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Decreasing Transaction Costs and Endogenous Fluctuations in a Monetary Model

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Abstract

We study an infinite horizon economy with a representative agent whose utility function includes consumption, real balances and leisure. Real balances enter the utility function pre-multiplied by a parameter reflecting the inverse of the degree of financial market imperfection, i.e. the inverse of the transaction costs justifying a positively valued fiat money. Indeterminacy arises both through a transcritical and a flip bifurcation: somewhat paradoxically, the amplitude of the indeterminacy region improves as soon as the degree of market imperfection is set lower and lower. Such results are robust with respect to the choice for the elasticity of the labor supply, both when the latter is set close to zero and to infinite. We also provide conditions for the existence, uniqueness and multiplicity of the steady states and finally, we asses the impact of the degree of market imperfection on the occurrence of such phenomena

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Abstract

We study an infinite horizon economy with a representative agent whose utility function includes consumption, real balances and leisure. Real balances enter the utility function pre-multiplied by a parameter reflecting the inverse of the degree of financial market imperfection, i.e. the inverse of the transaction costs justifying the introduction of money in the utility function. When labor is supplied elastically, indeterminacy arises through a transcritical and a flip bifurcation, for degree of financial imperfection arbitrarily close to zero. Similar results are observed when labor is supplied inelastically: indeterminacy occurs through a flip bifurcation for values of the degree of financial imperfection unbounded away from zero. We also study the existence and the multiplicity of the steady states.

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1 Introduction

It is well known that growth models may be locally indeterminate, i.e. they may possess a steady state such that the equilibrium dynamics converges toward it for infinitely many choices for the initial conditions, when some market imperfection is accounted for. This is the case when one assumes some external effects in production not mediated by markets, when participation to the latter is restricted, as it is the case in *OLG* models, or as a consequence of the frictions due to the introduction of money as a medium of exchange. When an equilibrium is indeterminate, it is possible to construct nearby stochastic sunspot equilibria, i.e. equilibria representing optimal responses to agents' revisions in their beliefs (see, among others, Woodford (1986)).

The extension of the standard neoclassical growth models effectuated in order to account for the role played by money includes the seminal Sidrauski (1967), Brock (1974) and Brock (1975) models in which real balances enter into the instantaneous utility function besides consumption good and, possibly, leisure. The inclusion of real balances in the utility function is motivated by the perception of money as one of many assets (some financial, some real) and the indirect utility money provides stemming from the liquidity allowing to carry out transactions and the possibility to save time compared to being illiquid. The money-in-the-utility function approach can be viewed indeed as a device to account for the liquidity services money provides by reducing the transaction costs (e.g. shopping time models): as it is shown in Feenstra (1986), each transaction cost formulation is equivalent to an appropriate money-in-the-utility function framework. Also the cash-in-advance constraint approach (see, among others, Clover (1967)) which compels agents to purchase the consumption good out of money balances previously accumulated, falls within a specification of the utility function in which consumption and real balances are assumed to be perfect complements. Another approach, as in Benhabib and Farmer (2000), supposes that money reduces the selling costs and can then be viewed as a productive input besides, e.g., physical capital. They show that a sufficient condition to get indeterminacy is money to be sufficiently productive. In absence of transaction or selling costs, money can be positively valued only in *OLG* models (see, among others, Grandmont (1985)) where *fiat* money is delivered from the old generation to the young one and can be therefore viewed as a speculative bubble, Tirole (1985)).

Brock (1974) and Brock (1975) are the first to discuss the possibility of indeterminacy in a model that includes real balances in the utility function. He shows how self-fulfilling hyperinflationary and deflationary equilibria can occur and under which conditions to rule out such phenomena. Calvo (1979), within a money-in-the-utility function approach, shows that indeterminacy is most easily obtained when changes in the stock of real balances have large effects on output, meanwhile Matsuyama (1991) proves that cyclical and even chaotic equilibria can be obtained when the rate of money growth supply is set high enough. Farmer (1997) considers a *RBC* model with money-in-the-utility function and calibrates the model to fit the first moments of *U.S*. by choosing a parametrization of the utility for which the model admits the existence of indeterminate equilibria in order to explain the monetary propagation mechanism. The mechanism that generates indeterminacy is that a small increase in real balances must be associated with a big increase, at equilibrium, of labor allocated to production.

