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A note on endogenous growth with public capital

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Abstract

This paper develops a two sector model of endogenous economic growth with public capital where private goods and public investment goods are produced with different production technologies. The government buys public investment goods produced by private producers; and the government is a monopsonist in this market to determine the price. The price of public investment good and the income tax rate are not two independent policy instruments for the government; and the government maximises its objective function with respect to one of them and can set the other to balance the budget. When growth rate is maximised in the steady state equilibrium, the corresponding income tax rate is equal to the elasticity of private good's output with respect to public capital but it is independent of the technology in public good production. The welfare maximising solution is not necessarily identical to the growth rate maximising solution even in the steady state equilibrium.

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1. Introduction

The literature on growth theory emphasises the pivotal role played by public capital in the process of fostering growth. World Bank (1994) identifies public capital as the 'wheels' of economic growth. In a seminal contribution, Barro (1990) makes the first attempt to incorporate the productive role of public infrastructure in an endogenous growth model; and also determines and analyses the properties of optimal income tax used to finance this productive public expenditure. Futagami et al. (1993) extends Barro (1990) model by considering public capital as a stock variable rather than a flow variable (as assumed by the latter). Other important contributions that followed these two models are Eicher and Turnovsky (2000), Tsoukis and Miller (2003), Turnovsky (2000) and Irmen and Kuehnel (2009). Interestingly, in all these models it is assumed that producers of both the public good and the final private good use identical technology. The state buys public goods at the competitive price of the final good using tax revenue; and then freely provides the whole stock of public good to producers as public input.[1] Moreover, the government chooses optimal tax rate such that the rate of growth and / or the welfare level is maximised.

This type of modelling has two major problems. First of all, these models assume that the aggregate production functions of both goods are identical. In Barro's own words, "*As long as the government and the private sector have the same production functions, the results would be the same if the government buys private inputs and does its own production, instead of purchasing only final output from the private sector, as I assume.*" However, this simplifying assumption is too simple to model the real world. Productive public capitals, such as, ports, roads, bridges, dams, rail etc. may have different input elasticities than from the input elasticities of other private goods, such as agricultural products, clothing, computers, bicycles etc. In fact, a fairly large number of contributions (see for example, Pereira and Roca-Sagales (2001), Pereira and Andraz (2007), Cantos et al. (2005), Ammad and Ahmed (2013), Annala et al. (2004) and Feng and Serletis (2013)) have empirically shown that output elasticities of public capital are very different for different sectors. For example, Pereira and Roca-Sagales (2001) has shown that the long term accumulated elasticities with respect to public capital are 0.81, 1.23 and 0.37 in manufacturing sector, construction sector and in service sector respectively.[2] This implies that production functions are different for different sectors. Since aggregate production functions of public capital and of final private goods are weighted averages of production functions of these sectors with different sets of weights; so identical aggregate production functions for final private good and public capital is hardly possible. This in turn implies that it is important to derive the properties of optimal income tax rate where private goods and public goods are produced with different production technologies. Few papers, such as Dasgupta (1999, 2001), Dasgupta and Shimomura (2006), Pintea and Turnovsky (2006), Turnsovsky and Pintea (2006) consider different production functions for producing private goods and public goods. However, Pintea and Turnovsky (2006) and Turnsovsky and Pintea (2006) do not derive the optimal tax rate analytically. On the other hand, Dasgupta (2001) and Dasgupta and Shimomura (2006) do not consider income taxation[3]. Only Dasgupta (1999) derives the optimal income tax rate. However,

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[1] In Barro's own words, "*But conceptually, it is satisfactory to think of the government as doing no production and owning no capital. Then the government just buys a flow of output (including services of highways, sewers, battleships, etc.) from the private sector. These purchased services, which the government makes available to households, correspond to the input that matters for private production ……..*".

[2] Though these are long term accumulated elasticities with respect to public capital, but their very different values clearly indicate that the direct output elasticities with respect to public capital are very different. Otherwise, the long term accumulated elasticities would have been the same.

[3] Dasgupta and Shimomura (2006) considers lump sum taxes but not per unit income tax.

Dasgupta (1999) shows that the optimal income tax rate is zero and the government should earn the entire revenue only by charging the private sector firms for usage of public services on a per unit basis. This may be impossible to implement when public services are non-rival and non-excludable in nature, since, firms will try to take a free ride. So Barro (1990) model's idea of freely distributing services of public capital and of charging income taxes to finance its cost is better from the viewpoint of implementation.

