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Optimal Mix of the Extended Nelson Siegel Model for Turkish Sovereign Yield Curve

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Abstract

The yield curve is one of the most fundamental tools used by central banks. One of the most popular methods to estimate yield curve by the central banks is Extended Nelson Siegel model. However, there are some technical differences in yield curve estimation. These differences are mainly related to the choice of objective function and the maturity spectrum of bonds in the data set. In this respect, this note aims to find out the optimal combination of the Turkish Treasury bond market yield curve based on the Extended Nelson Siegel model. Main findings indicate that the exclusion of long-term bonds results in a better in-sample fit for the short-term bonds. On the other hand, the inclusion of repo transactions leads to a worsening in in-sample fit across all maturity segments regardless of the choice of the objective function. Regarding the choice of objective function, weighted price minimization provides better in sample-fit of the yield curve for all different maturity segments when the repo transactions are excluded.

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Optimal Mix of the Extended Nelson Siegel Model for Turkish Sovereign Yield Curve

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Abstract

Since yield curve is one of the most fundamental tools used by the central banks, it is crucial to estimate spot rates precisely. The most popular method to estimate yield curve by the central banks is Extended Nelson Siegel (ENS) model. However, there are some technical differences in the implementation of the yield curve construction. These differences are mainly related to the choice of objective function and the maturity spectrum of bond in the data set. In this respect, this paper aims to find out the optimal combination of the Turkish Treasury bond market yield curve based on the ENS model. The results of the note indicate that the exclusion of long term bonds results in a better in-sample fit for the short-term bonds. On the other hand, the inclusion of repo transactions leads to worsening in-sample fit across all maturity segments regardless of the choice of objective function. In terms of the choice of objective function, weighted price minimization provides better in sample-fit of the yield curve for all different maturity segments when the repos are excluded. However, yield to maturity minimization results in better fit when repos are included in the data set.

CENTRAL BANK OF THE REPUBLIC OF TURKEY Head Office
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1. Introduction

The yield curve depicts the relationship between time to maturity and spot rates (zero rates). Since it contains information about the market expectations of future inflation and economic activity, it is widely used by many central banks as a guide for the monetary policy stance. However, yield curve can't be directly observed although the data about the prices and the yield to maturities of bonds are available. The problem here is that the bonds are issued across a finite set of maturities. One of the most common solutions to construct a continuous yield curve is through parametric models. These models describe the instantaneous forward rates or zero rates at all maturities as a single function of specific unknown parameters. One of the most popular parametric models is proposed by Nelson and Siegel (1987) and later extended by Svensson (1994), which is quite flexible to capture the different shapes of the yield curve and is widely used by central banks and many practitioners.

There are two main issues in the implementation of Extended Nelson Siegel (ENS); the maturity spectrum of bonds in the data set and the choices of the objective function. From the point of the maturity spectrum, if the short-term bonds are included, they tend to have higher weights in the optimization procedure since the bonds are generally weighted by the inverse of their durations. This leads to a better fit of the short end but not the rest of the curve. One solution is to ignore bonds with maturities shorter than some pre-specified maturities. However, it might result in poor fit of the short end of the curve. One way to solve this problem is to exclude some bonds and fit two different yield curves, generally for Treasury bills and Treasury bonds separately. Additionally, to increase the number of short-term securities, the inclusion of repo transactions to the data set for the yield curve estimation is proposed in the literature.

The second important point in the implementation of yield curve construction lies on the choice of the objective function. If the variance of the yield errors is assumed to be homoscedastic, then weighted price minimization or yield minimization is theoretically the same (Fong and Vasicek, 1982). In practice, the magnitude of the yield errors differs across bonds due to some factors such as liquidity (Berenguer et al., 2013). Therefore, the choice of objective function turns out to be an important point for yield curve estimation.