The goal of our paper is to enrich further the study of the dynamic properties of economies with positively valued *fiat* money. To this end, we assume an infinitely lived representative agent whose utility function is defined over consumption, real balances and leisure and that, besides money issued by the Central Bank, accumulates safe government bonds. The main feature of our model consists in introducing in the utility function defined over real balances a scaling parameter included between zero and infinite. Such a parameter can be viewed as the inverse of the degree of financial market imperfection: a larger value of the former reflects the existence of liquidity services provided by money very sensitive to the amount of the real balances held. Equivalently, increasing the value of such a parameter is consistent with the hypothesis of diminishing transaction costs and thus, *ceteris paribus*, of a larger marginal utility of real balances.

We show that indeterminacy may occur for a wide range of the degree of financial market imperfection: even when the former is set arbitrarily close to zero, sunspot fluctuations do emerge. As a matter of fact, indeterminacy appears first through a transcritical bifurcation and then through a flip one or only through the latter. Quite surprisingly, we show that the scope of indeterminacy improves as soon as the degree of financial market imperfection is progressively relaxed. This seems to confirm the results of Bosi *et al.* (2005) according to which, within a partial cash-in-advance economy, the plausibility of indeterminacy increases as soon as the amplitude of the liquidity constraint is set lower and lower. The underlying mechanism leading to this result is, roughly, the following: assuming for sake of simplicity an inelastic labor supply, let us suppose that the system is at the beginning at its steady state and that agents anticipate, say, an increase in the price level of the following period. Accordingly, they will react by decreasing the desiderated amount of money balances held at the end of the foregoing period and by increasing the current amount of nominal balances. But, if money balances are not too substitutable across time and if such an effect is magnified by the lower degree of financial imperfection, the investment in money balances will decrease only lightly. At the same time, the current price level will decrease sharply in order to re-establish equilibrium in money market, giving rise to an oscillatory dynamics with decreasing amplitudes and therefore converging back to the stationary solution.

We also carry out a complete steady state analysis in terms of existence, uniqueness and multiplicity. More in details, we provide sufficient conditions for the existence of exactly two stationary solutions and we prove that when such conditions are violated, uniqueness of the steady state, generically, is bound to prevail.

2 The model

We consider an infinite horizon discrete time economy populated by the government, the Central Bank, a large number of infinitely-lived households and a representative firm. In the sequel, we will describe the government and the Central Bank goal, the household behavior and the technology of the firm.

2.1 The Government and the Central Bank

Let us first consider the government behavior. We assume that there is no public spending and no taxes. Nevertheless, the government issues nominal bonds denoted by B_t^G and that in period 0 the nominal debt is equal to B_0^G. It follows that nominal bonds evolve through times according to

$$B_{t+1}^G = (1 + i_t)B_t^G \tag{1}$$

where i_t is the nominal interest rate. Let p_t be the price of the unique consumption good. Setting the real debt $b_t^G = B_t^G/p_t$, we have that its dynamics is given by

$$b_{t+1}^G = \frac{p_t}{p_{t+1}}(1 + i_t)b_t^G \tag{2}$$

The Central Bank issues money against the purchase of government bonds through open market operations. Denoting B_{t+1}^{CB} the amount of nominal government bonds purchased by the Central Bank in period t and M_{t+1} the stock of nominal balances available in the economy at the outset of period t, the dynamic budget constraint of the Central Bank is

$$B_{t+1}^{CB} = (1 + i_t)B_t^{CB} + M_{t+1} - M_t \tag{3}$$

which, setting $m_t \equiv M_t/p_t$ the real balances available at the beginning of period t, can be written in real terms as

$$b_{t+1}^{BC} = \frac{p_t}{p_{t+1}}(1 + i_t)b_t^{BC} + m_{t+1} - m_t\frac{p_t}{p_{t+1}}. \tag{4}$$

