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### Efficient taxation with differential risks of dependence and mortality

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#### Abstract

The purpose of this note is to analyze the optimal tax and transfer policies that should be conducted in a society where individuals differ according to their productivity and their risk of mortality and dependency. We show that according to the most reasonable estimates of correlation among these three characteristics, an optimal policy should consist of a tax on earning and second period consumption and of a subsidy on long term care spending. Also, the implicit tax on saving is positive.

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# 1 Introduction

Panel surveys of elderly people such as Survey of Health, Ageing and Retirement in Europe (SHARE) or the U.S. Health and Retirement Study (HRS) consistently point to three correlations: one positive between income and longevity, one also positive between dependence and longevity, and one negative between dependence and income. The purpose of this note is to see the implications of these stylized facts on the taxation of earnings, saving and long-term care (LTC) insurance. We start with a setting a la Atkinson and Stiglitz (1976) and Cremer et al. (2010), in which, without differential risks of mortality or dependence, a tax on earnings suffices to achieve efficiency. Introducing the risks of longevity and of dependence makes it desirable to interfere with saving and insurance choices. The setting we adopt is that of a society with two types of individuals differing in their earning capacity and their probability of dependence and of mortality. The government does not know these characteristics and tries to influence the choice of labor, saving and LTC consumption through non-linear taxes (or subsidies). We show that the tax structure closely depends on how these characteristics relate to each other. There exist some papers looking at the design of optimal taxation when both productivity and longevity are not observable. See, e.g., Cremer et al. (2010) and Diamond and Spinnewijn (2011). There does not exist work extending this setting to include an unobservable probability of dependency.

## 2 The model

Consider a two-period model, where individuals work and save in the first period and retire in the second. In the second period people face different risks of mortality and dependence. Following Stiglitz (1982) we consider a society comprising two types of individuals that we call unskilled (1) and skilled (2). The proportion of type  $i$  ( $i = 1, 2$ ) individuals is denoted by  $n_i$ , with  $n_1 + n_2 = 1$ . Each individual is characterized by three characteristics: (i)  $w_i$  (labor productivity in the first period), (ii)  $\pi_i$  (the probability to be alive in the second period), and (iii)  $p_i$  (the probability of becoming dependent in the second period). The skilled are more productive so  $w_2 > w_1$ . As to  $\pi_i$  and  $p_i$ , we assume the following, based on some stylized facts derived from the most recent waves of SHARE:<sup>1</sup>

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<sup>1</sup>Survey of Health, Ageing and Retirement in Europe. Own calculations.

- longevity increases with income:  $\pi_2 > \pi_1$ ;
- the probability of dependency decreases with income:  $\pi_2 p_2 < \pi_1 p_1$ ;

Notice that they also imply  $\pi_2(1 - p_2) > \pi_1(1 - p_1)$ : the probability of remaining autonomous increases with income.

Type  $i$ 's lifetime utility can be written as:

$$U_i = u(c_i) - v(\ell_i) + \pi_i(1 - p_i)u(d_i) + \pi_i p_i H(m_i)$$

where  $c_i$  and  $d_i$  denote first and second period consumption;  $m_i$ , LTC spending;  $\ell_i$ , labor supply; both  $u(\cdot)$  and  $H(\cdot)$  are strictly concave functions, and  $v(\cdot)$  is convex. We also assume that  $H(x) < u(x)$ .

## 2.1 Laissez faire

We first look at the laissez faire solution for an individual of type  $i$ . The problem of an individual of type  $i$  is to choose the labor supply, the saving  $s_i$  and the insurance premium  $I_i$  that maximize:

$$U_i = u(w_i \ell_i - s_i - I_i) - v(\ell_i) + \pi_i(1 - p_i)u(s_i/\pi_i) + \pi_i p_i H(I_i/(p_i \pi_i) + s_i/\pi_i)$$

where we implicitly assume no time preference, a rate of interest equal to 0, an actuarially fair annuity market and LTC insurance.

We easily verify the following conditions:

$$\frac{u'(d_i)}{u'(c_i)} = \frac{H'(m_i)}{u'(c_i)} = 1; \quad \frac{v'(\ell_i)}{u'(c_i)} = w_i$$

## 2.2 Optimum

To obtain the optimality conditions we maximize a weighted sum of individual lifetime utilities subject to two constraints: a resource constraint and a self-selection constraint in which we assume that the parameters are such that type 2 wants to mimick type 1 and not the other way around.<sup>2</sup>

<sup>2</sup>This latter alternative case would occur if the probability of dependence of the skilled were much higher than that of the unskilled. We exclude this case, by simply considering a case in which the skilled individuals result in higher lifetime utility than the unskilled at the laissez faire outcome.

