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Dynastic Altruism, Population, and R&D based Growth

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Abstract

We show that non-linear dynastic altruism toward future generations yields a non-monotonic relation between population growth and economic prosperity, which is polynomial in general. The exact shape of this non-monotonic relation depends on the concavity of parental altruistic utility. Hence, this work contributes to a recent line of modified R&D-based growth models aimed at aligning theory with empirical evidence on the non-linear relation between population growth and economic prosperity.

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1 Introduction

This note contributes to a recent line of modified R&D-based growth models aimed at aligning theory with the ambiguous empirical findings regarding the relations among fertility, innovation, and economic growth[1]. We show that when dynastic altruism follows Becker and Barro's (1988) specification, the relation between population growth rate and economic prosperity may have a humped shape, which is consistent with the empirical findings reported by Boikos et al.(2013) and Kelley and Schmidt (1995).

Recent modifications to R&D-based growth models have aimed to remove the so-called "weak scale effect", which was present in second- and third-generation models – i.e., the counterfactual positive relation between population growth and economic growth[2]. This line of research has incorporated human capital as a productive input in the R&D sector, thereby creating room for substitution between the quantity and quality of workers in the overall labor supply. That is, the effective labor supply can increase and enhance growth even if the working population is constant or declining.

Several works in this literature have emphasized the role of dynastic altruism toward future generations in determining the effect of population growth on economic prosperity; see, for example, Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2008), and Bucci (2013)[3]. These studies show that dynastic altruism stimulates saving and investment in human capital. This positive effect of altruism on saving is increasing with the fertility rate[4] and may overcome the negative diluting effect of population growth on human capital accumulation[5]. In these studies, parents' altruistic utility is linear in the fertility rate (for a given per child consumption level), and the effect of population growth on technological progress depends on the values of model parameters – i.e., it is monotonic given the parameter set.

We depart from this literature by introducing nonlinear parental altruism in the number of offspring, following the influential papers on fertility and economic growth by Barro and Becker (1989) and Becker et al. (1990). In their studies, as in our analysis, parents' "selfish" utility – i.e., that from their own consumption – is higher than their altruistic utility – i.e., that from the consumption of their offspring – and the degree of parental altruism for each child is decreasing with the number of children[6].

Our analysis extends Young's (1998) two-sector R&D model by incorporating population growth, endogenous human capital accumulation, and dynastic altruism. In an earlier related study, Diwakar and Sorek (2016), we establish similar non-monotonic relations between fertility and R&D-

[1] Recent summaries of the empirical literature can be found in Strulik et al. (2013) and Bucci (2015).

[2] See Jones (1999) for a compact summary of this literature.

[3] In Bucci (2008) and Bucci (2013), the total effect of population growth on economic prosperity also depends on the effect of technological progress on human capital accumulation and on the returns to specialization.

[4] We identify the population growth rate, which is exogenous throughout our analysis, with the fertility rate.

[5] If human capital is not purely non-rival, population growth works to decrease per capita human capital, as the human capital of new borns is lower than the average of existing workers.

[6] The microeconomic foundations of these works were laid in Becker and Barro (1988), and their broader implications to economic growth were summarized in Becker (1992).

based growth due to human capital spillovers from parents to their offspring that are subject to congestion in the number of children.

The remainder of this paper is organized as follows. Section 2 presents the detailed model. Section 3 analyzes the dynamic equilibrium and the effect of population growth on technological progress, and Section 4 concludes this study.

2 The Model

We extend Young's (1998) two-sector R&D model by incorporating population growth, human capital accumulation, and dynastic altruism. Time is discrete, and population grows at exogenous rate $n \geq 0$. Population size in each period is denoted $L_t = L_0(1+n)^t$, where L_0 is normalized to one. In each period, each worker is endowed with one unit of time.

