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Capital specialization and aggregate productivity

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Abstract

This paper makes use of a Dixit-Stiglitz framework to dissect the combined productivity e□ects of capital variety and average capital specificity under general equilibrium. The impact of key parameters like entry costs, market size or price elasticities of demand on these variables is shown to differ according to whether agglomeration effects are present or not and how strong they are.

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1. Introduction

Output per worker varies substantially across economies, as noted by Hall and Jones (1997). They show that differences in physical capital intensity are not sufficient to explain this, requiring the identification of additional determinants. For instance, Ciccone and Hall (1996) and Ciccone (2002) verify that locally increasing returns due to knowledge externalities or higher supplier density account for a larger share of productivity variation across U.S. states and European economies. Similarly, regional economics approaches like those in Abdel-Rahman (1988) or Rivera-Batiz (1988) make use of monopolistic competition models inspired by Dixit and Stiglitz (1977) to assert a positive link between the number of suppliers, productivity and welfare in the presence of agglomeration economies.

These neoclassical interpretations, built upon aggregrate production functions, fail however to address qualitative differences in physical capital. Unlike spatial representations on a line or circle, rooted on the Hotelling-Lancaster tradition, measurable technical distance is not featured here, making the endogenous treatment of capital fitness harder. This paper attempts to fill that gap, integrating upstream specialization choices into a Dixit-Stiglitz model and using them to draw inferences on the behavior of output per worker. The analysis is not concerned with the bargaining problems usually found in small numbers environments. The scope is broader, encompassing a large set of agents under general equilibrium. We find that once capital fitness is accounted for and endogenously adjusted, the relation between production efficiency and the number of intermediate goods is no longer straightforward. Moreover, since this characterization is contingent on industry specific parameters like entry costs, market size and market power, additional sources of productivity variation can be identified. As Ciccone (2002) notes, agglomeration effects benefits are likely to differ across sectors, an idea that is corroborated here.

2. The Model

2.1 Households

The representative household maximizes

$$U = \ln C, \tag{1}$$

where C is the consumption aggregator

$$C = \left(\int_0^N C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2}$$

N is the mass of consumption goods, C_i the quantity of each variety, and $\varepsilon > 1$.

This is subject to the constraint

$$E = \int_0^N P_i C_i di, \tag{3}$$

where E is the household's income and P_i the price of final good i.

The individual demand schedule becomes

$$C_i = \frac{E}{P}\left(\frac{P}{P_i}\right)^{\varepsilon}, \tag{4}$$

where P is a price index defined as usual by

$$P = \left(\int_0^N P_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} di. \tag{5}$$

2.2 Specialization and Productivity

Final goods are produced under constant returns to scale and monopolistic conditions, following

$$Y_i = L_i^{1-\alpha}\int_0^K \bar{x}_{i,k}^{\alpha} dk. \tag{6}$$

There are K capital goods with additively separable effects on output and diminishing marginal returns. The variety of capital is linked by Ethier (1982) to higher tiers of manufacturing complexity, yielding productivity gains through a finer division of labor.

We assume that each final good creates a distinct set of requirements over capital inputs. For the sake of simplicity, we take the heterogeneity of the former as given, so that we can focus on the specialization choice of suppliers. This comes in two steps. First, a supplier decides which of the final producers his intermediate good best matches. This depends on how many alternative users are in the market. We adopt a random pairing mechanism, defining the individual probability of a final producer i benefitting from a more tailored input k as

$$M_{i,k} = \theta(N). \tag{7}$$

Recall that N is the number of final producers. Similarly, the probability of final producer i not being the primary target of supplier k is

$$\tilde{M}_{i,k} = 1 - \theta(N). \tag{8}$$

No restrictions are imposed on the first derivative of $\theta(N)$. Notice that capital inputs are independent and their productivities unrelated, so matching does not depend on other suppliers or their number. Since firms are symmetric in their production functions and demand, suppliers are *ex-ante* indifferent to which user represents their best fit. With simultaneous decisions, $M_{i,k} \equiv M$ and $\tilde{M}_{i,k} \equiv \tilde{M}$.