The second problem with Barro (1990) type of modelling is more severe. In this entire genre of literature, it is assumed that the government buys public goods from private producers at a given price and this price is equal to the competitive price of the final good. However, why the government should act as a price-taker is not clear. The government is the only buyer; and so it should act as a monopsonist by setting the relative price in order to attain its objective.

These two issues motivate us to develop the present model. Otherwise building closely on Futagami et al. (1993), we assume a two sector economy with different production functions for producing the final good and a public investment good. Here, we attempt not only to analyse the properties of optimal income tax rate used to finance investment in public capital but also analyse the properties of the optimal buying price of the public investment good. In this model, the private sector produces public investment good and sells it to the government who has a monopsony power to set the buying price. The government balances its budget at each point of time by charging income tax. Thus, the price of public investment good and income tax rate are not two independent policy instruments for the government. The government uses one of them to maximise its objective function and the other is adjusted to maintain balance in the budget. In this paper, we use buying price of public investment good as government's policy instrument to control intersectoral allocation of private resources and the income tax rate is adjusted to maintain balance in the budget.[4]

We derive many interesting results from this model. First, the budget balancing government cannot change allocation of private capital between two sectors and thereby the performance of the economy if we assume identical production technology in both sectors and equal price of both goods like Barro (1990) and its one sector extension models. This is so because, in a two sector economy, resource allocation depends upon the marginal productivity of the resource across sectors. Since both sectors have identical production technology and income tax rate reduces the value of marginal productivity in the same ratio across all sectors, so the value of marginal productivity depends only upon the price of the good produced. So the government cannot alter the allocation of private resources in different sectors by setting price of public investment good equal to the price of final good. Secondly, growth rate maximising buying price of public investment good is not necessarily equal to the competitive price of final good. In fact, if we assume identical production functions for both goods like Barro (1990) type of modelling, then also this growth rate maximising buying price of public investment good is not necessarily equal to the competitive price of the final good. This result stems from the fact, that in case the productivity of private capital is lesser (greater) than the productivity of public capital in the final good producing sector, a benevolent state would choose an optimal price of the public investment good greater (lesser) than the price of the final good to attract (drive away) resources and to enhance its (final good's) production. Thus, the buying price of the public investment good is an instrument for promoting economic growth. Thirdly, the budget balancing income tax rate corresponding to the growth rate maximising solution is equal to the elasticity of output with respect to public capital in the production of final private goods only but is independent of the production

[4] Major results of this paper does not change qualitatively if we use income tax rate as policy instrument and the buying price of public investment good is set to balance budget.

technology to produce public investment good. This is so because, public investment good sector uses public capital as input only to produce additional public capital. There exists only one final good sector to receive the service of public capital free of cost. If there is exchange, it is optimal for the final good sector to buy public investment good at the competitive price. So in the absence of exchange, it is optimal to charge a tax rate which is equal to the competitive output share of public capital in the final good sector. As a result, optimal tax rate is independent of the production technology of public capital. Lastly, welfare maximising buying price of public capital and its corresponding income tax rate and allocation of private capital are different from their corresponding growth rate maximising values even in the steady state growth equilibrium. However, if we consider identical production functions for both goods, then welfare maximising solutions and growth rate maximising solutions become identical. Since Barro (1990) and Futagami et al. (1993) consider identical production functions for both goods, so in those models, growth rate maximising fiscal policies are identical with welfare maximising fiscal policies in the steady state equilibrium. Thus the present model generalises these previous results. These results are new in the literature of endogenous growth with public capital.

Rest of the paper is organised as follows. Section 2 describes the structure of the model. Section 3 deals with steady state growth equilibrium and growth rate maximising policies. Section 4 compares growth rate maximising fiscal policies to optimal (welfare maximising) fiscal policies in the steady state equilibrium; and section 5 concludes the paper.

2. The Model

The representative household-producer produces both final good and public investment good using private capital and public capital. Public investment good is defined as the additional stock of non-rival public capital. Production functions of two sectors with different technologies are given by

$$Y = A(\theta K)^{\alpha} G^{1-\alpha} \quad \text{where} \quad \alpha \in (0,1) \quad \text{and} \quad A > 0 \quad ; \tag{1}$$

and

$$\dot{G} = B[(1-\theta)K]^{\beta} G^{1-\beta} \quad \text{where} \quad \beta \in (0,1) \quad \text{and} \quad B > 0 \quad . \tag{2}$$

Here, Y, K, G and θ denote level of output of final good, stock of private capital, stock of public capital and the share of private capital allocated to production of final goods respectively. Neither type of capital depreciates over time. $\dot{G}$ represents the level of output of public investment good.