2.1 Preferences

The consumer's lifetime utility is given by

$$U = \sum_{t=0}^{\infty} \rho^t (1+\theta n)^t \ln(c_t) \tag{1}$$

where $\rho, \theta \in (0,1)$ are the time preference and degree of altruism, respectively. The current literature is focused on the linear specification of the altruism factor, implying that θ is scalar; See, for example, Strulik (2005), Bucci (2008, 2013)[7]. Here, we let the degree of altruism per child depend on the number of offspring, that is, on $\theta \equiv \theta(n)$. Following Barro and Becker (1988, 1989), Becker et al.(1990) and Becker (1992), we assume $\theta(n) = \theta_0 n^{-\gamma}$; hence, $\theta(n)\, n = \theta_0 n^{1-\gamma}$, where $\gamma, \theta_0 \in (0,1)$. The assumption $\theta_0 < 1$ implies that parents' "selfish" utility from their own consumption has a higher weight than their altruistic utility from per-child consumption, which is in line with the latter references. To ensure that (1) has finite values, we also assume $\rho(1+\theta n) < 1$. The per capita utilization level of consumption in (1), denoted c, is derived from M differentiated products (i.e., varieties), denoted c_i, subject to a CES utility function

$$c_t = \left(\sum_{i=1}^{M_t} c_{i,t}^{\frac{1}{\varepsilon}} \right)^{\varepsilon} \tag{1a}$$

with $\varepsilon = \frac{s}{s-1}$, and s is the elasticity of substitution across all varieties. The consumption level of each variety is defined as $c_i = q_i x_i$, where x_i and q_i designate utilized quantity and quality, respectively.

The assumed preferences imply the following instantaneous demand for each variety

$$x_{i,t}^d = q_{i,t}^{s-1} \left(\lambda p_{i,t}\right)^{-s} \left(\sum_{i=1}^{M_t} c_{i,t}^{\frac{1}{\varepsilon}} \right)^{\varepsilon} \tag{1b}$$

[7] At the two extremes, $\theta = 0$ or $\theta = 1$, preferences are of Millian or Benthamite type, respectively.

where λ is the Lagrange multiplier from the instantaneous utility maximization, i.e., the shadow value of the given periodic spending level. The logarithmic specification in (1) implies the standard Euler condition for optimal consumption smoothing, written in terms of aggregate spending and denoted E

$$\frac{E_{t+1}}{E_t} = \rho(1+\theta n)(1+r_{t+1}) \tag{2}$$

where $(1+r_{t+1})$ is the (gross) interest rate earned between periods t and $t+1$.

2.2 Production and innovation

The effective labor supply is the sole input for production and innovation, and the wage rate is normalized to one. One unit of labor produces one unit of consumption good (regardless of its quality). Following Young (1998), innovation is certain and is subject to the following cost function

$$f(q_{i,t+1}, \overline{q}_t) = \begin{cases} \exp\left(\phi \frac{q_{i,t+1}}{\overline{q}_t}\right) & q_{i,t+1} > q_{i,t} \\ \exp(\phi) & q_{i,t+1} \leq q_{i,t} \end{cases} \tag{3}$$

The innovation cost in sector i is increasing in the rate of quality improvement over the existing quality frontier – denoted $\overline{q}_t$, which is the highest quality already attained in the economy. As innovation is certain, vertical innovation (i.e., quality improvements) implies that the effective lifetime of each product is one period. Hence, each firm maximizes the following profit, denoted Π

$$\Pi_{i,t} = \frac{(p_{i,t+1}-1)x^d_{i,t+1}L_{t+1}}{1+r_{t+1}} - f(q_{i,t+1}, \overline{q}_t) \tag{4}$$