The Central Bank in each period creates or withdraws money by means of lump-sum transfers to the households. We denote the total supply of money in period t by M_t and the constant rate of money creation $\hat{\sigma}_t$. Thus we have

$$M_t = (1 + \hat{\sigma})M_{t-1} = \sigma M_{t-1}. \tag{5}$$

Let T_t be the lump-sum transfers received by the agents at period t as a consequence of money creation or withdrawal. It follows that

$$T_t = \hat{\sigma}M_{t-1}. \tag{6}$$

2.2 Households

The preferences of the representative agent are described by the following intertemporal utility function:

$$\sum_{t=0}^{+\infty}\beta^t\left[u\left(c_t, \mu\frac{M_t}{p_t}\right) - Av\left(l_t\right)\right] \tag{7}$$

where c_t is the unique consumption good, M_t the money balances held at the beginning of period t, l_t the supply of labor, p_t the price of consumption good and $\beta \in (0, 1)$ the discount factor. We assume in addition that real money balances enter the utility function in view of their liquidity services premultiplied by the parameter $\mu > 0$. Such parameter can be view as the inverse of the degree of financial market imperfection: a larger value of μ reflects, indeed, underlying diminishing transaction costs and as a consequence, for a given level of real balances, a larger marginal utility of money. Finally, $A > 0$ is a scaling parameter that will allow us to calibrate the model. When maximizing (7) agents must respect the dynamic budget constraint

$$p_t c_t + M_{t+1} + B_{t+1} = M_t + (1 + i_t) B_t + w_t l_t + T_t \tag{8}$$

where B_t denotes the safe nominal bonds held by the representative agent and w_t the nominal wage. The initial endowment of bonds held by the representative household is denoted B_0 and the initial amount of his nominal balances is denoted M_0. We assume that the instantaneous utility function of the representative agent satisfies the following standard Assumption.

Assumption 1. *$u(c, \mu m)$ is C^2 over $\mathbb{R}^2_+$, increasing with respect to each argument*[*]*, i.e. $u_c(c, \mu m) > 0$ and $u_m(c, \mu m) > 0$, and concave over $\mathbb{R}^2_{++}$. In addition, $\forall c > 0$ and $\forall m > 0$, $\lim_{c\to 0} u_c(c, \mu m) = 0$, $\lim_{c\to +\infty} u_c(c, \mu m) = +\infty$, $\lim_{m\to 0} u_m(c, \mu m) = +\infty$ and $\lim_{m\to +\infty} u_m(c, \mu m) = 0$. Moreover, $v(l)$ is C^2 over $\mathbb{R}_+$, strictly increasing and weakly convex.*

Setting $b_t = B_t/p_t$ the real governments bonds held by the representative household, $\omega_t = w_t/p_t$ the real wage and $\theta_t = T_t/p_t$ the real transfers, we finally have that the intertemporal maximization problem of the representative agent can be written as

$$\max_{\{c_t, m_t, l_t, b_t\}_{t=0}^{\infty}} \sum_{t=0}^{+\infty} \beta^t \left[u(c_t, \mu m_t) - A v(l_t)\right] \tag{9}$$

subject to the dynamic budget constraint

$$c_t + m_{t+1} \frac{p_{t+1}}{p_t} + b_{t+1} \frac{p_{t+1}}{p_t} = m_t + (1 + i_t) b_t + \omega_t l_t + \theta_t. \tag{10}$$

Denoting λ the Lagrangian multiplier associated to the dynamic budget constraint, one obtains the following first order conditions

$$\beta^t u_c(c_t, \mu m_t) = \lambda_t, A\beta^t v_l(l_t) = \omega_t \lambda_t, \mu\beta^t u_m(c_t, \mu m_t) = \lambda_{t-1} \tfrac{p_t}{p_{t-1}} - \lambda_t \tag{11}$$

and the Fisher equation

$$-\lambda_{t-1} \tfrac{p_t}{p_{t-1}} + \lambda_t (1 + i_t) = 0. \tag{12}$$

By exploiting appropriately (11)-(12), we obtain the following equations

$$A v_l(l_t) = \omega_t u_c(c_t, \mu m_t) \tag{13}$$

$$u_c(c_{t-1}, \mu m_{t-1}) = \beta u_c(c_t, \mu m_t)(1 + i_t) \tfrac{p_{t-1}}{p_t} \tag{14}$$