The government has the monopsony power to set the price of public investment good, $\dot{G}$, as the government is the sole buyer of it. It can determine this price either by maximising the steady state equilibrium growth rate or by maximising the social welfare. Since there is no money and there are two goods in this model, so the price of one good will be expressed in terms of the price of another good. Here we choose the price of final good as numeraire and express the price of public investment good in terms of it.[5] The household–producer determines the allocation of resources between two sectors once this price is set by the government.

The government buys all $\dot{G}$ at the relative price, μ; and freely provides the whole stock of G to the household-producers. So the government incurs an expenditure equal to $\mu\dot{G}$. On the other hand, the government charges an income tax at the rate, τ, on the representative household producer's total income, $(Y + \mu\dot{G})$. So the government's balanced budget equation becomes

$$\tau Y + \tau\mu\dot{G} = \mu\dot{G} \quad . \tag{3}$$

[5] If the price of public investment good is chose as numeraire, then also major results remain unchanged.

The representative household is infinitely lived; and she derives instantaneous utility from consumption of final goods only; and maximises her discounted present value of instantaneous utility subject to her intertemporal budget constraint after τ and μ are set by the government.. The optimisation problem of the household is given by the following.

$$Max \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt \tag{4}$$

subject to, $\dot{K} = (1-\tau)Y + (1-\tau)\mu\dot{G} - c$; (5)

$K(0) = K_0$;

and $\theta \in [0,1]$.

Here c is the level of consumption of the final good and K_0 is historically given initial private capital stock. σ represents the elasticity of marginal utility with respect to consumption and ρ denotes the constant rate of discount. Savings is always invested.

Here c and θ are two control variables and K is the only state variable. Solving this dynamic optimisation problem, we obtain[6]

$$(1-\tau)A\alpha\theta^{\alpha-1}K^{\alpha}G^{1-\alpha} = \mu(1-\tau)B\beta(1-\theta)^{\beta-1}K^{\beta}G^{1-\beta} \quad ; \tag{6}$$

and

$$\frac{\dot{c}}{c} = \frac{(1-\tau)A\alpha\theta^{\alpha}K^{\alpha-1}G^{1-\alpha} + \mu(1-\tau)B\beta(1-\theta)^{\beta}K^{\beta-1}G^{1-\beta} - \rho}{\sigma} \quad . \tag{7}$$

Equation (6) shows the efficient allocation of private capital between two sectors. It implies that the after tax value of the marginal product of private capital is same in both these two sectors. Equation (7) describes the demand rate of growth of consumption which is defined as the excess of after tax marginal return of private capital accumulation over the rate of discount normalised with respect to the elasticity of marginal utility.

3. The Steady State Equilibrium

The equations of motion of the system are given by equations (2), (5) and (7). In the steady-state growth equilibrium,

$$g = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \frac{\dot{c}}{c} \quad , \tag{8}$$

where g is the balanced growth rate of the economy.

Now, from equation (6), we obtain

$$\frac{(1-\theta)^{1-\beta}}{\theta^{1-\alpha}} = \frac{\mu B\beta}{A\alpha}\left(\frac{K}{G}\right)^{\beta-\alpha} \quad ; \tag{6a}$$

and from equations (1), (2) and (3), we obtain

$$\left(\frac{\tau}{1-\tau}\right) = \frac{\mu B(1-\theta)^{\beta}}{A\theta^{\alpha}}\left(\frac{K}{G}\right)^{\beta-\alpha} \quad . \tag{9}$$

In the case of identical production technologies, i.e., $A = B$ and $\alpha = \beta$, equations (6a) and (9) become

$$\left(\frac{1-\theta}{\theta}\right)^{1-\alpha} = \mu \quad ; \tag{6b}$$

and

$$\frac{\tau}{1-\tau} = \mu\left(\frac{1-\theta}{\theta}\right)^{\alpha} \quad . \tag{9a}$$

Equations (6b) and (9a) show that if we assume $\mu = 1$ along with identical production functions like Barro (1990) type of models, then $\theta = 1/2$; and the budget balancing income

[6] Derivation of equations (6) and (7) are shown in the appendix.

tax rate, τ = 1/2. In fact, in this case, the government cannot affect the sectoral allocation of inputs and thereby cannot affect growth or welfare of the economy. This shows that, if the government's objective is not satisfied at θ = ½, then the relative price will not be set at $\mu = 1$.

