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Portfolio allocation in actively managed funds

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Abstract

We consider the problem of an investor who allocates his wealth among a risky asset and a managed portfolio. We obtain the optimal strategies of the fund managers for two different incentive schemes. We discuss an example of comparison of the efficient frontiers for the investor, in a model with mean reverting returns.

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1 Introduction

Fund's managers receive direct and indirect incentives. Direct, or explicit, incentives are paid directly by the investor under the form of management fees. Indirect, or implicit, incentives, are provided by the inflows of funds coming from new investors. Such inflows are related to the performances of the fund and may also have a negative sign, that is they may actually be outflows, in case of bad performances. The objective of this paper is to analyze the impact of the incentives on fund's managers strategies and, consequently, on the portfolio allocation of the investors.

The empirical relation between fund's performances and inflows/outflows has been studied starting from the seminal papers by Chevalier and Ellison (1997) and Sirri and Tufano (1998). Chevalier and Ellison (1997) provided some evidence that the rate of inflow linearly increases in the the relative performances with respect to a benchmark, but it has a lower and an upper bound. Basak, Pavlova and Shapiro (2007) determined the optimal strategy for a fund's manager with a power utility and implicit incentives bounded-linear as in Chevalier and Ellison (1997) in a continuous time model where the assets follow a geometric brownian motion. Cuoco and Kaniel (2011) determined the equilibrium prices in a market where portfolio managers receive a direct compensation from the investors. Here we consider a partial equilibrium model, where prices are given and one investor allocates his wealth between a market index, an actively managed mutual fund and a risk-free asset. We assume that the mutual fund is managed by a portfolio manager who receives an indirect compensation depending on the performance over a period of length T. The fund's value is determined by the portfolio strategy of the manager during the period and by capital inflows/outflows at the end of the period. The fund flows depend on the fund's performance relative to a benchmark, that is a fixed portfolio of stocks and money market. In the related paper Nicolosi, Angelini and Herzel (2017) determined the optimal strategy for a portfolio manager subject to implicit incentives. Here we will elaborate on some of the results of that study.

2 The model

We consider a market model with one riskless and one risky asset. One fund manager follows a dynamic strategy to optimize the utility of his wealth at time T, which is determined by the relative performance of the fund with respect to a benchmark consisting of a fixed portfolio of the two assets. One investor, who does not have access to the market continuously in time, chooses a static allocation at time $t = 0$ to optimize his utility at time T. The investor can buy any combination of the two basic assets and of the managed fund.

2.1 Asset prices

Let us define a complete probability space $(\Omega, \mathcal{F}, P)$, with filtration $\{\mathcal{F}_t\}$ $t \geq 0$ generated by a standard Brownian motion Z. The market is composed by a riskless asset with dynamics

$$dB_t = B_t r dt \tag{1}$$

where r is a constant interest rate, and by one risky asset following

$$dS_t = S_t \left(\mu_t dt + \sigma dZ_t\right) \tag{2}$$

where μ_t is a bounded and $\{\mathcal{F}_t\}$-progressively measurable process and σ is constant. We assume that the market price of risk

$$X_t = \sigma^{-1}(\mu_t - r) \tag{3}$$

follows the dynamics

$$dX_t = \lambda_X(\bar{X} - X_t) + \sigma_X dZ_t \tag{4}$$

where $\bar{X}$, λ_X, and σ_X are given constants representing, respectively, the long run expected value of X, the strength of its attraction towards $\bar{X}$ and the uncertainty on its evolution. Therefore, the market is dynamically complete and arbitrage free, with the unique state-price density process satisfying the equation

$$\frac{d\xi_t}{\xi_t} = -r dt - X_t dZ_t. \tag{5}$$

2.2 The manager

A fund manager dynamically allocates the fund's assets, initially valued at W_0, through a self-financing strategy in the two basic assets. The value of the portfolio W follows

$$\frac{dW_t}{W_t} = (r + \theta_t \sigma X_t)dt + \theta_t \sigma dZ_t \tag{6}$$

where θ_t represents the fraction of the portfolio value invested in the risky asset at time t. Since the strategy is self financing, the process $\xi_t W_t$ is a martingale with respect to the natural measure P.