[*] u_c, u_m and v_l denote, respectively, $\partial u(c, \mu m)/\partial c$, $\partial u(c, \mu m)/\partial m$ and $\partial v(l)/\partial l$.

and

$$\mu u_m\left(c_t,\mu m_t\right)-\frac{p_t}{p_{t-1}\beta}u_c\left(c_{t-1},\mu m_{t-1}\right)+u_c\left(c_t,\mu m_t\right)=0. \tag{15}$$

2.3 Firms

We assume that there exists a representative firm producing the unique consumption good by using only labor according to the following linear technology:

$$y_t=l_t. \tag{16}$$

3 Intertemporal Equilibrium and Steady State Analysis

In the sequel, we define the intertemporal equilibrium of the economy and carry out a steady state analysis.

3.1 Equilibrium

At equilibrium all the markets must clear. Equilibrium in the good market requires $y_t = l_t = c_t$ in each period t, meanwhile in order to have equilibrium in the money market one needs $p_t/p_{t-1} = m_{t-1}\sigma/m_t$. In addition, one immediately verifies that at equilibrium the real monetary transfers satisfy $\theta_t = \hat{\sigma}m_t/\sigma$. Since the technology is linear in labor, one has that the real wage is constant and equal to one, i.e. $\omega_t = 1$, for every t. Finally, by Walras law also the bonds market clears. Since at equilibrium $y_t = l_t = c_t$, it is possible to rewrite the static first-order condition (13) as

$$Av_l(y_t)=u_c(y_t,\mu m_t). \tag{17}$$

From (17) it is possible, by applying the Implicit Function Theorem, to obtain around a given point a positively valued function $y_t = y(m_t)$. In order that such a function to be well defined, one needs the following Assumption to be satisfied:

Assumption 2. $\frac{v_{ll}}{v_l}-\frac{u_{cc}}{u_c}\neq 0.$

The existence of such a function is immediately verifiable once one notices that for a given positive m_t the left hand-side of (17) is increasing in y_t meanwhile its right hand-side is decreasing in it. Therefore, in view of the Inada conditions included in Assumption 1, the uniqueness of such y_t solving (17) is ensured. One immediately verifies that $y^{'}(m_t)$ is given by

$$dy_t=\frac{\frac{\mu u_{cm}}{u_c}}{\frac{v_{ll}}{v_l}-\frac{u_{cc}}{u_c}}dm_t. \tag{18}$$

By combining (15) with equilibrium conditions and taking into the definition of $y(m_t)$, we can define the intertemporal equilibrium of the economy in terms of the evolution of real balances.

Definition 1. *A sequence* $\{m_t\}_{t=0}^{\infty}$*, with* $m_t > 0$ *for all t, is a perfect-foresight competitive equilibrium if it satisfies*

$$\mu u_m\left(y(m_t), \mu m_t\right) - \frac{m_{t-1}\sigma}{\beta m_t} u_c\left(y(m_{t-1}), \mu m_{t-1}\right) + u_c\left(y(m_t), \mu m_t\right) = 0, \tag{19}$$

together with the transversality condition

$$\lim_{t\to+\infty} \beta^t u\left(y(m_t), \mu m_t\right)(m_t + b_t) = 0. \tag{20}$$

System defined by (19) is one-dimensional and includes the temporal evolution of real balances that represent a non-predetermined variable. It follows that local indeterminacy requires a stable steady state; in the opposite case, the system will be locally determinate.

3.2 Steady State Analysis

In this Section, we provide conditions for the existence, the uniqueness and the multiplicity of the stationary solutions of the dynamic system defined by equation (19). A non-monetary steady state of the dynamic system defined by (19) is an amount of real balances equal to zero, corresponding to an hyperinflationary equilibrium. A monetary steady state of the dynamical system is a positive m satisfying

$$u_m\left(y(m), \mu m\right) = \left(\frac{\sigma-\beta}{\beta\mu}\right) u_c\left(y(m), \mu m\right) \tag{21}$$

Notice from (21) that if σ is set equal to β the marginal utility of money is equal to zero and thus agents are satiated: this corresponds to the well know Friedman monetary rule.