Maximizing (4) for $p_{i,t+1}$ yields the standard optimal monopolistic price[8] $p^* = \varepsilon, \forall t, i$, and the first-order condition for optimal quality choice, combined with the free-entry (zero-profit) condition, yields the equilibrium rate of quality improvement $\forall_i : 1+g_q \equiv \frac{q^*_{t+1}}{\overline{q}_t} = \frac{s-1}{\phi}$. To enhance exposition, hereafter, we denote $f(q^*_{i,t+1}, \overline{q}_t) \equiv f$. Notice that under a symmetric equilibrium, demand for each variety is $x^d_t = \frac{E_t}{\varepsilon M_t L_t} \forall_i$, and thus, imposing the free entry condition on (4) implies

$$\frac{(1-\frac{1}{\varepsilon})\frac{E_{t+1}}{M_t+1}}{f} = 1+r_{t+1} \tag{5}$$

Combining (2) and (5) we obtain

$$E_t = \frac{fM_{t+1}}{(1-\frac{1}{\varepsilon})\rho(1+\theta n)} \tag{6}$$

and plugging (6) back into (5) yields the interest rate

$$1+r_{t+1} = \frac{1+g_{M,t+1}}{\rho(1+\theta n)} \tag{7}$$

[8] The asterisk superscript denotes optimally chosen values.

where $1+g_{M,t+1} \equiv \frac{M_{t+1}}{M_t}$.

2.3 Human capital formation

Human capital formation is subject to the conventional specification[9]

$$\begin{aligned} h_{t+1} &= \frac{(\xi e_t + 1 - \delta)\, h_t}{(1+n)} \\ &\Rightarrow \quad \triangle h_{t+1} \equiv h_{t+1} - h_t = \left[\frac{(\xi e_t + 1 - \delta)}{(1+n)} - 1\right] h_t \end{aligned} \tag{8}$$

where h is per capita human capital, and $e \in (0,1)$ is the time invested in human capital formation. The effective labor supply, denoted H, is defined as the product of population size and per capita human capital: $H_t = L_t h_t$. Following (8), we define the growth rate of per-capita human capital $(1+g_{h,t+1}) \equiv \frac{h_{t+1}}{h_t} = \frac{(\xi e_t + 1 - \delta)}{(1+n)}$ and the growth rate of effective labor supply

$$1 + g_{H,t+1} \equiv \frac{H_{t+1}}{H_t} = (1 + g_{h,t})\,(1+n) = (\xi e_t + 1 - \delta) \tag{9}$$

The return on investment in human capital should equal the return on R&D investment

$$1 + r_{t+1} = \frac{(\xi e_t + 1 - \delta)\, h_t}{e_t h_t} \tag{10}$$

Plugging the interest rate (7) in (10) yield time investment in education

$$\forall t : e^* = \frac{(1-\delta)}{\frac{(1+g_{M_{t+1}})}{\rho(1+\theta n)} - \xi} \tag{11}$$

3 Population Growth and Economic Prosperity

Our analysis is confined to the stationary (steady-state) equilibrium, implying that the growth rates of all variables are time invariant. Plugging (11) back into (9) yields

$$1 + g_H = \frac{(1-\delta)}{1 - \frac{\rho(1+\theta n)\xi}{1+g_M}} \tag{12}$$

The aggregate resources use constraint for the economy is defined by the allocation of the labor supply across production, education, and R&D investment

$$(1 - e^*)\, H_t = \frac{E_t}{\varepsilon M_t} + f M_{t+1} \tag{13}$$

Plugging (6) into (13) yields

[9] For $\delta, n = 0$ this formulation coincides with Lucas (1988).

$$
\begin{aligned}
\left(1-e^{*}\right) H_t &= \frac{f M_{t+1}}{(\varepsilon-1)\,(1+\theta n)\rho} + f M_{t+1} \\
&\Rightarrow \quad M_{t+1} = \frac{\left(1-e^{*}\right) H_t}{f\left[\frac{1}{(\varepsilon-1)(1+\theta n)\rho}+1\right]}
\end{aligned} \tag{14}
$$

Hence, the variety expansion rate equals the growth rate of the effective labor supply, that is, $(1+g_M)=(1+g_H)$, which following (9)–(10), implies

$$
1+g_M = (1-\delta)+\xi\rho(1+\theta n) \tag{15}
$$

Observe that under a symmetric equilibrium, equation (1a) can be written as

$$
c_t = \left(\sum_{i=1}^{M_t}\left(q_t x_t\right)^{\frac{1}{\varepsilon}}\right)^{\varepsilon} = M_t^{\varepsilon} q_t x_t = M_t^{\varepsilon} q_t \frac{E_t}{L_t M_t \varepsilon}
$$