At time T the manager receives an indirect compensation $F(W_T, Y_T)$, that depends on the relative performance of the fund with respect to a benchmark Y. The benchmark is a portfolio with a constant fraction β invested in the risky asset and the remaining fraction in the riskless one. Hence, the dynamics of the benchmark is

$$\frac{dY_t}{Y_t} = (r + \beta\sigma X_t)dt + \beta\sigma dZ_t. \tag{7}$$

The manager has a constant relative risk aversion utility function

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma > 0.$$

Since the market is complete, we can apply the martingale method proposed by Cox and Huang (1989) and solve the problem

$$\max_{W_T} E[u(F(W_T, Y_T))], \tag{8}$$

$$\text{s.t. } E\frac{\xi_T}{\xi_0} W_T = W_0. \tag{9}$$

We consider two different specifications of the compensation. The first one, which we will call spread, is proportional to the final wealth of the fund and is given by

$$F(W_T, Y_T) = f_T(W_T, Y_T) W_T,$$

where

$$f_T(W_T, Y_T) = \begin{cases} f_L & \text{if} \quad R_T^W - R_T^Y < \eta_L \\ f_L + \psi(R_T^W - R_T^Y - \eta_L) & \text{if} \quad \eta_L \leq R_T^W - R_T^Y < \eta_H \\ f_H := f_L + \psi(\eta_H - \eta_L) & \text{if} \quad R_T^W - R_T^Y \geq \eta_H \end{cases} \tag{10}$$

with $f_L > 0, \psi = (f_h - f_L)/(\eta_h - \eta_L) > 0$, $\eta_L \leq \eta_H$, and where $R_T^W = \log \frac{W_T}{W_0}$ and $R_T^Y = \log \frac{Y_t}{Y_0}$, and we assume $Y_0 = W_0 = 1$.

From Basak, Pavlova and Shapiro (2007), Proposition 2, if the following condition holds (henceforth Condition A): $\frac{\gamma}{1-\gamma}\left(\frac{f_H+\psi}{f_L}\right)^{1-1/\gamma} + \left(\frac{f_H+\psi}{f_H}\right) - \frac{1}{1-\gamma} \geq 0$, then the optimal final wealth relative to the benchmark is

$$\frac{W_T^*}{Y_T} = f_H^{1/\gamma-1} y^{-1/\gamma} \zeta^{-1/\gamma} \mathbf{1}_{\zeta<\zeta_\star} + e^{\eta_H} \mathbf{1}_{\zeta_\star \leq \zeta < \zeta^\star} + f_L^{1/\gamma-1} y^{-1/\gamma} \zeta^{-1/\gamma} \mathbf{1}_{\zeta \geq \zeta^\star} \tag{11}$$

where $\zeta_T = \xi_T Y_T^\gamma$ and y is the Lagrange multiplier that ensures that the optimal wealth satisfies $W_0 = E_0\{\xi_T W_T^*\}$. Furthermore, $\zeta_\star = f_H^{1-\gamma} e^{-\gamma\eta_H}/y$ and $\zeta^\star > \zeta_\star$ satisfies $\hat{g}(\zeta) = 0$, with $\hat{g}(\zeta) = \left(\gamma\left(\frac{y}{f_L}\zeta\right)^{1-1/\gamma} - (f_H e^{\eta_H})^{1-\gamma}\right)/(1-\gamma) + e^{\eta_H} y\zeta$.

We assume henceforth that Condition A holds. Nicolosi, Angelini and Herzel (2017) provide an explicit solution for the optimal strategy followed by the manager in this case.