Notice in addition that, in view of the first-order conditions (14) evaluated at the stationary solution, one has $1 = \beta\sigma(1+i)$ and therefore the stationary nominal interest rate is $i = (\beta\sigma)^{-1} - 1$. This puts an upper bound on the money growth rate since bonds dominate money in terms of returns if and only if $\sigma < \beta^{-1}$. Recall to mind that, at the same time, the money growth rate σ must be larger than β in order to ensure that all the relevant variables of the model evaluated at the steady state are strictly positive. Then we need to introduce the following Assumption.

Assumption 3. $\sigma \in (\beta, \beta^{-1})$.

In addition, by looking at (21), one immediately verifies that the marginal utility of real balances is equal to zero when $\mu = +\infty$. In other words, under such a condition, individuals are satiated with money holding. This is not surprising, once one inspects the instantaneous utility function $u\left(y, \mu m\right)$ in which it is easy to verify that μ influences the marginal utility of m: actually, as soon as μ is set larger and larger, the former decreases, entailing therefore a Pareto improvement.

3.2.1 Uniqueness and Multiplicity

Studying the number of steady states of our dynamical system consists in finding the number of solutions of (21). This boils down to study the number of solutions of

$$g(m) \equiv g(y(m), m) = u_m(y(m), \mu m) - \left(\frac{\sigma - \beta}{\beta\mu}\right) u_c(y(m), \mu m) = 0.$$

In order to carry out our analysis it is useful to compute the derivative $g'(m)$ which, after straightforward computations, yields

$$g'(m) = \frac{\mu\left[\frac{u_{cm}^2}{u_c} + u_{mm}\left(\frac{v_{ll}}{v_l} - \frac{u_{cc}}{u_c}\right)\right] - u_{cm}\frac{v_{ll}}{v_l}\frac{\sigma-\beta}{\beta}}{\frac{v_{ll}}{v_l} - \frac{u_{cc}}{u_c}}. \tag{22}$$

Notice that, under Assumption 2, $g'(m)$ is well defined. One immediately verifies that when $g(m)$ is monotonic, i.e. $g'(m) > 0$ or $g'(m) < 0$ for all m, there exists at most one steady state. Since the denominator of $g'(m)$ is positive, a sufficient condition for the uniqueness requires the numerator of (22) to have constant sign for every m and some boundary conditions to be respected, as it is stated in the following Proposition.

Proposition 1. *Under Assumptions 1-3, there exists one stationary solution of system defined by (19) if*

i) g'(m) is decreasing for all m, $\lim_{m\to 0} g(m) > 0$ *and* $\lim_{m\to +\infty} g(m) < 0$*;*
ii) g'(m) is increasing for all m, $\lim_{m\to 0} g(m) < 0$ *and* $\lim_{m\to +\infty} g(m) > 0$*.*

One immediately verifies that if the boundary conditions included in Proposition 1 are not satisfied there is no steady state for the dynamics system defined by (19). Taking into account that u_{cm} is positive and u_{mm} is negative, one has that $g'(m)$ is in particular negative for every m (and thus the first part of condition i) in Proposition 1 does hold) when $u_{mm} < -u_{cm}^2/[u_c(v_{ll}v_l^{-1} - u_{cc}u_c^{-1})]$, as it is claimed in the following Corollary.

Corollary 1. *Under Assumptions 1-3, there exists one steady state of system defined by (19) if* $u_{mm} < -u_{cm}^2/[u_c(v_{ll}v_l^{-1} - u_{cc}u_c^{-1})]$ *for all m,* $\lim_{m\to 0} g(m) > 0$ *and* $\lim_{m\to +\infty} g(m) < 0$.