The optimization choices of the yield curve estimation in major central banks have been covered in BIS (2005). Table I shows the choice of objective function (minimized error), shortest maturity in estimation and relevant maturity spectrum in main central banks. It is observed that although these major central banks have been using the Nelson Siegel or Extended Nelson Siegel, there are some differences in the optimization objective and the maturity of the bonds involved in the estimation. Countries like France and Italy use weighted prices whereas countries such as Germany, Norway, Sweden, and Switzerland choose to minimize yields to maturities. Additionally, the shortest maturity included in the yield curve estimation differs across the central banks. Although some countries use all bonds and bills such as Finland, Spain, and Sweden, some countries exclude the bonds with the maturities shorter than 3 months or 1 year.

Table I: The Optimization Routines Followed in the Yield Curve Estimation				
Central Bank	**Estimation Method**	**Minimized Error**	**Shortest Maturity in Estimation**	**Relevant Maturity Spectrum**
Belgium	ENS &NS	Weighted Prices	Treasury Certificates>few days Bond> 1 year	Couple of days to 16 years
Finland	NS	Weighted Prices	>1 day	1 to 12 years
France	ENS &NS	Weighted Prices	Treasury Bills: All Treasury Notes>1 month Bond> 1 year	Up to 10 Years
Germany	ENS	Yields	> 3 months	1 to 10 Years
Italy	NS	Weighted Prices	Money Market Rates: Overnight and Libor rates from 1 to 12 months Bonds>1 year	1 to 30 Years
Norway	ENS	Yields	Money Market Rates > 30 days Bonds >2 year	Up to 10 Years
Spain	ENS	Weighted Prices	>1 day	Up to 10 Years
Sweden	ENS	Yields	>1 day	Up to 10 Years
Switzerland	ENS	Yields	Money Market Rates> 1 day Bonds >1 year	1 to 30 Years
Source: BIS (2005).				

This study aims to find out the optimal combination for the Turkish Treasury bond market yield curve based on ENS model. It tries to answer the following questions: Does the exclusion of some bonds contribute to the fit at the short end of the curve? Does the inclusion of the repo transactions increase the goodness of fit for the yield curve estimation? Does the optimization through the yield to maturity minimization provide any improvement for the goodness of fit?

2. Literature Review

Besides the comprehensive BIS study, Anderson et al. (2001) apply spline based techniques for the UK government bond markets. They include repo transactions whose maturities range from overnight to one year. The results indicate the benefit of using data from repo market to derive estimates of the nominal yield curve at short maturities. Yu and Fung (2002) estimate zero-coupon yield curve for Hong Kong SAR government securities using Extended Nelson Siegel with the choices of weighted price and yield error minimization as objective functions. They find out that both objectives result in very good estimates of the zero-coupon yield curve. However, they state that yield error minimization approach is computationally more demanding since it requires one more iteration stage. Additionally, the findings of the paper indicate that convergence problems are more likely to occur in yield error minimization and the results are more sensitive to the choice of the initial parameter values. They state that since the primary interest for the yield curve analysis is in the interest rates, yield error minimization approach is suggested.

Hladikova and Radova (2012) estimate the yield curve of the Czech Treasury bond market employing both yield and price minimization approaches based on different weighting schemes with daily data from 2002 to 2011. According to the price minimization criteria, equally weighted and the inverse of modified duration weighted yield curves show the best performance. Regarding yield minimization, putting equal weights on bonds produces more accurate estimate of yield curve.

The literature related to estimating the yield curve of Turkish Treasury bond market is focused on the comparison of different yield curve estimation methods. One of the most comprehensive studies, Akinci et al. (2006), estimates ENS yield curve for Turkish sovereign bond market using only price minimization excluding the bonds with maturities shorter than 3 months. However, they do not consider the impact of the choice of the objective function. In this regard, this is the first study that addresses the optimal combination of the yield curve estimation in Turkey to the best of our knowledge.

3. Yield Curve Estimation Methodology

The yield curve construction requires estimating zero rates and forward rates. Although zero rates for specific maturities might be calculated using the bootstrap technique, all zero rates and forward rates cannot be observed directly. Since it is not possible to construct a continuous yield curve using the bootstrap technique, parametric methods have been developed to estimate the yield curve. The main idea behind these methods is that they assume a functional form consistent with the properties of interest rates and optimize the parameters in the functional form.