The above discussion can be summarised as the following proposition.

Proposition 1: In case of identical production functions, the government cannot affect intersectoral allocation of private capital and therefore cannot affect performance of the economy when the relative price of public investment good is set at unity.

Now, for the rest of the paper, we focus on endogenising relative price, μ. From equations (2) and (8), we obtain

$$\left(\frac{G}{K}\right) = \frac{(1-\theta)B^{\frac{1}{\beta}}}{g^{\frac{1}{\beta}}} \quad . \tag{2a}$$

Using equations (2a), (6), (7), (8) and (9), we have[7]

$$\rho + \sigma g = \frac{\beta B^{\frac{1}{\beta}} \mu g^{\frac{\beta-1}{\beta}}}{1 + \alpha \left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}} \beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu}{A\alpha}\right)^{\frac{1}{1-\alpha}} \right]} \quad . \tag{10}$$

Equation (10) solves for the balanced growth rate, g; and this equation also shows the nature of the relationship between the buying price of the public investment good, μ, and the balanced growth rate, g.

Now using equations (2a), (6a) and (9), we find that

$$\left(\frac{\tau}{1-\tau}\right) = \mu^{\frac{1}{1-\alpha}} g^{\frac{\beta-\alpha}{\beta(1-\alpha)}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{B^{\frac{\alpha}{\beta}}}{A}\right)^{\frac{1}{1-\alpha}} \quad . \tag{9b}$$

Equation (10) shows g as a function of μ; and then equation (9b) shows τ as a function of μ. So the two policy instruments μ and τ are not independent. If μ is used as the policy instrument, τ is automatically determined by the budget balancing condition. These two equations show how a change in the buying price of public investment good, μ, affects the tax rate, τ, in the steady – state equilibrium. This change in μ has a direct positive effect obtained for a given growth rate, g, and an ambiguous indirect effect working through change in g via equation (10). The final effect will depend on the relative strength of these direct and indirect effects and also on the nature of capital – intensity ranking between the two sectors, i.e., on the mathematical sign of (α - β).

However, μ is not a parameter in this model. μ is an independent policy instrument to solve the optimisation problem of the government; and the tax rate, τ, is adjusted to keep consistency with the optimum solution of μ.[8] Ideally, the government's objective should be to maximise the welfare level of the representative household, ω, given by

[7] Derivation of equation (10) is shown in appendix.

[8] The government can choose either τ or μ as policy instrument to maximise its objective and the other is set to balance its budget. In this paper, we use μ as the government's policy instrument. Results of this paper remain unchanged if we choose the alternate way where the government uses τ as the policy instrument and μ is set to balance the budget.

$$\omega = \underset{\{c\}}{Max} \int_0^\infty \frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} \, dt \qquad . \tag{11}$$

Unfortunately, we cannot explicitly solve for the welfare maximising buying price of the public investment good due to technical complications. Rather, we solve for its steady-state equilibrium growth rate maximising solution in this section; and then examine, in the next section, whether it deviates from its welfare maximising solution. So we now maximise g given by equation (10) with respect to the buying price of the public investment good, μ; and then, using the first order condition, we obtain the following solution.[9]

$$\mu = \frac{A(1-\alpha)^{1-\alpha}}{\alpha^{1-2\alpha} B^{\frac{\alpha}{\beta}} g^{\frac{\beta-\alpha}{\beta}} \beta^{\alpha}} \qquad . \tag{12}$$

Using equations (10) and (12), we have

$$(\rho + \sigma g) g^{\frac{1-\alpha}{\beta}} = A(1-\alpha)^{1-\alpha} \beta^{1-\alpha} \alpha^{2\alpha} B^{\frac{1-\alpha}{\beta}} \qquad . \tag{13}$$

Equation (13) solves for the maximum value of g, which is the endogenous rate of growth of the economy in the steady-state equilibrium.