Notice that $u_{mm} < -u_{cm}^2/[u_c(v_{ll}v_l^{-1} - u_{cc}u_c^{-1})]$ in Corollary 1 is always satisfied when the utility function is separable in consumption and real balances since in such a case one has $u_{cm} = 0$. From the previous results one thus has that in the particular case where $g(m)$ is constant, there is either a continuum of steady states or no steady state at all. In order to provide conditions for the multiplicity of the steady state let us observe that if $g'(m)$ changes its sign only once, then there are at most two steady states. A sufficient condition for the existence of exactly two stationary solutions is to be found in a single-peaked function $g(m)$ or in a single-caved function $g(m)$. In both cases, however, some additional boundary condition are needed. In the case of a single-peaked function $g(m)$ it is indeed also required $\lim_{m\to 0} g(m) < 0$, $\lim_{m\to +\infty} g(m) < 0$ for all m and $g(m^*) > 0$, where $m^* < +\infty$ is such that $g'(m^*) = 0$, meanwhile in the case of a single-caved $g(m)$ it is also required that $\lim_{m\to 0} g(m) > 0$, $\lim_{m\to +\infty} g(m) > 0$ for all m and $g(m^*) < 0$, where $m^* < +\infty$ is such that $g'(m^*) = 0$. All these findings are summarized in the following Proposition.

Proposition 2. *Under Assumptions 1-3, there are exactly two steady states for the dynamic system (19) if one of the following conditions is satisfied:*

i) $g'(m)$ is decreasing for all m, $\lim_{m\to 0} g(m) < 0$, $\lim_{m\to +\infty} g(m) < 0$ for all m and $g(m^*) > 0$, where $m^* < +\infty$ is such that $g'(m^*) = 0$;

ii) $g'(m)$ is increasing for all m, $\lim_{m\to 0} g(m) > 0$, $\lim_{m\to +\infty} g(m) > 0$ for all m and $g(m^*) < 0$, where $m^* < +\infty$ is such that $g'(m^*) = 0$.

3.2.2 Existence of a Normalized Steady State

We want now to calibrate the particular stationary solution $m = 1$. To this end, it useful to rewrite the first-order conditions (13) and (15) evaluated at the steady state as

$$u_m (y, \mu m) = \left(\tfrac{\sigma-\beta}{\beta\mu}\right) u_c (y, \mu m) \tag{23}$$

and

$$Av_l (y) = u_c (y, \mu m) . \tag{24}$$

Notice that, in view of Assumption 1, the left-hand side of (23) is increasing in y meanwhile its right-hand side is decreasing: it follows that for each m there exists a unique y solving (23). It is then possible to obtain a smooth function h such that $y = h(m)$ for each m. By plugging such a function in (24), one can then calibrate the particular stationary solution $m = 1$ by simply setting

$$A = \frac{u_c (h(1), \mu)}{v_l (h(1))}. \tag{25}$$

Under the hypothesis included in Assumption 1 and taking into account (18), the following Proposition is then immediately proved.

Proposition 3. *Under Assumptions 1-3, let $A > 0$ be the unique solution of (25). Then $m = 1$ is a stationary solution of the system (19).*

Notice that Proposition 3 ensures the existence and uniqueness of the normalized steady state $m = 1$, but does not entail any consequence in terms of the possible occurrence of other stationary solutions.

4 Local Stability

In this Section, we carry out the stability analysis of the dynamic system defined by equation (19). For the purpose of our study, it is useful to introduce at this stage of the paper the following elasticities: $\varepsilon_{cc} \equiv -u_{cc}c/u_c$ the elasticity of the marginal utility of consumption, $\varepsilon_{mm} \equiv -u_{mm}\mu m/u_m$

the elasticity of the marginal utility of real balances, $\varepsilon_{mc} \equiv u_{mc}c/u_c$ and $\varepsilon_{cm} \equiv u_{cm}\mu m/u_m$ the crossed elasticities of the marginal utility of real balances and consumption and $\varepsilon_{ll} \equiv v_{ll}l/v_l$ the inverse of the elasticity of the labor supply, all evaluated at the steady state under study and all belonging to $(0, +\infty)$. Furthermore, by exploiting the definition of ε_{mc} and of ε_{cm}, and taking into account (23) evaluated at the steady state, one derives the following relationship:

$$\varepsilon_{mc} = \left(\frac{\sigma - \beta}{\beta\mu^2}\right)\varepsilon_{cm}. \tag{26}$$

