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Fractal analysis revisited: The case of the US industrial sector stocks

Taro Ikeda
Graduate School of Economics, Kobe University

Abstract

In contrast to earlier studies of long memories, this paper indicates that most of the US industrial sector stocks have the long memories when we consider the structural changes for the Hurst exponents.

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Contact: Taro Ikeda - young.tubo315o@gmail.com.
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1 Introduction

Fractal geometry has a concern with a long memory property of time-series by a power law distribution. In this methodology, a wider degree of autocorrelations than the unitroot process (random walk) can be considered. In this respect, the fractal geometry is a method to detect generalized autocorrelations more than the unitroot process.

The generalization of the random walk model with the fractal geometry implies that the efficient market hypothesis is not valid anymore if the autocorrelation of asset price is higher than the unitroot process. This is because an independence among the sequence of the past shocks on the current price is destroyed by the higher order autocorrelations of the fractal series.

By using spectral regressions, Barkoulas and Baum (1996) have suggested that the US aggregate and industrial sector stock prices seem not to have the long memories. However in contrast to them, we insist that most of the US industrial sector stocks have long memories when we consider the structural breaks for the estimated Hurst exponents.[1][2][3] This result implies the failure of the efficient market hypothesis in the recent US industrial sector stocks.

This paper proceeds as follows. Section 2 introduces a fractal Brownian motion and R/S analysis to detect the long memories of the stock prices empirically. Section 3 presents obtained results. Section 4 concludes.

2 Methodology

This section introduces a fractional Brownian motion which provides mathematical backgrounds of the fractal geometry, and then the Hurst's R/S analysis to determine the generalized autocorrelations of stock prices.

2.1 A fractional Brownian motion

A fractional Brownian motion (fBm) of the fractal geometry is a key concept to generalize the ordinary Brownian motion (random walk). To reveal the relationship between the two, we denote the difference of the fractional Brownian motion $B_H(t)$ for any two times t and t_0 as $|B_H(t) - B_H(t_0)|$. Then, the second moment of the difference can be described as:

$$\langle |B_H(t) - B_H(t_0)|^2 \rangle \propto |t - t_0|^{2H}, \qquad (1)$$

[1] Onali and Goddard (2011) find the long range dependence of stock market indexes of Itary and Czech republic. Also, Goddard and Onali (2014) develop an empirical test for self-affinity of stock price returns.

[2] Also, Chimanga and Mlambo (2014) indicate that the Johannesburg stock exchange index exhibits the long rage dependence and suggest that the stock market indexes of emerging countries are more autocorrelated than those of developed countries.

[3] In addition, see Booth et al. (1982) for the case of exchange rates of developed European countries.

where $\langle\rangle$ is an ensemble average, and H is the Hurst exponent which takes a value 1/2 for the random walk case.[4] It is relevant that the variance of the ordinary Brownian motion is the special case of the fBm's variance (1).

Also, note that the variance of the fBm goes to infinity with t, and the rate is faster than the one of the random walk in the case $1/2 < H < 1$.

2.2 The Hurst's R/S analysis

The R/S analysis has been proposed by Hurst (1955) in order to determine the generalized degree of autocorrelations for any time-series.

We describe the increments of time-series in logarithms as $x(t) \in \big(x(1), ..., x(T)\big)$, and calculate the ensemble average over window-size τ as:

$$\langle x_t|\tau\rangle = \frac{1}{\tau}\sum_{\tau(l-1)+1}^{\tau l} x(t), \tag{2}$$

for $l = 1 : \text{ceil}(T/\tau)$.

Then, we calculate the accumulated sum of dispersion of $x(t)$ from the average as follows:

$$x(t,\tau) = \sum_{t=1}^{T}\big(x(t) - \langle x(t)|\tau\rangle\big). \tag{3}$$

By subtracting the local averages from $x(t)$, constant trends in the sample are excluded.