Denoting this maximum value of g by g^* and putting it in equation (12), we obtain[10]

$$\mu^* = \frac{A(1-\alpha)^{1-\alpha}}{\alpha^{1-2\alpha} B^{\frac{\alpha}{\beta}} (g^*)^{\frac{\beta-\alpha}{\beta}} \beta^{\alpha}} \qquad . \tag{14}$$

This equation (14) shows that the steady state equilibrium growth rate maximising μ is not necessarily equal to unity, i.e., the growth rate maximising price of public investment good is not necessarily equals to the competitive price of the final good. Even if we consider identical production technology in both the sectors, i.e., $A = B$ and $\alpha = \beta$, then also

$$\mu^* = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \qquad ; \tag{14a}$$

and hence $\mu^* = 1$ if and only if $\alpha = 1/2$, i.e., if and only if production function is symmetric in terms of its arguments. This result is stated in the following proposition.

Proposition 2: The steady-state equilibrium growth rate maximising buying price of public investment good is not necessarily equal to the competitive price of the final good. The equality is obtained if production technology in both the sectors are identical and symmetric.

In Barro (1990) and Futagami et al. (1993), where production functions are identical, this symmetry assumption is not made but the government's buying price of public investment good is set to be equal to the competitive price of the final good.

Equations (2), (6), (9) and (14) can be used to obtain

$$\theta^* = \frac{\alpha^2}{\alpha^2 + \beta(1-\alpha)} \qquad ; \tag{15}$$

and

$$\tau^* = 1 - \alpha \qquad . \tag{16}$$

θ^* represents the growth rate maximising allocation of private capital to the final goods producing sector in the steady state growth equilibrium. Equation (15) shows that θ^* varies inversely with β and positively with α. This is so because, as β (α) rises, productivity of private capital rises in the public investment good (final good) sector relative to the other

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[9] Derivation of equation (12) is shown in the appendix.

[10] The second order condition of maximisation of growth rate with respect to μ is satisfied. From equation (10), it can be shown very easily that $\frac{d^2 g}{d\mu^2} < 0$ when equation (12) holds.

sector; and, as a result, allocative share of private capital to public investment good (final good) sector goes up. In the case of identical production technology, $\theta^* = \alpha$. This is stated in the following proposition.

Proposition 3: The growth rate maximising allocative share of private capital to final good (public investment good) producing sector varies positively (inversely) with the private capital elasticity of output of final good and varies inversely (positively) with the private capital elasticity of output of public investment good.

Equation (16) shows the income tax rate corresponding to μ^*, i.e., when balanced growth rate, g, is maximised. (1 - α) represents the elasticity of output of final good with respect to public capital. This equation does not involve β. So this leads to the following proposition.

Proposition 4: When the steady state equilibrium growth rate is maximised with respect to the buying price of public investment good, income tax rate corresponding to that growth rate maximising solution is equal to the elasticity of output of final good with respect to public capital but is independent of the production technology in the public investment good producing sector.

Public investment good sector uses public capital as input only to produce additional public capital. There exists only one final good sector to receive the service of public capital free of cost. If there is exchange, it is optimal for the final good sector to buy public investment good at the competitive price. So in the absence of exchange, it is optimal to charge a tax rate which is equal to the competitive output share of public capital in the final good sector.

In Barro (1990) and Futagami et al. (1993), input elasticities of output are same in both the sectors. So this problem does not arise.

4. Welfare Maximisation

In this section, we examine identicalness between growth rate maximising solutions and welfare maximising solutions. We use equations (1), (2), (5), (6), (7), (9) and (11) to obtain the welfare level of the representative household, denoted by ω. This is identical to her discounted present value of instantaneous utilities over the infinite time horizon. It is derived as[11]

$$\omega = \frac{\left[\frac{\rho}{\alpha} + g\left(\frac{\sigma}{\alpha} - 1\right) + \frac{(\alpha-\beta) g^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} \mu^{\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right]^{1-\sigma}}{{K_0}^{\sigma-1}(1-\sigma)[\rho - g(1-\sigma)]} + constant \quad . \qquad (17)$$

If $\sigma > \alpha$ and if $\rho - g(1-\sigma) > 0$, then equation (17) shows that ω varies positively with g when $\alpha = \beta$. So the growth rate maximising solution is identical to the welfare maximising solution in the steady state equilibrium when $\alpha = \beta$, i.e., when production technologies are

[11] See appendix for derivation of equation (17).