We now turn to the linearization of the system defined by (19), taking into account (18), around the calibrated stationary solution $m = 1$. After tedious but straightforward computations one obtains the following expression for the derivative dm_t/dm_{t-1} evaluated at the calibrated steady state:

$$\frac{dm_t}{dm_{t-1}} = \left(\frac{\sigma\mu}{\beta}\right)\left(\frac{(\varepsilon_{cc} + \varepsilon_{ll})\,\mu\beta + \varepsilon_{ll}\varepsilon_{cm}(\sigma - \beta)}{\varepsilon_{cm}^2(\sigma - \beta)^2 + \varepsilon_{cm}\varepsilon_{ll}(\sigma - \beta)\mu + (\varepsilon_{cc} + \varepsilon_{ll})\left[\sigma - (\sigma - \beta)\varepsilon_{mm}\right]\mu^2}\right). \tag{27}$$

In order to provide an exhaustive picture of the stability properties of the calibrated steady state of the dynamic system under study, we first look at the bifurcation points of (27). Let us first notice that the numerator of (27) is strictly positive and that its denominator is equal to zero when

$$\varepsilon_{mm}^0 = \frac{\sigma}{\sigma - \beta} + \frac{\varepsilon_{cm}\left[(\sigma - \beta)\varepsilon_{cm} + \mu\varepsilon_{ll}\right]}{(\varepsilon_{cc} + \varepsilon_{ll})\mu^2}. \tag{28}$$

One immediately verifies that the existence of such bifurcation is guaranteed for any parameters configuration. A transcritical bifurcation may occur when dm_t/dm_{t-1} is equal to 1. This is true when ε_{mm} is set equal to ε_{mm}^t whose expression is easily derived as

$$\varepsilon_{mm}^t = \frac{\varepsilon_{cm}(\sigma - \beta)\left[\beta\varepsilon_{cm} - \mu\varepsilon_{ll}\right]}{(\varepsilon_{cc} + \varepsilon_{ll})\beta\mu^2}. \tag{29}$$

Notice that ε_{mm}^t is economically meaningful when it is positive. As it is easy to check, this requires $\mu < \mu^t$, where μ^t is defined as follows:

$$\mu^t = \frac{\beta\varepsilon_{cm}}{\varepsilon_{ll}}. \tag{30}$$

As it is well known, the transcritical bifurcation entails generically the existence of multiple steady states which change their stability properties as soon as the system undergoes such a bifurcation. The existence of such ε_{mm}^t represents thus a useful piece of information for the study of the occurrence of multiple stationary solutions of system defined by (19). At the same time, in order to get a flip bifurcation, the expression (27) must be equal to -1, which is true when

$$\varepsilon_{mm}^f = \frac{\sigma}{\sigma - \beta} + \frac{\varepsilon_{cm}\left[(\sigma - \beta)\varepsilon_{cm} + (\sigma + \beta)\mu\varepsilon_{ll}\right]}{(\varepsilon_{cc} + \varepsilon_{ll})\mu^2}. \tag{31}$$

One immediately verifies that the existence of a flip bifurcation is guaranteed for any parameters configuration. One may now wonder whether it is possible to rank the critical elasticities ε^t_{mm}, ε^0_{mm} and ε^f_{mm} under the domain of definition of ε^t_{mm}. Straightforward computations show that the following inequalities hold: $\varepsilon^t_{mm} < \varepsilon^0_{mm} < \varepsilon^f_{mm}$ when $\mu < \mu^t$. In order to characterize the stability properties of system we just need to exploit the additional information that (27) is increasing in ε_{mm}, it is lower than one for $\varepsilon_{mm} < \varepsilon^t_{mm}$, it tends to $+\infty$ when ε_{mm} converges to ε^0_{mm-}, it tends to $-\infty$ when ε_{mm} converges to ε^0_{mm+}, and converges to 0 when ε_{mm} tends to $+\infty$. On the other hand, when $\mu > \mu^t$, the following inequality holds: $\varepsilon^0_{mm} < \varepsilon^f_{mm}$ and ε^t_{mm} does not exist anymore. It follows that in such a case (27) never belongs in the interval $(0, 1)$. Thus the following Proposition is immediately proved.