The intermediate producer also controls to what extent capital is fine-tuned with regard to a given final producer. The resulting specificity level is labeled as s. Non-matching users can still use this input, though necessarily incurring in losses. These can be formalized in a variety of ways. For instance, Kim (1989) imposes costly training whenever human capital skills differ from a firm's job requirement. McLaren (2000) uses distinct cost reduction parameters to capture varying degrees of systems compatibility. Grossman and Helpman (2002) require firms to add labor to make less specialized inputs fit their purposes.

Let then $x_{i,k}$ be the quantity of intermediate good k used by producer i and $\bar{x}_{i,k}$ its quality adjusted level, with

$$\bar{x}_{i,k} = \varphi\left(s_k\right)^{\frac{1-\alpha}{\alpha}} x_{i,k}. \tag{9}$$

$\varphi\left(s_k\right)$ shifts the productivity schedule of intermediate good k according to its specificity level.[1]

This adjustment depends on the matching outcome. Define this as

$$\varphi\left(s_k\right) = \begin{cases} \varphi\left(s_k|m\right), & \textit{if there is a match with supplier } k \\ \varphi\left(s_k|\tilde{m}\right), & \textit{if there is no match with supplier } k. \end{cases} \tag{10}$$

These functions are continuous and differentiable. They obey

$$\varphi\left(s_k|m\right) > \varphi\left(s_k|\tilde{m}\right), \forall k, s_k > 0. \tag{11}$$

This means that input k is more productive in the firm best matching its specifications. Since this function also drives marginal costs, it becomes isomorphic to the literature examples just discussed.

Finally, the following assumption completes the characterization of the productivity schedule.

Assumption 1 *Specificity increases productivity for a matching user with decreasing marginal gains, that is,* $\varphi'\left(s_k|m\right) > 0$ *and* $\varphi''\left(s_k|m\right) < 0$*. Conversely, more generality affords other users higher productivity with decreasing marginal gains, that is,* $\varphi'\left(s_k|\tilde{m}\right) < 0$ *and* $\varphi''\left(s_k|\tilde{m}\right) < 0$*.*

The second derivative signs also guarantee that the equilibrium choice of suppliers is well-behaved. Figure 1 provides a representation of the resulting function $\varphi\left(s_k\right)$.

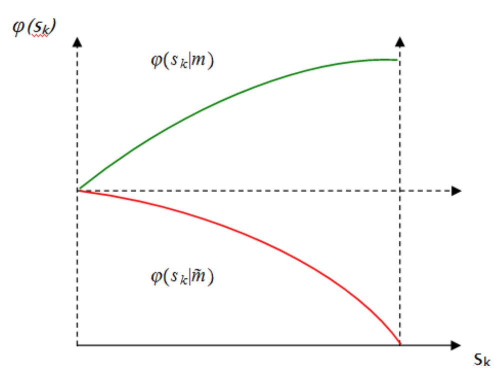

Fig. 1: Effective Productivity

2.3 Final Producers

Fixed costs for final producers, F_Y, are expressed in units of labor. Labor is the numeraire and

[1] This is raised to a power of $\frac{1-\alpha}{\alpha}$ without loss of generality in order to ensure that expected capital demand is linear in this function.