identical in these two sectors. However, when $\alpha \neq \beta$, i.e., when production technologies are not identical, then the welfare maximising solution is not identical to the growth rate maximising solution even in the steady state equilibrium. From equation (17), we differentiate ω with respect to μ and then evaluate it at $\mu = \mu^*$. Hence we obtain[12]

$$\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*} = \left\{\frac{\left[\frac{\rho}{\alpha} + g^*\left(\frac{\sigma}{\alpha} - 1\right) + \frac{A(\alpha-\beta){g^*}^{\frac{\alpha-1}{\beta}} B^{\frac{1-\alpha}{\beta}} \beta^{1-\alpha}}{[\alpha^2 + \beta(1-\alpha)](1-\alpha)^{\alpha-2}\alpha^{1-2\alpha}}\right]^{-\sigma}}{{K_0}^{\sigma-1}[\rho - g^*(1-\sigma)]}\right\} \cdot \left\{\frac{(\alpha-\beta){g^*}^{\frac{\beta-1}{\beta}} B^{\frac{1}{\beta}} \alpha^2 \beta}{[\alpha^2 + \beta(1-\alpha)]^2}\right\} . \qquad (18)$$

We assume $\sigma > \alpha$ and $\rho > g^*(1-\sigma)$. This ensures that the right hand side of equation (18) is positive (zero) (negative) when α > (=) (<) β[13]. This implies that the welfare maximising value of μ is higher (lower) than the growth rate maximising value of μ even in the steady state equilibrium when the final private good sector is more (less) private capital intensive than the public investment good sector. We refer welfare maximising μ as $\bar{\mu}$.

Now, we compare growth rate maximising solutions τ^* and θ^* to welfare maximising solutions $\bar{\tau}$ and $\bar{\theta}$. When $\alpha > \beta$, then $\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*}$ is positive and as a result, $\bar{\mu} > \mu^*$. So the growth rate corresponding to $\bar{\mu}$, i.e., $\bar{g}$, is less than g^* as g^* is the maximum value of balanced growth rate. As a result, $\bar{\mu}\bar{g}^{\frac{\beta-\alpha}{\beta}} > \mu^* {g^*}^{\frac{\beta-\alpha}{\beta}}$.

Using equations (2a), (6a) and (9), we obtain[14]

$$\theta = \frac{1}{1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}} \quad ; \qquad (19)$$

and

$$\tau = \frac{\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}} + \beta} < 1 \quad . \qquad (20)$$

Since equations (19) and (20) show that θ and τ vary inversely and positively with $\mu g^{\frac{\beta-\alpha}{\beta}}$ respectively, so welfare maximising θ, i.e., $\bar{\theta}$, is less than θ^* but welfare maximising τ, i.e., $\bar{\tau}$, is higher than τ^*. Similarly, when $\beta > \alpha$, then $\mu^* > \bar{\mu}$ and $g^* > \bar{g}$. This implies that, $\bar{\mu}\bar{g}^{\frac{\beta-\alpha}{\beta}} < \mu^* {g^*}^{\frac{\beta-\alpha}{\beta}}$; and as a result, $\bar{\theta}$ is greater than θ^* but $\bar{\tau}$ is less than τ^*.

Barro (1990) and Futagami et al. (1993) show that growth rate maximising income tax rate is identical to the welfare maximising income tax rate in the steady state equilibrium. However, we find that the welfare maximising solution is different from the growth rate

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[12] See appendix for derivation of equation (18).

[13] When $\beta > \alpha$, then also the first term in the R.H.S. of equation (18) is positive as c_0 cannot be negative.

[14] See appendix for derivation of equations (19) and (20).

maximising solution even in the steady state equilibrium when we consider different production functions for different goods. However, two solutions are always identical with identical production technology. So our result generalises the result of Barro (1990) and Futagami et al. (1993). This result is stated in the following proposition.

Proposition 5: When the final good sector is more (less) private capital intensive than the public investment good sector, welfare maximising buying price of public investment good, corresponding budget balancing income tax rate and corresponding allocation share of private capital to the public investment good sector exceed (fall short of) their corresponding growth rate maximising values even in the steady state equilibrium.

5. Conclusions

This paper constructs a simple two sector endogenous growth model with public capital; and derives the properties of optimal fiscal policies in the steady state equilibrium. Both final good and public investment good are produced by the private sector using different production technologies. However, in this model, the government buys public good from private producers and the government has the monopsonistic power to set this relative buying price. This buying price is a policy instrument of the government to control allocation of resources between these two sectors. Tax rate is not an independent policy instrument. Tax rate is adjusted to balance the budget when the relative price is determined. This is how the present model differs from models like Barro (1990), Futagami et al. (1993) etc.