Any bond price traded in the market can be expressed as follows:

$$P_t^i = \sum_{j=1}^{M} CF_{t_j}^i \ B_{t,t_j} \tag{1}$$

where CF denotes the matrix of the cash flows corresponding to bonds and M stands for the number of different cash flow dates for bond i. Let B_{t,t_j} be M-dimensional vector of zero coupon bond prices at time t for the maturities of t_j where $j = 1, 2,. ..M$. If we model the zero coupon bond price B_{t,t_j} as a function of unknown parameters, then fitted bond prices will be as follows:

$$P_t^{i,fitted} = \sum_{j=1}^{M} CF_{t_j}^i \ h(\beta, t, t_j) \tag{2}$$

where $h(\beta, t, t_j)$ is any parametric function describing the zero coupon bond price and β is a matrix consisting of parameters. The objective is to minimize the weighted difference between the fitted prices and market prices of bonds. The general application is to weight the bonds with the inverse of their durations since the estimation errors for short-term zero rates do not lead to substantial price differences. The objective function can be written as:

$$\min_\beta \sum_{i=1}^{K} \left(\frac{P_t^i - P_t^{i,fitted}}{D_t^i} \right)^2 \tag{3}$$

where D_t^i denotes the Macaulay durations at time t and K stands for the number of bonds.[1] Instead of comparing the bond prices, an alternative method to obtain the parameters is to minimize the squared deviation between estimated and observed yield to maturities. In this case, the estimation procedure involves two steps. The first step is to find out the fitted prices based on the parameter estimates. Then, yields to maturity for the bonds traded in a given day are estimated. However, the relation between bond price and yield to maturity (y) is nonlinear, so the yield to maturity for each bond should be solved numerically[2].

$$\mathrm{P_t^{i,fitted}} = \sum_{j=1}^{M} \mathrm{CF_{t_j}^i}\, e^{-y_t^{i,fitted}(t_j-t)} \tag{4}$$

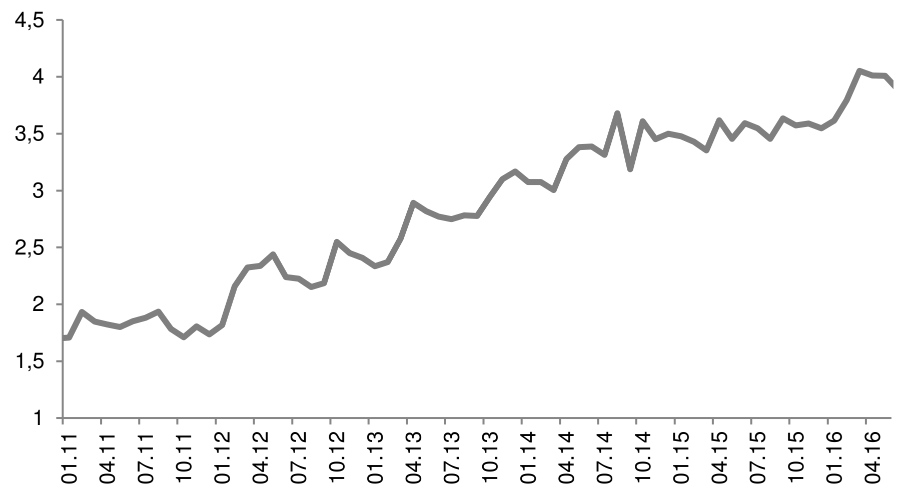
After we find out the estimated yield to maturities, the next step is to minimize the difference between the observed yields and estimated yields choosing the optimal parameters.

$$\mathrm{min_\beta} \sum_{i=1}^{K} (\mathrm{y_t^i} - \mathrm{y_t^{i,fitted}})^2 \tag{5}$$

4. Data and Methodology

We use daily weighted average prices of Turkish Treasury bonds in Borsa Istanbul (BIST) Debt Securities Market Daily Bulletin. The data set used in the estimation of the yield curves includes zero coupon bonds and fixed coupon bonds. We exclude floating rate coupon bonds due to unknown future coupon payments. Since the long-term maturity bonds are issued after 2010, the sample period between January 2011 and May 2016 is used. The average maturity of the bonds traded in BIST has been increasing over time (Figure 1).