$W = 1$. The cost minimization problem is

$$\min_{L_i, x_{i,\,k}|_{k=1,\ldots,K}} L_i + \int_0^K P_{x_k} x_{i,k} dk + F_Y \tag{12a}$$

$$s.t.\ Y_i = L_i^{1-\alpha} \int_0^K \varphi(s_k)^{1-\alpha} x_{i,k}^{\alpha} dk. \tag{12b}$$

For a sufficiently large number of agents, each producer takes s_k as given. The first order conditions with respect to L_i and $x_{i,k}$ yield

$$x_{i,k} = \left[\frac{\alpha L_i}{(1-\alpha) Y_i P_{x_k}}\right]^{\frac{1}{1-\alpha}} \varphi(s_k) L_i. \tag{13}$$

Naturally, effective capital demand is not necessarily the same, depending rather on the outcome of the matching process, as expressed by (10). This uncertainty is addressed *ex-ante* by determining its expected productivity. Using (7) and (8), we may define

$$E[\varphi(s_k)] = \theta(N) \varphi(s_k|m) + [1-\theta(N)] \varphi(s_k|\tilde{m}). \tag{14}$$

As will be shown in the next section, upstream specificity choices and pricing decisions are symmetric, that is $s_k = s$ and $P_{x_k} = P_x, \forall k$. From (6), expected output, $E(Y_i)$, becomes a linear function of expected productivity, $E[\varphi(s)]$. From here, algebraic manipulation yields each input's demand:

$$x_i(Y_i, P_x) = \left[\frac{\alpha}{(1-\alpha) P_x K}\right]^{1-\alpha} \frac{E(Y_i)}{KE[\varphi(s)]^{1-\alpha}}; \tag{15}$$

$$L_i(Y_i, P_x) = \left[\frac{(1-\alpha) P_x K}{\alpha}\right]^{\alpha} \frac{E(Y_i)}{KE[\varphi(s)]^{1-\alpha}}. \tag{16}$$

Hence, the anticipated total cost is

$$TC_i = E(Y_i) \{(1-\alpha) KE[\varphi(s)]\}^{\alpha-1} \left(\frac{P_x}{\alpha}\right)^{\alpha} + F_Y. \tag{17}$$

Market clearing imposes $Y_i = LC_i$. Using (4) and (17), expected profits are

$$\pi_i = \left\{P_i - \{(1-\alpha) KE[\varphi(s)]\}^{\alpha-1} \left(\frac{P_x}{\alpha}\right)^{\alpha}\right\} \frac{LE}{P} \left(\frac{P}{P_i}\right)^{\varepsilon} - F_Y. \tag{18}$$

Profit maximization yields the anticipated price level

$$P_i = \frac{\varepsilon}{\varepsilon - 1} \{(1-\alpha) KE[\varphi(s)]\}^{\alpha-1} \left(\frac{P_x}{\alpha}\right)^{\alpha}. \tag{19}$$

Note also that $P_i = N^{\frac{1}{\varepsilon-1}} P, \forall i$. Substituting this in (18) gives

$$\pi_i = \frac{LE}{\varepsilon N} - F_Y. \tag{20}$$

Actual profits may be different across firms in *ex-post* terms, depending on the matching outcome with suppliers. The previous functions define just their anticipated value, used here to guide decisions in a simultaneous one-shot equilibrium. Accordingly, free entry pins down the expected number of final producers as

$$N = \frac{LE}{\varepsilon F_Y}. \tag{21}$$

2.4 Intermediate Producers

Production technology is linear in labor, following

$$X_k = \frac{1}{\mu} L_k. \tag{22}$$

Profits are

$$\pi_k = \int_0^N \left(P_{x_k} - \mu\right) x_{i,k} di - F_K, \tag{23}$$

where F_K are fixed costs expressed in labor units. Using (13), the optimal monopoly price is

$$P_{x_k} = \frac{\mu}{\alpha}. \tag{24}$$

Substituting this in the profit function, along with (13) and (14), results in

$$\pi_k = N \left\{\theta(N) \varphi(s_k|m) + [1 - \theta(N)] \varphi(s_k|\tilde{m})\right\} \Omega - F_K, \tag{25}$$

where

$$\Omega = \frac{\mu(1-\alpha)}{\alpha} \left[\frac{\alpha^2 L_i}{(1-\alpha)\mu Y_i}\right]^{\frac{1}{1-\alpha}} L_i. \tag{26}$$

With a sufficiently large number of suppliers, labor, price and output levels among final producers are treated as given, no matter what the matching outcome for one single input comes to be.