In addition, we calculate $R(\tau)$ of the max-min range over the window-size τ and $S(\tau)$ of the standard deviations in each segment as follows:

$$R(\tau) = \max_{\tau(l-1)+1\le t\le \tau l} x(t,\tau) - \min_{\tau(l-1)+1\le t\le \tau l} x(t,\tau), \tag{4}$$

and

$$S(\tau) = \sqrt{\frac{1}{\tau}\sum_{\tau(l-1)+1}^{\tau l}\big(x(t) - \langle x(t)\rangle\big)^2}. \tag{5}$$

Clearly, the range R is an increasing function for τ.

Finally, the R/S statistics is defined as a fraction of (4) and (5), which obeys the power law distribution as:

$$E_\tau\big(R(\tau)/S(\tau)\big) \propto \tau^H, \tag{6}$$

where H is the Hurst exponent as already introduced in Section 2.1.

Using Hurst's H, we can categorize autocorrelations of any time-series into three cases. First, the case $0 < H < 1/2$ corresponds to *antipersistence* or *short correlation*, where

[4] The variance equation of the random walk takes well-known formulation as:

$$\langle |B_H(t) - B_H(t_0)|^2\rangle \propto |t - t_0|.$$

the time-series increasing today must decrease tomorrow. Second, the case $H = 1/2$ implies the random walk as frequently used in financial market analysis, implying the shock terms of the series are not mutually interdependent over time. Third, $1/2 < H < 1$ denotes *long dependence* or *long memory* where the past shocks do not disappear and do affect for a very long time on the current series, and therefore trend-reinforcing.[5]

3 Empirical results

This section estimates the statistical Hurst exponents for the US industrial sector stock prices. To this end, we utilize the American sector stock indexes categorized by Dow Jones. All data are weekly-frequency and obtained from Thomson Reuters Datastream.[6]

Here, we implement 5555 times bootstrapping to construct confidence intervals for estimated H, since the variance for H goes to infinity in the case $1/2 < H < 1$.

3.1 Benchmark results

Table 1 provides the statistical Hurst exponents for the US industrial sector stocks. Almost all point estimates of the exponents exceed the threshold of 0.50. Also, the long memories are strongly significant in 7/15 industries: Auto and Parts, Consumer Goods, Consumer Services, Media, Real Estate, Technology, Telecommunications. This is because the lower confidence intervals for the Hurst exponents of these stocks take values more than 0.50.[7]

Figure 1 confirms these results. In most of the figures, the fitted lines replicated by the bootstrapping contain realized R/S values, which grantees an appropriateness for our regressions. However, there are only two exceptions: Farming and Fishing and Oil and Gas. These realized values do not fall within the bootstrapped lines, implying a possibility of structural changes for H.

3.2 Structural changes for the Hurst exponents

This section examines whether the time-dependent structural changes occurs on the stock price indexes by sequential estimations of H.[8]

Figure 2 suggests the structural changes of H for all sectors. These results are amazing in the sense that most of the sectors (11/15 sectors) exhibit the long memories.

In sum, the recent US stock markets by industrial sectors are not efficient when we revisit the fractal market hypothesis by Benoit B. Mandelbrot.

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[5] See Feder (1988, Chap. 9) for this classifications.

[6] All results are robust even if we use the daily observations (see Supplemental Appendix).

[7] The result of Real Estate is consistent with Ikeda (2016), who suggests the fractality of the US stock market.

[8] See Sibbertsen (2004) for econometric assessments for the structural breaks of the long range dependence.

4 Conclusion

This paper found that most of the US industrial sector stocks are fractal, and therefore have long memories.

