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Optimal income taxation without commitment: policy implications of durable goods

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Abstract

This paper examines the design of non-linear tax policies applied to the consumption of durable goods. These tax policies involve an issue of time inconsistency, which the government can re-optimize its tax policies in the future period based on taxpayers' information revealed in the current period. We consider situations in which the government cannot commit to a future tax policy. If a type of taxpayers is unrevealed and a durable good consumption is complementary to a non-durable good consumption, it is optimal to tax the durable goods consumption of high-income earners and subsidize that of low-income earners. Under the additional assumption that taxpayers' disutility of labor supply is iso-elastic, if a type of taxpayers is revealed, a positive marginal tax rate on high-income earners' durable goods consumption and negative marginal tax rate on low-income earners' durable goods consumption are desirable. These imply that the government should design taxes on durable goods consumption to be progressive and supplement its optimal tax policies.

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1. Introduction

The appropriate treatment of durable goods is an important and intractable issue for value-added tax (VAT). Under VAT, durable goods that provide a flow of services over a long time period should be taxed on the values of the services that the goods subsequently provide. In the case of housing services, we can distinguish between construction activities and the lease and sales of properties. Many OECD countries tax building materials at the standard rate, but the rental values of owner-occupied properties are not taxed.¹ For taxation on properties, studies on the optimal capital income tax problem contribute to policy discussion. An important study in this context is Fischer (1980), which demonstrates the issue of time inconsistency under capital income taxation and shows that imposing a positive tax on capital income is desirable. The purpose of this study is to propose a system of optimal taxation on durable goods consumption, taking into account the time inconsistency problem that arises when non-linear tax schedule is applied.

We construct a two-period non-linear income taxation à la Mirrlees (1971), in which there is non-linear taxation on consumption of a durable good. Using non-linear taxation on consumer durables is natural, because the government can gather information on the market's durable good consumption activities. In this context, we refer to Atkinson and Stiglitz (1976) who deny the role of non-linear commodity taxes when consumers' preference is weakly separable over consumption and leisure. Note that, under a multi-period setting, the Atkinson-Stiglitz theorem depends on the assumption that the government can commit to its tax policies. Under the two-period model in this study, the government has an incentive to re-optimize its tax schedule in the second period using information the taxpayers revealed in the first period. Thus, the government's commitment is not time consistent. When the taxpayers know this re-optimization, they might reconsider their first-period decision making in order to conceal their information, which is called a ratchet effect. We know that if the assumption that the government can commit to its tax policy is relaxed, the results of Atkinson-Stiglitz theorem no longer hold.² Previous studies that analyze dynamic optimal non-linear income taxation when the government cannot commit to its tax policies include Roberts (1984), Apps and Rees (2006), Bisin and Rampini (2006), Brett and Weymark (2008), Krause (2009), Guo and Krause (2011, 2013), Berliant and Ledyard (2014), and Morita (2016). To the best of our knowledge, our study is the first study to examine the implications of lack of commitment for the optimal tax treatment of durable consumption goods. First, we show that when the government can commit to its tax policy, the optimal non-linear commodity tax on a durable good is useless, which is consistent with the Atkinson-Stiglitz

¹Exceptions are Ireland where a 12.5% tax rate which is lower than the standard tax rate is applied and Italy where specific materials are taxed at a lower rate 9%.

²Saez (2002) illustrates the conditions under which the Atkinson-Stiglitz result is robust in the context of homogeneity of preference for consumption. Note that the lack of commitment is a factor that Saez (2002) does not refer to.

theorem In other words, it is optimal to tax consumer durables at the same rate as that of a non-durable good. If such a commitment is not possible, we consider two cases of candidates for equilibrium in the model: the complete pooling case and the complete separation case. In the complete pooling case, all taxpayers are completely pooled in the first period, while in the complete separation case, all taxpayers are separated in the first period. We show that in both cases, it is optimal to tax high-income earners for durable goods consumption while subsidizing low-income earners. Note that these policy outcomes depend on assumptions about consumer's preference.

The remainder of this paper is organized as follows. Section 2 describes the setting of the model. Section 3 presents our analytical results. We conclude in Section 4. We derive analytical results in the Appendix A to D.

