depends on the size of the subsequent increase in the implicit domestic return relative to the size of the subsequent increase in the implicit foreign return. Indeed, the implicit domestic return should increase sufficiently so that the domestic demand for domestic sovereign bonds increases more than foreign demand following the increase in the debt, which depends on the relative slopes of both demand functions in the initial equilibrium point. Therefore, whether such a switch occurs depends on the determinants that are noted in the previous case and on the shapes of domestic and foreign demands with respect to the interest rate on sovereign bonds i. Appendix B derives those results mathematically.
Therefore, two cases in the model enable the replication of the increase in the share of debt held domestically following an increase in the debt level. These cases are associated with constant or increasing probabilities of default.

3. Conclusion

In a simple and stylized manner, the model illustrates the mechanism through which an exogenous variation in public debt can affect the share of debt held at home. The model makes explicit the variables that impact the likelihood of an increase in the share of domestic holdings of public debt and illustrates how this increase – which was observed recently in the Euro area – impacts default risk.

Thus, although an initial positive debt to GDP ratio shock can cause the market interest rate to rise, it may not be purely destabilizing for sovereign debt sustainability because an increasing presence of domestic creditors simultaneously reduces the government's incentives to default. In addition, the model provides a simple framework to explain how the increase in the share of debt held by the resident sector can also result from exogenous self-fulfilling factors that lead to switches in equilibria and variations in default risk.

Those results could suggest that economic policies that target both containing the increase in public debt ratios and boosting domestic investment in sovereign bonds, which ensure the credibility of the government's willingness to repay its debt, may provide temporary fiscal breathing space if further episodes of stress on the sovereign debt markets arise.

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A Multiple equilibria zones

In order to characterize what equilibria can arise for a given value of d, depending on agents' expectations, it is necessary to determine how the boundaries of each equilibrium zone (which represents, for each equilibrium, the interval of value of d such that for those values of d this equilibrium is a rational expectations equilibrium) are found relative to each other. Equilibrium (2), which can be seen as a pessimistic equilibrium, holds for any value of d. The boundaries of the equilibrium region (1) (the optimistic equilibrium) and the bounds of the equilibrium region (3) (the semi-optimistic equilibrium) depend on the market interest rate and on the equilibrium domestic share of public debt (through its impact on the cost of default and the cost of taxation), which both vary with the expected probability of default.

What differentiates the upper boundary of the equilibrium (1) region and the lower boundary of the equilibrium (3) region is that the latter is characterized by a higher domestic share of public debt ($H^{*3} > \frac{1}{2}$) and a higher interest rate on domestic sovereign bonds ($i^{*3} > 0$). The higher domestic share of public debt in equilibrium (3) tends to make the upper boundary of the equilibrium (1) region lower than the lower boundary of the equilibrium (3) region – the stabilizing effect, through the impact on the cost of default and cost of taxation. On the contrary, the higher market interest rate in equilibrium (3) tends to make the upper boundary of the equilibrium (1) region higher than the lower boundary of the equilibrium (3) region – the destabilizing effect, through a higher service of the debt.

The relative location of the boundaries of both regions thus depends on whether the destabilizing impact of the higher market interest rate dominates or not the possibly stabilizing impact of the higher share of debt held by domestic creditors.

The following figure provides an illustration of the distinct rational expectations equilibria that can arise depending on d in the case where the destabilizing impact of a higher market interest rate dominates the stabilizing impact of a higher share of debt held by domestic creditors. The horizontal axis represents the level of debt d, going from 0 to $+\infty$. The green arrow delimits the range of values of d such that equilibrium (1) holds when agents expect the probability of default to be 0. The black arrow delimits the range of values of d such that equilibrium (3) holds when agents expect the probability of default to be p_1. Finally, the red arrow delimits the range of values of d such that equilibrium (2) holds when agents expect the probability of default to be 1. It is true for all d.