At this point a word on our informational setting is in order. We assume that the government proposes two menus of taxes/subsidies at the beginning of the first period, not knowing the individual types. It then commits to these menus. At the same time, we posit that private insurers observe the probability of longevity and that of dependence, and can thus offer actuarially fair LTC insurance and retirement annuities. Under commitment, the government does not redesign the tax/transfer of the second period, making use of the observed insurance purchases.

We use the multipliers  $\mu$  and  $\lambda$  for the resource constraint and the self-selection constraint, respectively. We now write the following Lagrange expression:

$$\begin{aligned} \mathcal{L} = & \sum n_i \{ \alpha_i (u(c_i) - v(\ell_i) + \pi_i(1 - p_i)u(d_i) + \pi_i p_i H(m_i)) \\ & - \mu [c_i - w_i \ell_i + \pi_i(1 - p_i)d_i + \pi_i p_i m_i] \} \\ & + \lambda [u(c_2) - v(\ell_2) + \pi_2(1 - p_2)u(d_2) + \pi_2 p_2 H(m_2) \\ & - (u(c_1) - v(w_1 \ell_1 / w_2) + \pi_2(1 - p_2)u(d_1) + \pi_2 p_2 H(m_1))] \end{aligned}$$

where  $\alpha_1$  and  $\alpha_2$  are individual non negative weights that guarantee a Pareto optimal solution ( $\alpha_1 \geq \alpha_2$ ).

Setting  $y_i \equiv w_i \ell_i$ , the FOC's are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_2} &= n_2 \alpha_2 u'(c_2) - \mu n_2 + \lambda u'(c_2) = 0 \\ \frac{\partial \mathcal{L}}{\partial d_2} &= [n_2 \alpha_2 u'(d_2) - \mu n_2 + \lambda u'(d_2)] \pi_2 (1 - p_2) = 0 \\ \frac{\partial \mathcal{L}}{\partial m_2} &= [n_2 \alpha_2 H'(m_2) - \mu n_2 + \lambda H'(m_2)] \pi_2 p_2 = 0 \\ \frac{\partial \mathcal{L}}{\partial y_2} &= -n_2 \alpha_2 v'(\ell_2) / w_2 + \mu n_2 - \lambda v'(\ell_2) / w_2 = 0 \end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_1} &= n_1 \alpha_1 u'(c_1) - \mu n_1 - \lambda u'(c_1) = 0 \\
\frac{\partial \mathcal{L}}{\partial d_1} &= [n_1 \alpha_1 u'(d_1) - \mu n_1] \pi_1 (1 - p_1) - \lambda u'(d_1) \pi_2 (1 - p_2) = 0 \\
\frac{\partial \mathcal{L}}{\partial m_1} &= [n_1 \alpha_1 H'(m_1) - \mu n_1] \pi_1 p_1 - \lambda H'(m_1) \pi_2 p_2 = 0 \\
\frac{\partial \mathcal{L}}{\partial y_1} &= -n_1 \alpha_1 v'(\ell_1)/w_1 + \mu n_1 + \lambda v'(y_1/w_2)/w_2 = 0
\end{aligned}$$

From the first set of FOC's we derive the following expressions:

$$\frac{u'(d_2)}{u'(c_2)} = \frac{H'(m_2)}{u'(c_2)} = 1; \quad \frac{v'(\ell_2)}{u'(c_2)} = w_2$$

These equalities express the standard no distortion at the top. In other words, there is no need to distort the choices of saving, LTC and labor of type 2 individuals.

We now turn to the unskilled individuals. With the evidence mentioned in the beginning of this section, we can interpret the tax formulas for the unskilled. Starting with the demand for LTC, we have:

$$\frac{H'(m_1)}{u'(c_1)} - 1 = \frac{\lambda H'(m_1)}{\mu n_1} \left[ \frac{\pi_2 p_2}{\pi_1 p_1} - 1 \right] < 0 \iff \frac{\pi_2 p_2}{\pi_1 p_1} < 1$$

Namely, long term care ought to be subsidized as the probability of dependency of the unskilled is higher than that of the skilled individuals. As to second period consumption, we have:

$$\frac{u'(d_1)}{u'(c_1)} - 1 = \frac{\lambda u'(d_1)}{\mu n_1} \left[ \frac{\pi_2 (1 - p_2)}{\pi_1 (1 - p_1)} - 1 \right] > 0 \iff \frac{\pi_2 (1 - p_2)}{\pi_1 (1 - p_1)} > 1$$

In words, second period consumption ought to be taxed as the probability of keeping autonomous is higher for the skilled than for the unskilled individuals.