The second specification of the compensation function, which we call "fulcrum", following Cuoco and Kaniel (2011) is

$$F(W_T, Y_T) = bW_T + c(W_T - Y_T).$$

If $b = 1$ and $c = 0$, that is the case when the manager is managing his own money, the optimal final wealth can be found directly from the first order conditions for problem (8), (9), and the corresponding wealth at time T is

$$\hat{W}_T = \frac{\xi_T^{-1/\gamma}}{H_0(-1/\gamma)} \tag{12}$$

where

$$H_0(-1/\gamma) = E^Q\{\zeta_T^{-1/\gamma}\}$$

and E^Q is the expectation under the forward risk neutral measure with respect to the benchmark. For more details on this argument and on the computation of the function $H_0(-1/\gamma)$ see Nicolosi, Angelini and Herzel (2017). We remark that, for a deterministic market price of risk, this solution is the optimal strategy provided by Merton (1971), while for a mean-reverting market price of risk it is the one found by Wachter (2002). Henceforth we will refer to such strategy as to the "Normal" one, because it is not influenced by external incentives.

The optimal final wealth for the fulcrum specification is given by the following result:

Proposition 1 *The optimal wealth at time T of problem (8), (9) with compensation function*

$$F(W_T, Y_T) = bW_T + c(W_T - Y_T)$$

is

$$W_T^* = \frac{1}{b+c}\left(b\hat{W}_T + cY_T\right),$$

where $\hat{W}$ is the Normal portfolio (12), that is the solution for $b = 1$ and $c = 0$.

Proof. The First Order Condition for problem (8), (9) yields to

$$W_T^* = \frac{1}{b+c}\left[\left(\frac{y\xi_T}{b+c}\right)^{-1/\gamma} + cY_T\right].$$

The Lagrange multiplier y can be determined by imposing the budget constraint (9): $y^{-1/\gamma} = (b+c)^{-1/\gamma} b/H_0(-1/\gamma)$. The result follows by using definition (12). □

2.3 The investor

At time $t = 0$ the investor decides the allocation of his wealth in a buy and hold strategy with a time horizon T, taking his decision by considering the mean and the variance of a portfolio consisting of the two basic assets and of the managed fund. The objective of the following examples is to show how the optimal allocation of the investor is affected by the different kind of incentives of the manager.

To compute the mean and the variance of the random variable (S_T, W_T^*) we simulate the dynamics (5) and (2) and then we substitute in the optimal final wealth W_T^* of the manager. Figure 1 shows the efficient frontiers produced by two different specification of

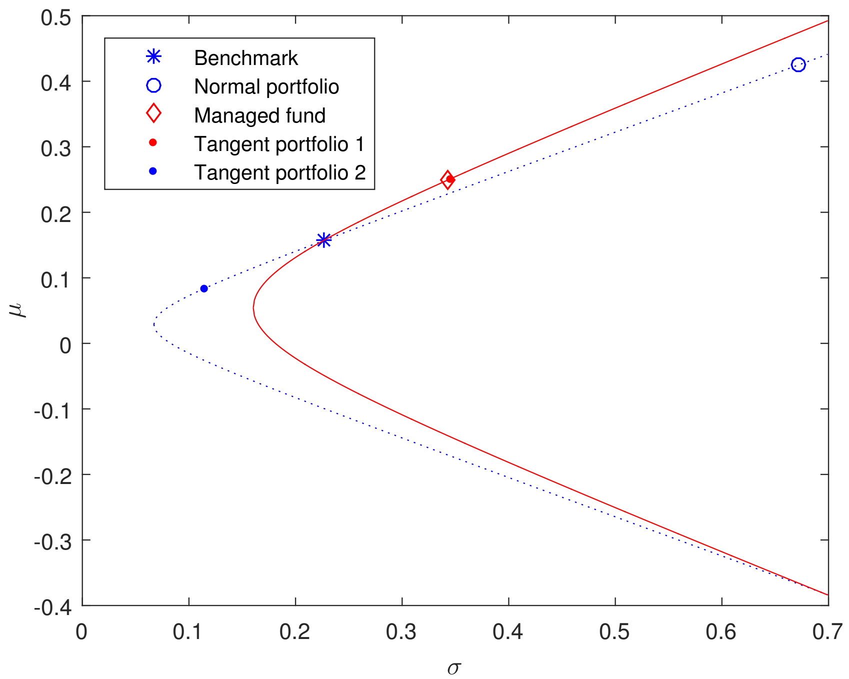

Figure 1: Efficient frontiers for an investor who allocates his wealth between the risky asset and the managed portfolio with convex kind incentive (continuous line) or with linear incentive (dotted line)