Figure 1: Average Maturity of Turkish Treasury Bonds (Years)



Source: BIST, Authors' Calculations.

We test the performance of the yield curve estimation with the inclusion of the repo transactions in BIST Interbank Repo Market. Since repo agreements are seemed to be as a

[1] The Macaulay duration is the weighted average term to maturity based on the cash flows of a bond. The weight of each cash flow is calculated by dividing the discounted value of the cash flow by the price of a bond.

[2] The yield to maturity calculation here is continuously compounded. The actual yield to maturity is also converted to the continuously compounded yields.

secured loan and a satisfactory substitute for T-bills, we complement our dataset with repo transactions. The interbank repo data in BIST Debt Securities show that the maturity of the significant portion of the repo transactions is one or three days.

Many central banks widely use the Svensson (1994) model because of its parsimonious nature of the functional form. Specifically, ENS model assumes that the instantaneous forward rates can be described explicitly by the following functional form:

$$f(m,\beta,\tau) = \beta_0 + \beta_1 e^{\frac{-m}{\tau_1}} + \frac{m}{\tau_1}\beta_2 e^{\frac{-m}{\tau_1}} + \frac{m}{\tau_2}\beta_3 e^{\frac{-m}{\tau_2}} \qquad (6)$$

where m denotes the time to maturity, $\beta=(\beta_0,\beta_1,\beta_2,\beta_3,\tau_1,\tau_2)$ is the parameter set to be estimated. The parameters of ENS model have economically interpretable meanings. β_0 reflects a level of the curve or long run expectations about the level of interest rates. β_1 represents the slope of the yield curve that can be defined as the difference between long-term and short-term instantaneous forward rate. β_2 and β_3 can be interpreted as magnitude and direction of the hump. τ_1 and τ_2 determine the location of the humps in the curve. The zero coupon or spot interest rate can be derived by integrating the instantaneous forward rates.

Spot rates are estimated by assigning some initial values for the parameters set $\beta=(\beta_0,\beta_1,\beta_2,\beta_3,\tau_1,\tau_2)$ and then theoretical spot rates and discount factors are calculated. Finally, ENS model parameters are computed through the minimization of the squared difference between weighted price errors or yield errors between fitted and actual prices. We consider the in-sample fit to evaluate the performance of different alternatives. In this respect, Root Mean Squared Error (RMSE) is used. RMSE is calculated as follows:

$$RMSE = \sqrt{\sum_{i=1}^{K} \frac{\left(P_i - P_i^{fit}\right)^2}{K}} \qquad (7)$$

5. Empirical Evidence

One of the focus points of the note is whether the exclusion of the bonds with the maturities greater than pre-specified years contributes to the in-sample fit of the short end of the yield curve. Therefore, we exclude the bonds with maturities greater than 2 years and 5 years.[3] Then, we compare the in-sample fit of the curve across different maturities using weighted price minimization as an objective function. For instance, if none of the bonds are excluded, then RMSE for the bonds with the maturity between 1 year and 2 years is around 0.114 (Table II).[4] However, if we exclude the bonds with maturities greater than 2 years, RMSE for the bonds with the maturity between 1 year and 2 years drops to 0.077. Generally, it is observed that when the bonds with longer maturity are excluded, the in-sample fit at the short end of the yield curve becomes better. This results from the fact that higher weights are assigned to the shorter maturity bonds. However, excluding longer maturity bonds leads to loss of information due to the coupon payments of long-term bonds. The empirical evidence here suggests that the effect of assigning higher weights to the shorter maturity bonds outweighs the loss of information due to the exclusion of long-term bonds.

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[3] 2 years and 5 years can be regarded as medium-term of the Turkish Treasury yield curve. The results are robust when we consider other medium-term maturities.