The equilibrium specificity choice (s_k^*) comes from (25) as

$$\theta(N) \left.\frac{d\varphi(s_k|m)}{ds_k}\right|_{s_k = s_k^*} + [1 - \theta(N)] \left.\frac{d\varphi(s_k|\tilde{m})}{ds_k}\right|_{s_k = s_k^*} = 0. \tag{27}$$

Next, (4) and (19) can be substituted in (16) to determine Ω, making use of the condition $Y_i = LC_i$. This leads to

$$\pi_k = \frac{LE(\varepsilon - 1)(1-\alpha)\alpha}{\varepsilon K} - F_K. \tag{28}$$

This equilibrium does not depend on variable s. That is a simple result of external effects generated by the combined actions of all intermediate producers. Individual supplier choices, taken isolatedly, are negligible enough so as not to carry any effect on downstream prices. However, once the aggregate and symmetric behavior of every supplier is considered, marginal costs are impacted, causing the price of final goods to adjust and cancel out any productivity changes derived from s.

Finally, a zero profit condition sets the anticipated number of suppliers as

$$K = \frac{LE\left(\varepsilon - 1\right)\left(1 - \alpha\right)\alpha}{\varepsilon F_K}. \tag{29}$$

2.5 Resource Constraint

Labor is required for upstream and downstream production, besides supporting fixed costs at both stages. Market clearing implies

$$K\mu\left(Nx_k\right) + KF_K + N\left(L_i + F_Y\right) = L. \tag{30}$$

Using (15), (16) and (21), in combination with (19) and the final goods market clearing condition, yields households expenditures as $E = 1$.

3. Results and Discussion

We now proceed to evaluate the effects of key parameters on productivity and marginal costs. These translate directly into pricing decisions, which ultimately help shape welfare outcomes.

Proposition 1 *A larger market size* (L) *decreases expected marginal production costs when* $\theta'(N) > 0$. *This effect is indeterminate when* $\theta'(N) < 0$, *but negative under the sufficient condition* $-\frac{\theta'(N)N}{\theta(N)} \leq 1$. *Higher entry costs for final producers* (F_Y) *increase expected marginal costs when* $\theta'(N) > 0$, *while decreasing them for* $\theta'(N) < 0$. *Finally, a higher price elasticity of demand* (ε) *has an indeterminate impact on expected marginal costs when* $\theta'(N) > 0$ *and a negative one when* $\theta'(N) < 0$. *The former is negative under the sufficient condition* $\frac{\theta'(N)N}{\theta(N)} \leq \frac{1}{\varepsilon - 1}$.

Proof. See appendix. ■

The scale effect associated with market size brings to mind the famous theorem advanced by Smith (1776), postulating that the division of labor is merely limited by the extent of the market. The resulting firm productivity gains have been explained in different ways in the literature. These have included, for instance, trade-induced vertical restructuring of production (McLaren, 2000), learning by exporting (De Loecker, 2013) or sequential production chains supported by specialized teams (Chaney and Ossa, 2013). In our case, a larger market leads to higher aggregate production, enabling more suppliers to break even and increasing the availability of capital goods. In other words, specialization on the extensive margin, K, is enhanced. This is a common marshallian externality, but one that is now reinforced by a similar gain on the intensive margin, $E(\varphi)$, when firms see their matching likelihood increase as a result of agglomeration effects, associated with $\theta'(N) > 0$. Even if that is not the case, productivity may still go up (and marginal costs down) under a sufficiently inelastic matching function. This implies that the drop in the frequency of good pairings, due to a higher number of alternate users, is relatively small. Under such conditions, the benefits of accessing more capital goods outweigh the loss from adopting less adequate solutions.