the incentive functions. We use the following model parameters: $T = 1$, $r = 0$, $\sigma = 0.2$, $X_0 = 0.5$, $\bar{X} = 2X_0$, $\sigma_X = 0.1$, $\lambda_X = log(2)/0.5$. The continuous line represents the frontier of the opportunity set given by investing in the risky asset and in the managed fund with the spread incentives scheme (10) where following Basak, Pavlova and Shapiro

(2007), we used $f_L = 0.8$, $f_H = 1.5$, $\eta_L = -0.08$, $\eta_H = 0.08$ and assumed that the benchmark coincides with the risky asset (i.e. we set $\beta = 1$). The dotted line represents the frontier when the portfolio manager receives fulcrum kind incentive. As shown by Proposition 1, in this case the optimal wealth is a linear combination of the Normal portfolio and of the benchmark. Hence the corresponding frontier depends only on the Normal portfolio and the benchmark and does not change with the contract parameters' specification.

We observe that for levels of standard deviation greater than that of the benchmark one, the introduction of the spread incentive positively affect the opportunity set of the investor. However, this effect disappears when introducing also the risk free asset, since the slopes of the linear frontiers are very close to each other. However, the two tangent portfolios are very different. For the spread incentive scheme the tangent portfolio almost coincides with the managed fund, while for the linear case it is obtained by taking a deep short position in the managed fund and a deep long position in the benchmark.

In other experiments which we do not report here for brevity, we observe that for $\bar{X} = X_0$, that is when, over the long run, the market price of risk is expected to remain at the same level as today, there is a significant improvement of the opportunity set even after introducing the risk-free asset. Differently, when the long run expected value of the market price of risk $\bar{X}$ is lower than the current value X_0, the two frontiers get closer and the effect of the spread incentives becomes less evident.

3 Conclusion

We showed that different incentives affect the choices of portfolio managers and, consequently, of the investors who optimally decide how to allocate their wealth. We considered two different kind of indirect incentives, the spread and the fulcrum, introduced into the literature by Basak, Pavlova and Shapiro (2007) and by Cuoco and Kaniel (2011). The effect of the two different incentive schemes has been shown by an example where the corresponding efficient frontiers have been plotted after obtaining the relevant distributions by simulation. A challenging and interesting research topic is the determination of the optimal contract for this instance of the principal-agent problem.

References

Basak, S., A., Pavlova and A., Shapiro (2007) "Optimal asset allocation and risk shifting in money management" *The Review of Financial Studies* **20**(5) 1583-1621.

Chevalier, J. and G., Ellison (1997) "Risk Taking by Mutual Funds as a Response to Incentives" *Journal of Political Economy* **105** 1167-200.

Cox, J.C. and C.F. Huang (1989) "Optimal Consumptions and Portfolio Policies when Asset Prices Follow a Diffusion Process" *Journal of Economic Theory* **49** 33-83.

Cuoco, D. and R., Kaniel (2011) "Equilibrium prices in the presence of delegated portfolio management" *Journal of Financial Economics* **101**(2) 264-296.

Merton, R.C. (1971) "Optimum Consumption and Portfolio Rules in a Continuous-Time Model" *Journal of Economic Theory* **3** 373-413.

Nicolosi, M., F., Angelini and S., Herzel (2017) “Portfolio management with benchmark related incentives under mean reverting processes” *Annals of Operations Research* doi:10.1007/s10479-017-2535-y.

Sirri, E. R. and P. Tufano (1998) “Costly Search and Mutual Fund Flows” *Journal of Finance* **54** 359-75.

Wachter, J. A. (2002) “Portfolio and Consumption Decisions under Mean-Reverting Returns: An Exact Solution for Complete Markets” *Journal of financial and quantitative analysis* **37**(1) 63-91.