2. Model

2.1 Environment

A continuum of consumers exists with a unit measure of which a proportion n^1 are low-skilled workers and the remainder n^2 high-skilled workers. They live in two periods, $t = 1, 2$ and differ only in labor productivity. The skill level is given by θ^i where $\theta^1 < \theta^2$. These productivities are assumed constant over the time periods. We call the low-skilled workers “type 1 consumers,” and the high-skilled workers “type 2 consumers.”

Assume, in both periods, that type- i consumers supply l_t^i units of labor. Their labor income in the period t is defined as $y_t^i \equiv \theta^i l_t^i$. There are two classes of goods: a durable good and a non-durable good. In the first period, consumers consume d_1^i units of the durable good and c_1^i units of the non-durable good. Each unit of the non-durable good can be stored in the first period and produces $(1 + r)$ units of the non-durable goods in the second period. This means that r is interpreted as the return on savings. In the second period, consumers choose their consumption of the non-durable good, c_2^i . For simplicity, there is no depreciation and no maintenance investment for the durable good. Then, the durable good consumption evolves according to the following equation:

$$d_1^i = d_2^i = d^i \tag{1}$$

Consumer preference is assumed additive and separable between consumption and labor supply, but consumer preference is not separable in terms of the durable good consumption and the non-durable good consumption. Then, the utility function of type- i consumers is

$$U^i(c_1^i, c_2^i, d^i, l_1^i, l_2^i) = \sum_t \beta^{t-1} \{u(c_t^i, d^i) - v(l_t^i)\} \tag{2}$$

where β is the discount factor for $\beta \in (0, 1)$. Both the sub-utility function $u(\cdot)$ and the sub-disutility function $v(\cdot)$ are twice continuously differentiable and increasing in each argument. It is assumed that $u(\cdot)$ is a strictly concave function and $v(\cdot)$ is a strictly convex function. It is known that whether a consumption good is complementary with or a substitute for leisure in the Edgeworth sense is crucial in considering the optimal commodity taxation (See, Christiansen (1984)). We focus on the term Edgeworth dependence to characterize the cross-derivative of the sub-utility function. If the cross-derivative is positive (negative), then the durable good and non-durable good are Edgeworth complement (Edgeworth substitutes). Eichenbaum and Hansen (1990) point out that the separability between the service from durable goods and non-durable goods in preference is sensitive to the overall specification of consumer preference.³ Here, we assume the Edgeworth complement⁴, that is,

Assumption 1.

$$\frac{\partial^2 u}{\partial c_t^i \partial d^i}(c_t^i, d^i) > 0 \quad i, t = 1, 2$$

Given the non-linear labor income tax $T_t(y_t^i)$, the non-linear tax on durable good consumption $\tau(d^i)$, and the non-linear capital income tax $\Psi(rs^i)$, the type- i consumers' budget constraints in each period can be written as follows:

$$y_1^i - T_1(y_1^i) = c_1^i + d^i + s^i + \tau(d^i) \quad (3)$$

$$y_2^i - T_2(y_2^i) + (1 + r)s^i - \Psi(rs^i) = c_2^i \quad (4)$$

Note that we consider a durable good without resale values. As shown in Appendix A, the first-order condition with respect to d^i yields the following expression for the marginal tax rate on the durable good consumption:

$$MTRD^i \equiv \tau'(d^i) = -1 - \left(-\frac{\frac{\partial u}{\partial d^i}(c_1^i, d^i) + \beta \frac{\partial u}{\partial d^i}(c_2^i, d^i)}{\frac{\partial u}{\partial c_1^i}(c_1^i, d^i)} \right), \quad i = 1, 2 \quad (5)$$

³Eichenbaum and Hansen (1990) find substantial evidence against the hypothesis that consumer preference is completely separable between durable goods consumption and non-durable goods consumption if the preference is represented by the quadratic form. By contrast, for S-branch preference, the authors find very little evidence against the hypothesis that consumer preference is completely separable between services from durable goods and non-durable goods.

⁴For the utility function such that the elasticity of intertemporal substitution and the coefficient of relative risk aversion are inversely linked, several studies estimate the parameters of utility function. Fauvel and Samson (1991) find that the elasticity of substitution over time is positive and varies between 1.5 and 2.3. Ogaki and Reinhart (1998) point out that ignoring the intratemporal substitution between a durable good and non-durable good causes a misspecification bias and find that the elasticity of intertemporal substitution ranges from 0.32 to 0.45. These empirical evidences are in support of the Edgeworth complement.