Set-up costs for final producers increase market concentration, but do not impact the number of suppliers. That happens because downstream aggregate output levels remain the same. In the absence of specificity choices, there would be no variation in productivity. That is no longer the case now. Depending on whether the matching likelihood increases or decreases as market structure changes, marginal production costs evolve in the opposite direction.

The more elastic demand for each variety is, the smaller mark-ups become. Lower equilibrium prices increase downstream production, along with demand for intermediate goods. As a result, more suppliers can be sustained, adding to the capital variety. This link between price elasticities of demand, competition toughness, higher firm size and the adoption of more productive technologies is also explored by Desmet and Parente (2010), for instance, in the context of internal research and development mechanisms. In addition, we find here that $E(\varphi)$ also goes up if the matching likelihood increases with the higher concentration now observed. Overall productivity gains might still be attained otherwise, provided that the odds of a good match decline only slightly. This constraint can be relaxed for lower price elasticities of demand, under which capital variety is limited to start with. The marginal benefit out of its expansion is then more pronounced and likely to offset any loss in capital specificity.

These varied effects are summed up by Table 1.

	$\theta'(N) > 0$		$\theta'(N) < 0$	
Comparative Statics	ΔK	$\Delta E(\varphi)$	ΔK	$\Delta E(\varphi)$
ΔL	+	+	+	-
ΔF_Y	0	+	0	-
$\Delta \varepsilon$	+	-	+	+

Table 1: Specialization Outcomes

Two points are worth highlighting. First, both types of specialization examined here generate conflicting outcomes in at least some of the comparative statics exercises, regardless of the assumption made on the matching function. Second, conclusions hinge, as well, on the properties of this function. In this respect, understanding how the dispersion of technical or locational characteristics evolves with the number of firms may provide additional insights on the behavior of $\theta(N)$. That is beyond the scope of this short note, though.

The changes in productivity and marginal production costs are passed along to consumers, bearing a direct effect on their welfare. Using the consumption aggregator (2), utility may be captured as the combined product of variety and quantity,

$$C = N^{\frac{1}{\varepsilon - 1}} T, \tag{31}$$

where $T = NC_i$ represents total consumption. A more elastic demand (ε) is associated with higher cross-product substitutability, which decreases the returns from variety.

The individual consumption for each good can be determined from equations 19, 24, 21 and 29 (with $E = 1$) as

$$C_i = \Psi \left\{ \frac{(\varepsilon - 1) E [\varphi(s^*)]}{\varepsilon F_K} \right\}^{1-\alpha} \frac{F_Y (\varepsilon - 1)}{L^\alpha}, \tag{32}$$

where

$$\Psi = \frac{(1-\alpha)^{2(1-\alpha)} \alpha^{1+\alpha}}{\mu^\alpha}. \tag{33}$$

Applying this to the consumption aggregator yields the utility flow

$$C = \Psi \left(\frac{L}{\varepsilon F_Y} \right)^{\frac{1}{\varepsilon - 1}} \left\{ \frac{L (\varepsilon - 1) E [\varphi(s^*)]}{\varepsilon F_K} \right\}^{1-\alpha} \frac{\varepsilon - 1}{\varepsilon}. \tag{34}$$

These results may now be used to evaluate the welfare effects of different structural parameters.

Proposition 2 *A larger market size* (L) *increases consumer welfare when* $\theta'(N) > 0$ *and has an indeterminate impact when* $\theta'(N) < 0$. *The effect is positive under the sufficient condition* $-\frac{\theta'(N)N}{\theta(N)} \leq \frac{1}{(\varepsilon-1)(1-\alpha)} + 1$. *Higher entry costs for final producers* (F_Y) *decrease consumer welfare when* $\theta'(N) > 0$ *and have an indeterminate impact when* $\theta'(N) < 0$. *The effect is negative under the sufficient condition* $-\frac{\theta'(N)N}{\theta(N)} \leq \frac{1}{(\varepsilon-1)(1-\alpha)}$. *Finally, a higher price elasticity of demand* (ε) *has an indeterminate impact on consumer welfare for any sign of* $\theta'(N)$.