Plugging (6) into the above expression yields the following (stationary) per capita consumption growth rate

$$
1+g_c \equiv \frac{c_t}{c_{t-1}} = \frac{L_{t-1} M_t^{\varepsilon-1} q_t M_{t+1}}{L_t M_{t-1}^{\varepsilon-1} q_{t-1} M_t} = \frac{\left(1+g_q\right)\left(1+g_M\right)^{\varepsilon}}{1+n} \tag{16}
$$

Which can be also written as

$$
1+g_c = \frac{\left(1+g_q\right)\left[(1-\delta)+\xi\rho(1+\theta\,(n)\,n)\right]^{\varepsilon}}{1+n} \tag{16a}
$$

Plugging the explicit expression for $\theta\,(n)$ into (16a) and then differentiating for n shows that the sign of $\frac{\partial g_c}{\partial n}$ depends on the sign of $\frac{\varepsilon(1-\gamma)\theta_0 n^{-\gamma}(1+n)}{\frac{(1-\delta)}{\xi\rho}+1+\theta_0 n^{1-\gamma}} - 1$. The latter expression is positive (negative) if the following (reverse) inequality holds

$$
\varepsilon\,(1-\gamma)\,n^{-\gamma} - \left[1-\varepsilon\,(1-\gamma)\right] n^{1-\gamma} > \frac{1}{\theta_0}\left(\frac{1-\delta}{\xi\rho}+1\right) \tag{17}
$$

Proposition 1 *For a sufficiently large γ, the relation between g_c and n is* hump shaped.

Proof. If γ is large enough to ensure that $\varepsilon\,(1-\gamma) < 1$, the left side of (17) is decreasing with n: starting from plus infinity for $n \to 0$, and becoming negative for $n > \frac{\varepsilon(1-\gamma)}{1-\varepsilon(1-\gamma)}$. Hence, for $\varepsilon\,(1-\gamma) < 1$, the sign of $\frac{\partial g_c}{\partial n}$ is positive (negative) for low (high) fertility rates, and thus, $g_c(n)$ is hump shaped. ∎

Following (17), with $\varepsilon\,(1-\gamma) \leq 1$ (i.e., $\gamma \geq \frac{1}{s}$), the per capita consumption growth rate is maximized for $n = \frac{\varepsilon(1-\gamma)}{1-\varepsilon(1-\gamma)}$. As γ increases (decreases), the range of n for which $\frac{\partial g_c}{\partial n} > 0$ is shrinking (widening). The result presented in proposition 1 summarizes the total impact of the two contradictory effects of population growth on economic work presented in equation (16a):

the numerator shows the positive effect of population growth on savings and investment in the presence of altruism, i.e., for any $\theta(n) > 0$, as presented in equations (11)–(12). The denominator in (16) shows the standard diluting effect of population growth on human capital accumulation and, thereby, on economic growth presented in equation (8). However, under the current specification, the positive effect of altruism on growth depends on the rate of population growth, as $\frac{\partial \theta(n) n}{\partial n} = \frac{\partial \theta_0 n^{1-\gamma}}{\partial n} = (1-\gamma)\theta_0 n^{-\gamma}$. Hence, for high (low) levels of n, the positive (negative) effect dominates the overall impact of population growth on economic growth. This result is similar to the one we presented in Diwakar and Sorek (2016), albeit through a different mechanism, which is congestion in dynastic spillovers of human capital.

4 Conclusion

This study contributes to the literature on the role of population in R&D-driven growth by adding to a few recent studies that have established non-monotonic relations between population growth and economic prosperity. We have demonstrated that if parental altruism toward each child is decreasing with the number of offspring, a hump-shaped relation between population growth and economic prosperity may arise, which is consistent with the aforementioned empirical findings.

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