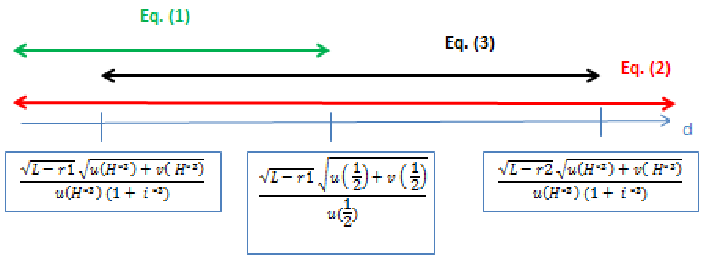

Figure 2: Multiple equilibria zones.

B Proof of results of section 2.3

Let's define $A = \frac{\sqrt{(L-r_1)(u(\frac{1}{2})+v(\frac{1}{2}))}}{u(\frac{1}{2})}$, $B = \frac{\sqrt{(L-r_2)(u(H^{*3})+v(H^{*3}))}}{u(H^{*3})(1+i^{*3})}$,
$C = \frac{\sqrt{(L-r_1)(u(H^{*3})+v(H^{*3}))}}{u(H^{*3})(1+i^{*3})}$, $D = \frac{\sqrt{(L-r_2)(u(H'^{*3})+v(H'^{*3}))}}{u(H'^{*3})(1+i'^{*3})}$ and $E = \frac{\sqrt{(L-r_1)(u(H'^{*3})+v(H'^{*3}))}}{u(H'^{*3})(1+i'^{*3})}$.

In mathematical terms, the results displayed in section 2.3 can be rewritten more precisely as:

- H^* increases from $\frac{1}{2}$ to $H'^{*3} > \frac{1}{2}$ when d changes to $d' = d + \varepsilon$ if and only if $A - \varepsilon < d \leq A$ and $B - \varepsilon \leq d \leq C - \varepsilon$.

- H^* goes from H^{*3} to H'^{*3} with $H'^{*3} > H^{*3}$ when d goes to d' if and only if $\frac{\partial H^{*3}}{\partial d} > 0 \Leftrightarrow f'(i^{*3}) \geq \frac{f(i^{*3})(p_1+(p_1-1)f(i^{*3}))}{g(i^{*3})(p_1+(p_1-1)g(i^{*3}))} g'(i^{*3})$ and $D \leq d \leq E$ and $B - \varepsilon \leq d \leq C - \varepsilon$.

Indeed, first, when the initial equilibrium is equilibrium (1) ($\{p = 0, C_1^*, \omega_D^{*1}, \omega_F^{*1}, H^{*1} = \frac{1}{2}, i^{*1}\}$), the only case in which the domestic share of debt rises following the marginal increase in the debt level is when beliefs that $p = 0$ are no longer validated in equilibrium and switch to $E[p] = p_1$.
The initial equilibrium being equilibrium (1), this implies that $d \leq A$, as shown in the paragraph about the optimistic equilibrium. Equilibrium (1) no longer holds (meaning that it no longer is a rational equilibrium) following the marginal increase in d if and only if $d > A - \varepsilon$. Equilibrium (3) holds following the marginal increase in debt if and only if $B - \varepsilon \leq d \leq C - \varepsilon$. Therefore, there is a switch from equilibrium (1) to equilibrium (3) following a marginal increase in the debt to GDP ratio when the two following inequalities are simultaneously verified for an initial debt to GDP ratio d:

$$A - \varepsilon < d \leq A$$

and

$$B - \varepsilon \leq d \leq C - \varepsilon.$$

As ε is an infinitesimal quantity, $\varepsilon \to 0$. Therefore, the first inequality implies that $d \to A$ is a necessary condition for a switch from equilibrium (1) to equilibrium (3) following the marginal increase in d.
If $A \leq B$ (meaning that the stabilizing impact of a higher share of debt held at home exceeds the destabilizing impact of a higher interest rate) and if $A \leq C$, we have: $B - \varepsilon \leq d \leq A \leq B$. Therefore, there is a switch if and only if $d \to A$ and $B = A$.
If $A \leq B$ and $C < A$, this reduces to $A - \varepsilon \leq B - \varepsilon \leq d \leq C - \varepsilon < A$. There is a switch if and only if $d = A$ and $C = B = A$.
If $B < A$ (meaning that the destabilizing impact of a higher interest rate exceeds the stabilizing impact of a higher share of debt held at home) and $A \leq C$, this reduces to $A - \varepsilon < d \leq A$. There is a switch if and only if $d \to A$.
When $B < A$ and $C < A$, this reduces to $A - \varepsilon < d \leq C - \varepsilon < A$. The switch occurs if and only if $d = A$ and $C = A$.
Second, when the initial equilibrium is equilibrium (2): $\{p = 1, C_1^*, \omega_D^{*2}, \omega_F^{*2}, H^{*2} = 0, i^{*2}\}$, initial beliefs are still validated in equilibrium following a marginal increase in d whatever the level of d. Therefore, we still have $H^* = 0$ following the increase.
Third, when the initial equilibrium is equilibrium (3): $\{p = p_1, C_1^*, \omega_D^{*3}(d), \omega_F^{*3}(d), H^{*3} > \frac{1}{2}, i^{*3}\}$, the share of debt held domestically cannot increase when the equilibrium switches to equilibrium (1) or to equilibrium (2). However, in equilibrium (3), H^{*3} and i^{*3} depend on the debt level d. Therefore, when d increases marginally, H^{*3} and i^{*3} are modified. As $i^{*3} = Q_T^{-1}(d)$, it is an increasing function of d with no restriction (as Q_T is an increasing function of i).
$H^{*3}(d) < H^{*3}(d')$ if and only if:

$$\frac{\partial H^{*3}}{\partial d} > 0 \Leftrightarrow f'(i^{*3}) \geq \frac{f(i^{*3})(p_1 + (p_1 - 1)f(i^{*3}))}{g(i^{*3})(p_1 + (p_1 - 1)g(i^{*3}))} g'(i^{*3}). \tag{5}$$

Indeed, we can rewrite H^{*3} as a function of d:

$$H^{*3}(d) = \frac{(p_1 - (1 - p_1)f[Q_T^{-1}(d)])g[Q_T^{-1}(d)]}{p_1 g[Q_T^{-1}(d)] + f[Q_T^{-1}(d)](p_1 - 2(1 - p_1)g[Q_T^{-1}(d)])}.$$

Eventually, this yields:

$$\frac{\partial H^{*3}}{\partial d} > 0 \Leftrightarrow f'[Q_T^{-1}(d)] \geq \frac{f[Q_T^{-1}(d)](p_1 + (p_1 - 1)f[Q_T^{-1}(d)])}{g[Q_T^{-1}(d)](p_1 + (p_1 - 1)g[Q_T^{-1}(d)])} g'[Q_T^{-1}(d)],$$

or, written as a function of i^{*3} again, we get condition (8) if and only if $f(i^{*3}) \geq \frac{p_1}{1-p_1}$ (as this determines the sign of the inequality), which always holds given that $\omega_D^{*3}(d) \geq 0$. Indeed, as the denominator of $\omega_D^{*3}(d)$ is positive, this implies that the numerator is also positive, meaning that $f(i^{*3})(1 - p_1) \geq p_1$).
Given that $\omega_F^{*3}(d) \geq 0$ as well, $\frac{f[Q_T^{-1}(d)](p_1+(p_1-1)f[Q_T^{-1}(d)])}{g[Q_T^{-1}(d)](p_1+(p_1-1)g[Q_T^{-1}(d)])} > 0$. Therefore, the economic interpretation of this condition is that a marginal increase in the debt to GDP ratio causes the domestic share of sovereign debt to increase if and only if the marginal rate of growth of the implicit return on domestic sovereign bonds for domestic creditors $f(i^*)$ increases suf-

ficiently in the equilibrium interest rate relative to the implicit return for foreign creditors $g(i^*)$.
For the initial and the new equilibrium to be equilibrium (3), it is necessary and sufficient to have simultaneously: $D \leq d \leq E$ and $B - \varepsilon \leq d \leq C - \varepsilon$.
If condition (5) is satisfied and if the destabilizing effect due to a higher interest rate is higher, that is $B \leq D$ and $C \leq E$, this reduces to $D \leq d \leq C - \varepsilon$. On the contrary, if $D < B$ and $E < C$, this reduces to $B - \varepsilon \leq d \leq E$.