[4] 0.114 RMSE can be considered as % 0.114 error since prices of the bonds are around face value (100).

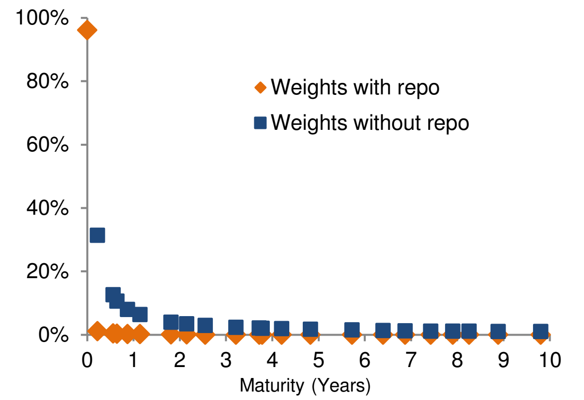
Table II. The In-Sample Fit Across Different Maturities without Repos						
The Period Between January 2011 and May 2016						
			Up to 1 Year	**1 to 2 Years**	**2 to 5 Years**	**5 to 10 Years**
No Filter	RMSE	Average	0.056	0.114	0.211	0.334
		Stdev.	0.036	0.068	0.133	0.330
5 Years Filter	RMSE	Average	0.055	0.113	0.196	--
		Stdev.	0.036	0.068	0.137	--
2 Years Filter	RMSE	Average	0.053	0.077	--	--
		Stdev.	0.035	0.043	--	--
"2 Years Filter" and "5 Years Filter" indicate that the bonds with the maturities greater than 2 years and 5 years are excluded from the dataset.						

We also repeat the previous analysis using the repo transactions. In this respect, we include the repo transactions to the yield curve estimation and calculate the price errors for the bonds with different maturity segments. Similar to the previous finding, the short end of the yield curve up to 1 year are estimated better when long-term bonds are excluded. However, when the yield curves estimated with repo and without repos are compared, it is observed that the inclusion of repo transactions worsens the in-sample fit of the yield curve significantly (Table III).

Table III. The In-Sample Fit Across Different Maturities with Repos						
The Period Between January 2011 and May 2016						
			Up to 1 Year	**1 to 2 Years**	**2 to 5 Years**	**5 to 10 Years**
No Filter	RMSE	Average	0.092	0.209	0.491	1.259
		Stdev.	0.050	0.123	0.351	1.243
5 Years Filter	RMSE	Average	0.085	0.198	0.563	--
		Stdev.	0.047	0.119	0.455	--
2 Years Filter	RMSE	Average	0.078	0.207	--	--
		Stdev.	0.042	0.143	--	--
"2 Years Filter" and "5 Years Filter" indicate that the bonds with the maturities greater than 2 years and 5 years are excluded from the dataset.						

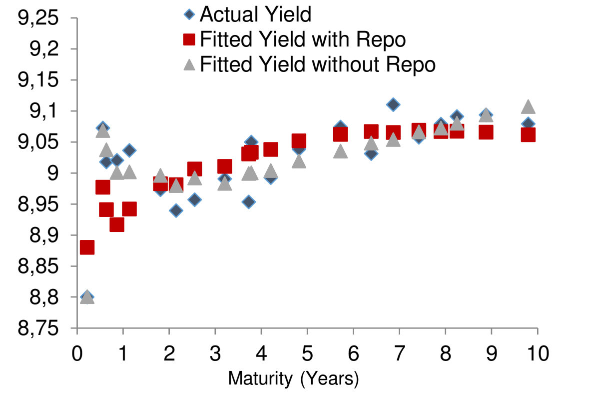
Although we include more observation to the data set, the results show that repo transactions do not improve the in-sample fit. This can be attributed to the maturity of the repo transactions. Since the maximum maturity of the repo transactions included in the estimation is 3 days, the repos tend to have very large weights in the optimization. For instance, Figure 2 shows the effect of the repo transactions in the yield curve estimations on April 25, 2016. The repo transactions tend to have % 96 weights in the optimization, and this situation results in decreases in the weights of the other bonds. Figure 3 shows that the yield curve estimation including the repo transaction results in larger price errors, especially on short-term bonds. In fact, there exists empirical evidence that repo transactions improve the fit of the yield curve in some countries. However, the maturities of the repo transactions in other countries are much longer than one or three days.