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Figures and Tables

Table 1: Hurst exponents by industrial sectors

No.	Name	Hurst exponent	Lower	Upper
1	The Americas Auto and Parts	0.64570	0.54456	0.75352
2	The Americas Basic Resources	0.50475	0.46536	0.60480
3	The Americas Consumer Goods	0.60409	0.53906	0.65110
4	The Americas Consumer Services	0.62969	0.58543	0.71856
5	The Americas Farming and Fishing	0.49060	0.39037	0.53957
6	The Americas Financial Services	0.56600	0.47879	0.63285
7	The Americas Food and Beverages	0.56925	0.48216	0.60622
8	The Americas Media	0.68863	0.55370	0.83186
9	The Americas Oil and Gas	0.49178	0.41657	0.50922
10	The Americas Real Estate	0.57230	0.53531	0.67868
11	The Americas Retail	0.57423	0.47975	0.59372
12	The Americas Technology	0.68715	0.57431	0.75518
13	The Americas Telecommunications	0.65490	0.52544	0.74653
14	The Americas Travel and Leisure	0.52539	0.41885	0.58782
15	The Americas Utilities	0.52157	0.45394	0.62740

Note: ‘Lower’ and ‘Upper’ denote bootstrapped 95% confidence intervals.

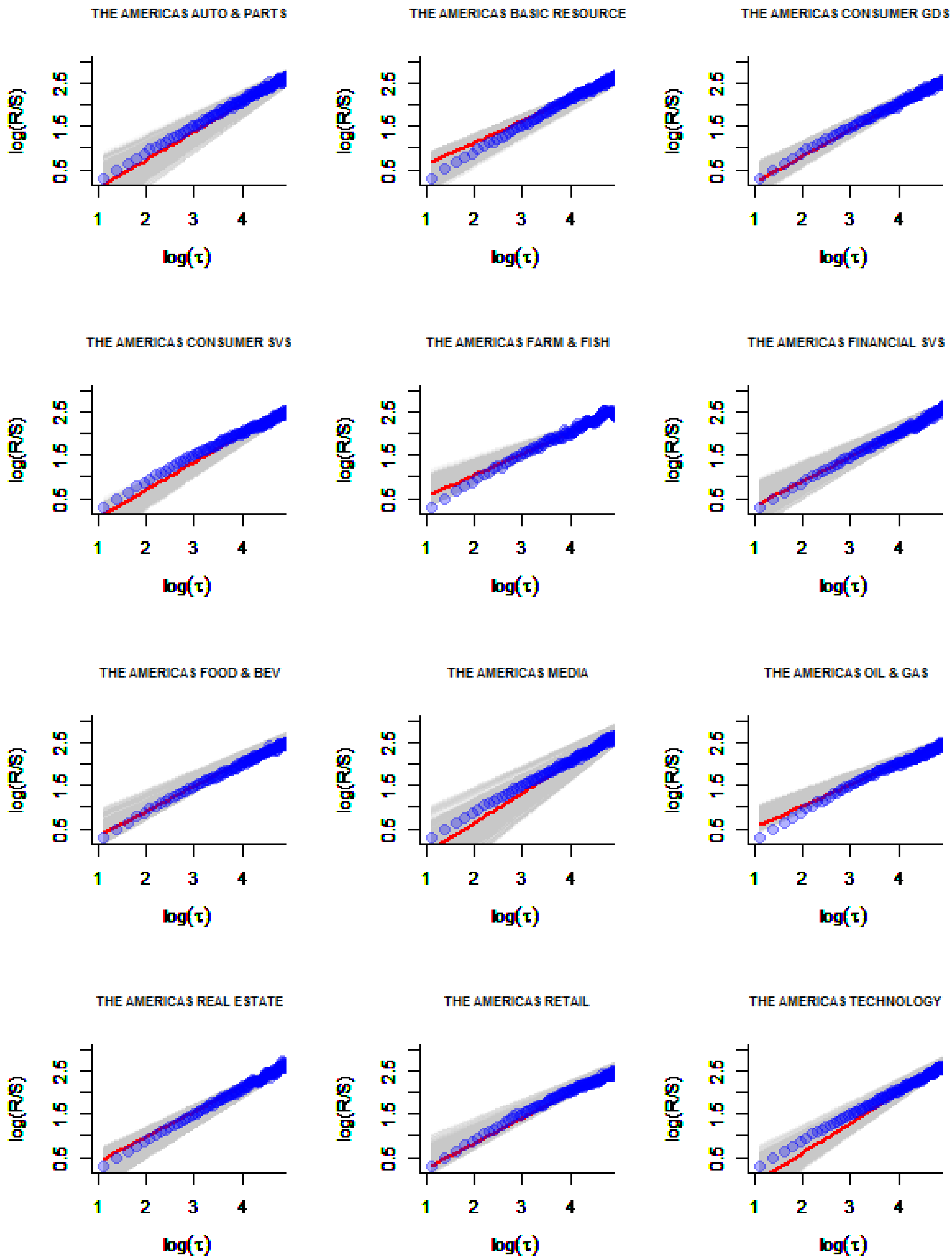
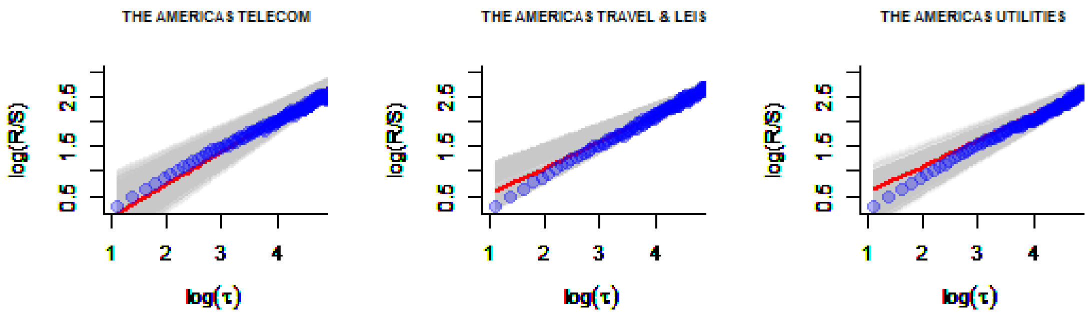