Proof. See appendix. ■

A larger market size brings more final producers into the market (the term N in equation 31) and expands the range of varieties available to households. In addition, total consumption (the term T in equation 31) also increases when $\theta'(N) > 0$. This is supported by dual specialization gains, which push down marginal production costs and final prices (see Proposition 1). The welfare outcome becomes uncertain when $\theta'(N) < 0$ due to the ambiguous effect observed in marginal costs. If the expected level of specialization on the intensive margin declines by too much in response to a higher number of downstream producers, prices may conceivably increase high enough so as to diminish aggregate consumption. That is however not the case when the matching function is relatively more inelastic. The sufficient condition for welfare gains under this scenario is less restrictive than the one required for a simple reduction of marginal costs, since these can now be reinforced by positive returns from variety. That condition is also more likely to hold the lower the price elasticity of demand is. Reduced substitutability attaches more weight in welfare calculations to a rise in the number of available varieties (see equation 31). Furthermore, as noted before, any loss on the intensive margin is more easily traded-off against complementary gains on the extensive margin when demand is inelastic.

We now turn to the effect of sunk costs for final producers. These discourage market entry and imply lower returns from variety. Without specialization on the intensive margin, no changes would occur in aggregate consumption, given that the drop in firm numbers is exactly offset by larger market shares and output for each one of them. If instead we now have $\theta'(N) > 0$, the odds of benefitting from a supplier's tailoring decision fall, while intermediate goods also become more generic. The resulting decline in productivity and rise in marginal costs (see Proposition 1) pulls down aggregate consumption, amplifying the consumer welfare loss. This conclusion might potentially be reversed in case $\theta'(N) < 0$. A sufficiently elastic matching function would afford easier access to tailored inputs due to greater downstream concentration. The benefits yielded over productivity and aggregate consumption could then countervail the drop in the number of varieties and generate higher consumer welfare. This event becomes less feasible, though, when price elasticities are lower, for similar reasons to those described in the analysis of market size.

Lastly, a higher price elasticity of demand prompts conflicting effects. This reduces markups for final producers, pushing some of them out of the market. Fewer and more substitutable varieties curtail the welfare returns on this dimension (that is, the term $N^{\frac{1}{\varepsilon-1}}$ in equation 31 decreases). On the other hand, specialization displays opposing variations on each of its margins when $\theta'(N) > 0$, rendering the aggregate consumption outcome uncertain (the term T in equation 31). This consumption may conceivably increase in the event that $\theta'(N) < 0$, due to unambiguous productivity gains, but the overall welfare assessment remains ambiguous.

The results just presented may be summarized by Table 2 below.

	Benchmark Dixit-Stiglitz	Extended Dixit-Stiglitz	
	No $\theta(N)$	$\theta'(N) > 0$	$\theta'(N) < 0$
$\partial C/\partial L$	$+$	$+$	indeterminate
$\partial C/\partial F_Y$	-	-	indeterminate
$\partial C/\partial \varepsilon$	indeterminate	indeterminate	indeterminate

Table 2: Welfare Outcomes

4. Conclusion

This paper introduced a measure of capital fitness into a model of monopolistic competition with love of variety preferences. The model offers aggregate trends, overlapping two concepts of specialization: one is quantitative, reflecting the number of available capital goods from which a finer division of tasks is enabled, while the other is qualitative, capturing the adequacy of each input to differentiated users. One key finding is that regardless of the assumption made on the matching function behavior, specialization vectors are bound to evolve in opposite directions in some of the comparative statics exercises. Recall that in a traditional Dixit-Stiglitz-Ethier formulation the productivity and welfare effects induced by parameters like market size or sunk costs are consistently monotonic. A finer characterization of capital specialization is shown here to introduce ambiguity in this assessment. In this sense, aggregate models relying on expanding varieties of capital goods as the engine of productivity change, as observed in some growth, urban and regional economics frameworks, may be lacking one relevant dimension.