Proposition 4. *Under Assumptions 1-3, there exist $\varepsilon^0_{mm}, \varepsilon^f_{mm}$, ε^t_{mm} and μ^t such that the following results generically hold:*

i) Let $\mu \in (0, \mu^t)$;

a) if $\varepsilon_{mm} < \varepsilon^t_{mm}$, the steady state is stable (locally indeterminate);

b) if $\varepsilon_{mm} \in (\varepsilon^t_{mm}, \varepsilon^f_{mm})$, the steady state is unstable (locally determinate);

c) if $\varepsilon_{mm} > \varepsilon^f_{mm}$, the steady state is stable (locally indeterminate);

ii) Let $\mu \in (\mu^t, +\infty)$;

a) if $\varepsilon_{mm} < \varepsilon^f_{mm}$, the steady state is unstable (locally determinate);

b) if $\varepsilon_{mm} > \varepsilon^f_{mm}$, the steady state is stable (locally indeterminate).

In addition, when ε_{mm} goes through ε^f_{mm} a flip bifurcation generically occurs and a stable or unstable cycle (according to the direction of the bifurcation) arises near the calibrated stationary solution meanwhile when ε_{mm} goes through ε^t_{mm} a transcritical bifurcation generically occurs and one may expect a change in stability between two nearby steady states.

It could be now interesting to analyze the two particular cases widely studied in economic theory, i.e. the case corresponding to an inelastic labor supply ($\varepsilon_{ll} = +\infty$) and to an infinitely elastic one ($\varepsilon_{ll} = 0$). By direct inspection of (29), one immediately observes that, if $\varepsilon_{ll} = +\infty$, part ii) of Proposition 4 holds, meanwhile, if $\varepsilon_{ll} = 0$, part i) of Proposition 4 does apply. A final remark could concern the effect of the amplitude of ε_{cm} to the local dynamic of our system. Here the answer is straightforward: as it is immediately verifiable by checking (27), the impact of ε_{cm} is only quantitative but not qualitative and thus results of Proposition 4 still hold.

For the purpose of the paper, the analysis of the impact of the amplitude of μ on the length of the indeterminacy region turns out to be compelling. Such an analysis is easily carried out by inspecting how the critical elasticities $\varepsilon^t_{mm}, \varepsilon^0_{mm}$ and ε^f_{mm} do vary as soon as μ is made to increase. As it is immediately verifiable, the transcritical bifurcation rapidly disappears for high values of μ, meanwhile ε^0_{mm} and ε^f_{mm} are decreasing and both converge to $\sigma/(\sigma - \beta)$ when μ tends to $+\infty$. In view of part ii) of Proposition 4, this implies that the indeterminacy region widens as soon as the inverse μ of the degree of financial market imperfection is progressively increased. This result, at first sight counter-intuitive, seems to confirm the findings of Bosi *et al.* (2005) within an economy with partial cash-in-advance constraint according to which local indeterminacy becomes more and more plausible as soon as the amplitude of the liquidity constraint is set lower and lower.

5 Conclusion

In this paper we have studied the behavior of an infinite horizon economy where the utility function of the representative agent is defined over consumption, real balances and leisure. We have assumed that real balances enter in the utility function pre-multiplied by a parameter which is to be interpreted as the inverse of the degree of financial market imperfection of the economy. Equivalently, a larger value for such a parameter can be seen as reflecting a lower underlying transaction cost. We have shown that indeterminacy and sunspot fluctuations may occur for a wide range of the degree of financial market imperfection even when the latter is set arbitrarily close to zero. Indeterminacy appears first through a transcritical bifurcation and then through a flip one and its scope improves as soon as the degree of market imperfection decreases. Our results are robust with respect to the specification of the elasticity of labor supply whose size has the unique effect of establishing whether or not the transcritical bifurcations exists. We have also provided the conditions for the existence, the uniqueness and the multiplicity of the steady states, and established some relationships between such phenomena and the existence of the transcritical bifurcation.

A suitable extension of the model should take into account capital accumulation, as Sidrauski (1967) and Farmer (1997): such a departure should better emphasize the arbitrages intervening between physical assets, government bonds and money balances and to appraise the dynamics of the economy in terms of the behavior of the nominal interest rate, by exploiting the Fisher equation. It could be also interesting, within such a framework, to assess the impulse response functions on *GDP* of the sunspot shocks.

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