From these formulas one can obtain the implicit tax on saving (which is

the capital income tax in the New Dynamic Public Finance).<sup>3</sup>

$$\pi_1 \left( p_1 \frac{H'(m_1)}{u'(c_1)} + (1 - p_1) \frac{u'(d_1)}{u'(c_1)} - 1 \right) = \pi_1 \left( \frac{p_1}{1 - \tilde{\lambda} \left( \frac{\pi_2 p_2}{\pi_1 p_1} - 1 \right)} + \frac{1 - p_1}{1 - \tilde{\lambda} \left( \frac{\pi_2(1-p_2)}{\pi_1(1-p_1)} - 1 \right)} - 1 \right) \quad (1)$$

where  $\tilde{\lambda} = \frac{\lambda u'(c_1)}{\mu n_1}$ . We show in the Appendix that the implicit tax on saving is positive.

Finally, we have the tax formula on labor:

$$1 - \frac{v'(\ell_1)}{u'(c_1)w_1} = \tilde{\lambda} \left[ \frac{v'(\ell_1)}{u'(c_1)w_1} - \frac{v'(y_1/w_2)}{u'(c_1)w_2} \right] > 0$$

As in the conventional optimal taxation problem,  $\frac{y_1}{w_2} < \ell_1$  and  $\frac{1}{w_2} < \frac{1}{w_1}$  imply the positive marginal income tax rate for the unskilled individuals.

To decentralize this optimum one can use a tax on earnings and saving and a subsidy on the insurance premium that imply the same distortions as those found in the above inequalities. Note that in the absence of private insurance for long term care, the above policy would consist of a LTC public benefit different for the two types that would be financed by a tax on the saving and the earnings of the unskilled and a lump sum tax paid by the skilled individuals.

### 3 Conclusion

We have shown in this note that under the assumption of higher probability of survival for the skilled and a lower probability of turning dependent, assumptions that are verified in most societies, the optimal policy towards long term care is to subsidize long-term care insurance and tax the second period consumption. The sign on the tax on saving seems ambiguous as saving is also used for dependence. However, we showed that the implicit tax on saving is positive. In the case where there is no market for private insurance,

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<sup>3</sup>If both types had the same survival probability but different probability of dependency, the outcome is not much different but less exacerbated. However, we note that  $\pi_2 > \pi_1$  is a stylized fact from the SHARE database. Needless to say, when  $\pi_1 = \pi_2$  and  $p_1 = p_2$ , the Atkinson-Stiglitz theorem holds so that the tax on saving is zero.

the government can supply long-term care benefits that would vary between the two types of households.

## Appendix

We show that the implicit tax on saving in (1) is positive. Let  $a \equiv \tilde{\lambda} \left( \frac{\pi_2 p_2}{\pi_1 p_1} - 1 \right) < 0$  and  $b \equiv \tilde{\lambda} \left( \frac{\pi_2(1-p_2)}{\pi_1(1-p_1)} - 1 \right) > 0$ . From the first order conditions, we have  $\frac{H'(m_1)}{u'(c_1)} = \frac{1}{1-a} > 0$  and  $\frac{u'(d_1)}{u'(c_1)} = \frac{1}{1-b} > 0$ . Also, since  $\pi_2 > \pi_1$ ,

$$\pi_2(1-p_2) - \pi_1(1-p_1) = \pi_1 p_1 - \pi_2 p_2 + \pi_2 - \pi_1 > \pi_1 p_1 - \pi_2 p_2,$$

so that  $b > -\frac{p_1}{1-p_1}a$ . Therefore,

$$\begin{aligned} p_1 \frac{H'(m_1)}{u'(c_1)} + (1-p_1) \frac{u'(d_1)}{u'(c_1)} - 1 &= p_1 \frac{a}{1-a} + (1-p_1) \frac{b}{1-b} \\ &= \frac{p_1 a(1-b) + (1-p_1)(1-a)b}{(1-a)(1-b)} = \frac{p_1 a + (1-p_1-a)b}{(1-a)(1-b)} \\ &> \frac{p_1 a - \frac{p_1}{1-p_1} a(1-p_1-a)}{(1-a)(1-b)} = \frac{\frac{p_1}{1-p_1} a^2}{(1-a)(1-b)} > 0. \end{aligned}$$

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