Appendix

Proof of Proposition 1

Substituting (24) and (29) in (17), expected marginal cost becomes

$$MC = \left\{ \frac{(1-\alpha)^2 L\alpha (\varepsilon - 1)}{\varepsilon F_K} E\left[\varphi(s)\right] \right\}^{\alpha - 1} \left(\frac{\mu}{\alpha^2} \right)^{\alpha}. \tag{35}$$

$E\left[\varphi(s)\right]$ is defined by (14) and evaluated at equilibrium. Applying the envelope theorem and using (21) results in

$$\frac{\partial E\left[\varphi(s)\right]}{\partial L} = \frac{1}{\varepsilon F_Y} \theta'(N) \left[\varphi(s|m) - \varphi(s|\tilde{m})\right]. \tag{36}$$

Additional manipulation yields

$$\frac{\partial MC}{\partial L} = \Gamma \left\{ \left[\frac{\theta(N)}{L} + \frac{\theta'(N)}{\varepsilon F_Y} \right] \left[\varphi(s_k^*|m) - \varphi(s_k^*|\tilde{m})\right] + \frac{\varphi(s_k^*|\tilde{m})}{L} \right\}, \tag{37}$$

$$\Gamma = \frac{\alpha - 1}{E\left[\varphi(s^*)\right]} \left\{ \frac{(1-\alpha)^2 L\alpha(\varepsilon - 1)}{\varepsilon F_K} E\left[\varphi(s^*)\right] \right\}^{\alpha - 1} \left(\frac{\mu}{\alpha^2} \right)^{\alpha}. \tag{38}$$

Given that $\Gamma < 0$ and $\varphi(s_k^*|m) > \varphi(s_k^*|\tilde{m})$, it follows that $\frac{\partial MC}{\partial L} < 0$ when $\theta'(N) > 0$. If $\theta'(N) < 0$, the sign of $\frac{\partial MC}{\partial L}$ is indeterminate. Using (21), a sufficient condition for $\frac{\partial MC}{\partial L} < 0$ is

$$-\frac{\theta'(N) N}{\theta(N)} \leq 1. \tag{39}$$

Concerning F_Y,

$$\frac{\partial E\left[\varphi(s)\right]}{\partial F_Y} = -\frac{L}{\varepsilon F_Y^2} \theta'(N) \left[\varphi(s|m) - \varphi(s|\tilde{m})\right]. \tag{40}$$

This leads to

$$\frac{\partial MC}{\partial F_Y} = -\Gamma \frac{L}{\varepsilon F_Y^2} \theta'(N) \left[\varphi(s_k^*|m) - \varphi(s_k^*|\tilde{m})\right]. \tag{41}$$

It is clear that $\frac{\partial MC}{\partial F_Y} > 0$ when $\theta'(N) > 0$ and $\frac{\partial MC}{\partial F_Y} < 0$ when $\theta'(N) < 0$.

The effect of ε comes through

$$\frac{\partial E\left[\varphi(s)\right]}{\partial \varepsilon} = -\frac{L}{\varepsilon^2 F_Y} \theta'(N) \left[\varphi(s|m) - \varphi(s|\tilde{m})\right]. \tag{42}$$

Next,

$$\frac{\partial MC}{\partial \varepsilon} = \Gamma \left\{ \begin{array}{c} \frac{\left[\varepsilon F_Y \theta(N) - L(\varepsilon - 1)\theta'(N)\right]\left[\varphi\left(s_k^*|m\right) - \varphi\left(s_k^*|\tilde{m}\right)\right]}{\varepsilon^2 (\varepsilon - 1) F_Y} \\ + \frac{\varphi\left(s_k^*|\tilde{m}\right)}{\varepsilon(\varepsilon - 1)} \end{array} \right\}. \tag{43}$$