Various interesting findings are obtained here. First, the budget balancing government cannot change allocation of private capital between two sectors and thereby the performance of the economy if we assume identical production technology in both sectors and equal price of both goods like Barro (1990) and its one sector extension models. Secondly, growth rate maximising buying price of public investment good is not necessarily equal to the competitive price of the final good even in the case with identical production technologies. Thirdly, the budget balancing income tax rate corresponding to steady state equilibrium growth rate maximisation is equal to the elasticity of output of final good with respect to public capital but is independent of the production technology of public investment good. Finally, welfare maximising solutions are different from growth rate maximising solutions even in the steady state equilibrium when production technologies are different in these two sectors.

This model can be extended in various possible directions. One very pertinent direction will be to incorporate the congestion effect of capital on productivity. Moreover, non-productive public services can directly affect households' utility. Political incentives remain a powerful alternative that can replace our assumption of a benevolent government in this set up. All these remain as possible projects for our future research.

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Appendix:

Derivation of equations (6) and (7):

Using equations (4) and (5), we construct the Current Value Hamiltonian as given by

$$H_c = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda\left[(1-\tau)Y + (1-\tau)\mu\dot{G} - c\right] \quad . \qquad (A.1)$$

Here λ is the co-state variable. Incorporating equations (1) and (2) in equation (A.1); and then maximising it with respect to c and θ, we obtain following first order conditions.

$$c^{-\sigma} - \lambda = 0 \quad ; \qquad (A.2)$$

and

$$\lambda(1-\tau)A(K)^{\alpha}G^{1-\alpha}\alpha\theta^{\alpha-1} = \lambda\mu(1-\tau)B[K]^{\beta}G^{1-\beta}\beta(1-\theta)^{\beta-1} \quad . \qquad (A.3)$$

From equation (A.3), we obtain equation (6) in the body of the paper.
Again from equation (A.1), we have

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1-\tau)AK^{\alpha-1}G^{1-\alpha}\alpha\theta^{\alpha} - \mu(1-\tau)BK^{\beta-1}G^{1-\beta}\beta(1-\theta)^{\beta} \quad ; \qquad (A.4)$$

and from equation (A.2), we have

$$\frac{\dot{\lambda}}{\lambda} = -\sigma\frac{\dot{c}}{c} \quad . \qquad (A.5)$$

Using equations (A.4) and (A.5), we have equation (7) in the body of the paper.

<u>Derivation of equation (10):</u>

From equation (7), we have

$$\rho + \sigma g = (1-\tau)A\alpha\theta^{\alpha}\left(\frac{G}{K}\right)^{1-\alpha} + \mu(1-\tau)B\beta(1-\theta)^{\beta}\left(\frac{G}{K}\right)^{1-\beta} \quad . \qquad (A.6)$$

From equation (2), we have

$$\left(\frac{G}{K}\right) = \frac{B^{\frac{1}{\beta}}(1-\theta)}{g^{\frac{1}{\beta}}} \quad . \qquad (A.7)$$

From equations (2), (6), (9), (A.6) and (A.7), we obtain equation (10) in the body of the paper.

<u>Derivation of equation (12):</u>

Taking log on both sides of equation (10) and then differentiating it with respect to μ and assuming $\frac{dg}{d\mu} = 0$, we obtain

$$\frac{1}{\mu} = \frac{\frac{\alpha}{1-\alpha}\mu^{\frac{\alpha}{1-\alpha}}\left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}\beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{1}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}{1+\alpha\left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}\beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \quad . \qquad (A.8)$$

From equation (A.8), we obtain equation (12) in the body of the paper.

<u>Derivation of equation (17):</u>

From equation (11), we obtain

$$\omega = \frac{{c_0}^{1-\sigma}}{[\rho - g(1-\sigma)](1-\sigma)} + constant \quad . \qquad (A.9)$$

Here, $c(0) = c_0$.
From equation (5), we obtain

$$c_0 = K_0\left\{(1-\tau)A(\theta)^{\alpha}\left(\frac{G_0}{K_0}\right)^{1-\alpha} + (1-\tau)\mu B(1-\theta)^{\beta}\left(\frac{G_0}{K_0}\right)^{1-\beta} - g\right\} \quad . \qquad (A.10)$$

Using equations (7) and (A.10), we obtain

$$c_0 = K_0\left\{\frac{\rho+\sigma g}{\alpha} + (1-\tau)\mu B(1-\theta)^{\beta}\left(\frac{G_0}{K_0}\right)^{1-\beta}\left(\frac{\alpha-\beta}{\alpha}\right) - g\right\} \quad . \qquad (A.11)$$