Figure 1: Bootstrapped regressions

Note: Gray lines are generated by regressions using 5555 bootstrapping.


THE AMERICAS TELECOM
THE AMERICAS TRAVEL & LEIS
THE AMERICAS UTILITIES
log(R/S)
log(τ)

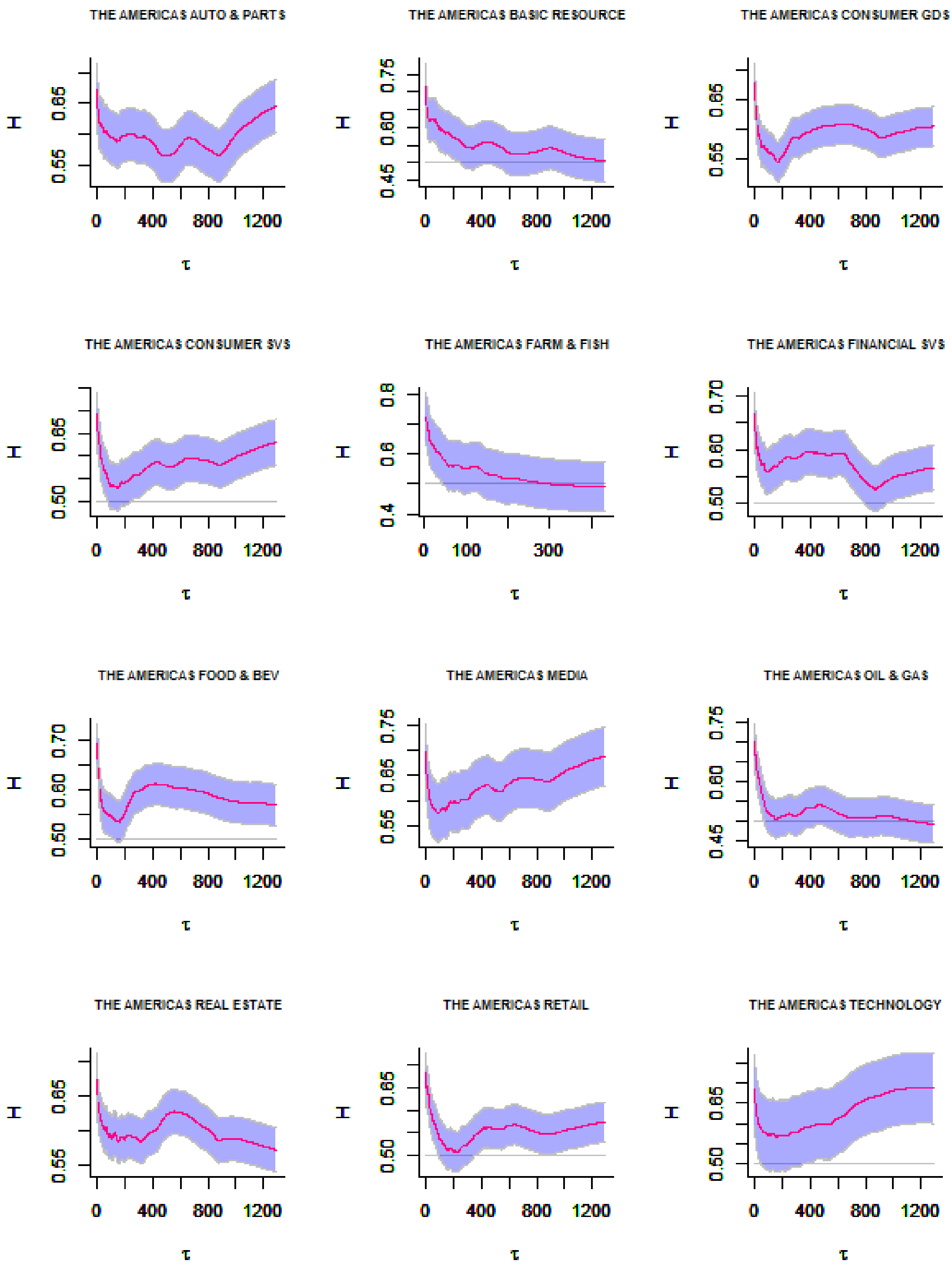
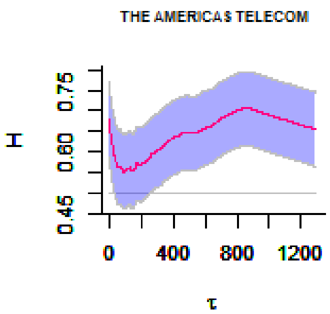
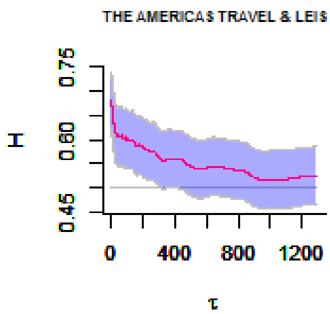


Figure 2: Sequential estimation of H

Note: Shaded area denotes 95% confidence interval for H.


THE AMERICAS TELECOM
H
0.45
0.60
0.75
0
400
800
1200
τ


THE AMERICAS TRAVEL & LEIS
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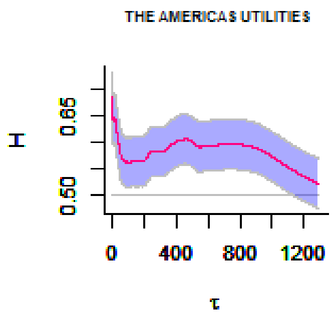
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