It is clear that $\frac{\partial MC}{\partial \varepsilon} < 0$ when $\theta'(N) < 0$. The result is indeterminate when $\theta'(N) > 0$. A sufficient condition for $\frac{\partial MC}{\partial \varepsilon} < 0$ in such case is

$$\frac{\theta'(N)\,N}{\theta(N)} \leq \frac{1}{\varepsilon - 1}. \tag{44}$$

Proof of Proposition 2

The partial effect of market size on welfare is determined from equation (34). Using also equation (36), we obtain

$$\frac{\partial C}{\partial L} = \Lambda\left[1 + (\varepsilon - 1)(1-\alpha)\right] \times \tag{45}$$

$$\left\{ \frac{\left[\varepsilon F_Y \theta(N) + \frac{(\varepsilon-1)(1-\alpha)}{1+(\varepsilon-1)(1-\alpha)} L\theta'(N)\right]\left[\varphi(s|m) - \varphi(s|\tilde{m})\right] + \varepsilon F_Y \varphi(s|\tilde{m})}{\varepsilon F_Y E\left[\varphi(s)\right]} \right\}, \tag{46}$$

with

$$\Lambda = \frac{\Psi}{\varepsilon L}\left(\frac{L}{\varepsilon F_Y}\right)^{\frac{1}{\varepsilon-1}} \left\{\frac{L(\varepsilon-1)E\left[\varphi(s)\right]}{\varepsilon F_K}\right\}^{1-\alpha}. \tag{47}$$

It can easily be shown that $\frac{\partial C}{\partial L} > 0$ when $\theta'(N) > 0$. The result is however indeterminate when $\theta'(N) < 0$. In such a case, a sufficient condition for $\frac{\partial C}{\partial L} > 0$ is to have the coefficient of $\varphi(s|m) - \varphi(s|\tilde{m})$ be positive, that is,

$$-\frac{\theta'(N)\,N}{\theta(N)} \leq \frac{1}{(\varepsilon-1)(1-\alpha)} + 1. \tag{48}$$

Comparative statics with respect to F_Y results in

$$\frac{\partial C}{\partial F_Y} = -\frac{\Lambda L}{F_Y}\left\{ \frac{\left[\theta(N) + \frac{(\varepsilon-1)(1-\alpha)}{\varepsilon F_Y} L\theta'(N)\right]\left[\varphi(s|m) - \varphi(s|\tilde{m})\right] + \varphi(s|\tilde{m})}{E\left[\varphi(s)\right]} \right\}. \tag{49}$$

It is clear that $\frac{\partial C}{\partial F_Y} < 0$ when $\theta'(N) > 0$. The sign is once again uncertain when $\theta'(N) < 0$. As in the previous case, a sufficient condition for $\frac{\partial C}{\partial F_Y} < 0$ is to have the coefficient of $\varphi(s|m) - \varphi(s|\tilde{m})$ be positive, that is,

$$-\frac{\theta'(N)\,N}{\theta(N)} \leq \frac{1}{(\varepsilon-1)(1-\alpha)}. \tag{50}$$

Finally, the marginal effect of ε is

$$\frac{\partial C}{\partial \varepsilon} = \Lambda L \left\{ \begin{array}{c} 1 - \alpha - \log\left(\frac{L}{\varepsilon F_Y}\right)^{\frac{1}{\varepsilon-1}} \\ -\frac{(1-\alpha)(\varepsilon-1)L\theta'(N)\left[\varphi(s|m)-\varphi(s|\tilde{m})\right]}{\varepsilon^2 F_Y E\left[\varphi(s)\right]} \end{array} \right\}. \tag{51}$$

The sign of this derivative is uncertain, depending on the rate of change of its multiple terms.

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