Using equations (2) and (A.11), we obtain

$$c_0 = K_0\left\{\frac{\rho+\sigma g}{\alpha} + (1-\tau)\mu B(1-\theta)\left(\frac{B^{\frac{1}{\beta}}}{g^{\frac{1}{\beta}}}\right)^{1-\beta}\left(\frac{\alpha-\beta}{\alpha}\right) - g\right\} \quad . \qquad (A.12)$$

Using equations (2), (6), (9) and (A.12), we obtain

$$c_0 = K_0\left\{\frac{\rho}{\alpha} + g\left(\frac{\sigma}{\alpha}-1\right) + \frac{(\alpha-\beta)g^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}}A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha-2}{1-\alpha}}\mu^{\frac{2-\alpha}{1-\alpha}}B^{\frac{1}{\beta(1-\alpha)}}\beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right\} \quad . \qquad (A.13)$$

Using equations (A.9) and (A.13), we obtain equation (17) in the body of the paper.

Derivation of equation (18):

Differentiating equation (17) with respect to μ and evaluating it at $\mu = \mu^*$, we obtain

$$\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*}$$

$$= \left\{\frac{\left[\frac{\rho}{\alpha} + g^*\left(\frac{\sigma}{\alpha}-1\right) + \frac{(\alpha-\beta)g^{*\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}}A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha-2}{1-\alpha}}\mu^{*\frac{2-\alpha}{1-\alpha}}B^{\frac{1}{\beta(1-\alpha)}}\beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right]^{-\sigma}}{{K_0}^{\sigma-1}[\rho-g^*(1-\sigma)]}\right\}$$

$$\cdot \left\{\frac{(\alpha-\beta)g^{*\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}}A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha-2}{1-\alpha}}\mu^{*\frac{2-\alpha}{1-\alpha}}B^{\frac{1}{\beta(1-\alpha)}}\beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right\}$$

$$\cdot \left\{\left(\frac{2-\alpha}{1-\alpha}\right)\left(\frac{1}{\mu^*}\right) - \frac{\frac{\mu^{*\frac{\alpha}{1-\alpha}}}{1-\alpha}\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} - \frac{\frac{\mu^{*\frac{\alpha}{1-\alpha}}}{1-\alpha}\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right\} \quad . \qquad (A.14)$$

Now, from equations (2), (6) and (9), we find that the last bracket term is equal to $\left(\frac{1}{\mu^*}\right)\left\{\frac{2-\alpha}{1-\alpha} - \frac{1}{1-\alpha}[(1-\theta^*)+\tau^*]\right\}$. Again, from equations (2) and (6), it appears that $\Big[1+$

$\left.\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]$ is equal to $\left(\frac{1}{\theta^*}\right)$; and from equations (2), (6) and (9), we find that $\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]$ is equal to $\frac{\alpha(1-\theta^*)}{\theta^*\tau^*}$. Incorporating all these equalities and putting values of μ^*, θ^* and τ^* from equations (14), (15) and (16), we obtain equation (18).

<u>Derivations of equations (19) and (20):</u>

From equation (2), we obtain

$$\frac{G}{K} = B^{\frac{1}{\beta}}(1-\theta)g^{-\frac{1}{\beta}} \quad . \tag{A.15}$$

Using equations (6) and (A.15), we obtain

$$\frac{(1-\theta)^{1-\alpha}}{\theta^{1-\alpha}} = \mu g^{\frac{\beta-\alpha}{\beta}}\frac{B^{\frac{\alpha}{\beta}}}{A}\frac{\beta}{\alpha} \quad . \tag{A.16}$$

From equation (A.16), we obtain equation (19) in the body of the article.

Now, from equation (9) and (A.15), we obtain

$$\left(\frac{\tau}{1-\tau}\right) = \frac{\mu(1-\theta)^{\alpha}}{A\theta^{\alpha}}g^{\frac{\beta-\alpha}{\beta}}B^{\frac{\alpha}{\beta}} \quad . \tag{A.17}$$

Using equations (A.16) and (A.17), we obtain

$$\left(\frac{\tau}{1-\tau}\right) = \frac{\alpha}{\beta}\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}} \quad . \tag{A.18}$$

From equation (A.18), we obtain equation (20) in the body of the paper.