Figure 2: The Weights Assigned to Each Bond on 25.04.2016



Source: BIST, Authors' Calculations.

Figure 3: In Sample Fit on 25.04.2016



Source: BIST, Authors' Calculations.

We also examine whether the yield error minimization results in better estimates than weighted price minimization. In this respect, yield to maturity for each bond is numerically calculated using estimated prices with the ENS method and the squared difference between estimated and actual yield to maturities is minimized. Since there is no analytical solution for the yield to maturity given bond prices, the optimization through yield to maturity minimization is computationally burdensome. Therefore, we have only reported the yield minimization results for the period between January 2015 and May 2016. The optimal parameters under the yield minimization are used to calculate the price errors for each bond. The results in Table IV show that the price minimization gives better in-sample fit for the Turkish Treasury bond data than the yield minimization. Additionally, the price errors tend to be larger in yield minimization compared to price minimization for all the bonds with different maturity segments.

Table IV. Comparison of Yield and Price Minimization Approaches without Repos

The Period Between January 2015 and May 2016

			Up to 1 Year	**1 to 2 Years**	**2 to 5 Years**	**5 to 10 Years**	**Overall**
Yield Minimization	RMSE	Average	0.091	0.148	0.301	0.513	0.334
		Stdev.	0.050	0.094	0.152	0.394	0.180
Price Minimization	RMSE	Average	0.075	0.102	0.234	0.310	0.220
		Stdev.	0.051	0.066	0.131	0.205	0.112

The last analysis conducted in the study is to investigate the effect of repo transactions in the yield minimization. The results indicate that the yield minimization provides better sample fit than price minimization when the repo transactions are included. However, the inclusion of repo transactions worsens the in-sample fit for both yield minimization and price minimization.

Table V. Comparison of Yield and Price Minimization Approaches with Repos							
The Period Between January 2015 and May 2016							
			Up to 1 Year	**1 to 2 Years**	**2 to 5 Years**	**5 to 10 Years**	**Overall**
Yield Minimization	RMSE	Average	0.103	0.187	0.393	0.656	0.414
		Stdev.	0.049	0.114	0.246	0.513	0.227
Price Minimization	RMSE	Average	0.111	0.212	0.424	0.994	0.577
		Stdev.	0.050	0.132	0.271	0.799	0.358

6. Conclusion

This technical note investigates the optimal combination for the Turkish sovereign yield curve estimation using the ENS method. First, it aims to test whether the exclusion of long term bonds results in better fits at the short end of the curve. Second, it focuses on whether the inclusion of repo transactions improves the in-sample fit. Finally, it compares the performance of yield minimization with that of weighted price minimization.

Our results suggest that fitting two different yield curves for Treasury bond market provides a better fit for short-term bonds. In other words, the exclusion of long-term bonds improves the fit of the short end of the yield curve. The positive effect of assigning higher weights to the short-term bonds outweighs the adverse effect due to the loss of information from the coupon payments of long-term bonds. The inclusion of repos leads to worsening of the in-sample fit for all the bonds across different maturity segments. This result might be attributed to very high weights assigned to the repo transactions due to their very short-term nature. Lastly, we apply yield to maturity minimization rather than weighted price minimization and find that the weighted price minimization provides more plausible results for the Turkish Treasury bond market.

The inclusion of repos does not provide better results since the maturity of repos is quite short. However, the poor performance of the yield curve estimation with the inclusion of repo transactions might be enhanced if the repo market becomes more concentrated around longer maturities like in advanced countries. Additionally, it should be kept in mind that the optimal mix for the yield curve estimation might differ across time depending on the maturity spectrum of the bonds traded, the lengthening of the maturities of repo transactions and increasing homogeneity in terms of liquidity premiums for different maturities.

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