The first term of equation (5) is the marginal rate of transformation of durable good consumption for non-durable good consumption. The second term is the marginal rate of substitution between durable good consumption and non-durable good consumption in the first period. Equation (5) is the difference between them, that is, the distortion of durable good consumption. By combining the first-order condition with respect to s^i , the marginal capital income tax rate is expressed as follow:

$$MTRC^i \equiv \Psi'(rs^i)r = (1+r) - \frac{\frac{\partial u}{\partial c_1^i}(c_1^i, d^i)}{\beta \frac{\partial u}{\partial c_2^i}(c_1^i, d^i)}, \quad i = 1, 2 \quad (6)$$

2.2 The commitment case

The assumption about information is conventional. While the government cannot observe the abilities of consumers, it can observe consumers' labor income, savings, and consumption of the durable good. Thus, the government can impose non-linear taxes on these variables. Moreover, it is assumed that the government can engage in savings by storing an amount s^G of the non-durable good with this saving technology available to the government being the same as the technology available to consumers.

If the government can commit to the tax policies in both periods, then it cannot re-optimize these policies using the information revealed by consumers in the first period. The government chooses c_t^i , d^i , s^i , y_t^i , and s^G for $i, t = 1, 2$ to maximize

$$\sum_i U^i(\cdot)n^i \quad (7)$$

subject to

$$\sum_i \{y_1^i - c_1^i - d^i - s^i\}n^i - s^G = 0 \quad (8)$$

$$\sum_i \{y_2^i + (1+r)s^i - c_2^i\}n^i + (1+r)s^G = 0 \quad (9)$$

$$U^2(c_1^2, c_2^2, d^2, \frac{y_1^2}{\theta^2}, \frac{y_2^2}{\theta^2}) \geq U^2(c_1^1, c_2^1, d^1, \frac{y_1^1}{\theta^2}, \frac{y_2^1}{\theta^2}) \quad (10)$$

Equation (7) is an objective function, the utilitarian social welfare function. Equation (8) and (9) are resource constraints in each period. Hereafter, we concentrate on the normal case in which a redistribution occurs from the type-2 consumers to the type-1 consumers. Then, the incentive compatible constraint for the type-2 consumer, that is, equation (10) is binding. The optimal conditions for this problem are shown in Appendix B. Since the preference for consumption represented by $u(\cdot)$ does not depend

on the type of consumer, the government has no motive to distort the decision making in both the durable good consumption and the non-durable good consumption. Thus, we obtain

$$MTRD^i = 0 \quad i = 1, 2 \quad (11)$$

$$MTRC^i = 0 \quad i = 1, 2 \quad (12)$$

These are consistent with the outcome in the standard non-linear income tax problem with multi-periods, as in the Atkinson-Stiglitz theorem.

3. The non-commitment case

It is not credible for the government to extend its first-period policy commitment to the second period. Given that the labor income, capital income, and durable good consumption are observable, the government is able to infer the identities of consumers at the end of the first period and to re-optimize its tax policies in the second period so that the distortion of the second period can be eliminated. Therefore, tax policies in the commitment case are not time consistent. When consumers know that there will be a re-optimization process of tax policies, they may adjust their own decision making in the first period, especially if there are incentives for type-2 consumers to conceal the information of their skill levels. To assess the analysis we consider two extreme cases on how to adjust their decision making: complete pooling and complete separation. In the complete pooling case, consumers are completely pooled in the first period. The complete separation case is the opposite in that all type-2 consumers are separated from type-1 consumers in the first period.

Such an analysis generates a question as to whether it is optimal to use separating or pooling non-linear taxes. The level of social welfare depends on the parameters of the model. Several previous studies, including Bisin and Rampini (2006), Guo and Krause (2013), and Morita (2016), report that the level of social welfare achieved in the complete separation case is higher than that in the complete pooling case.

3.1 The complete pooling case

If consumers are completely pooled in the first period, then the tax contract assigned in the first period does not depend on the type of consumers $(\bar{c}_1, \bar{d}, \bar{s}, \bar{y}_1)$. In the second period, given as $\bar{\mathbf{v}} = (\bar{d}, \bar{S})$, the government chooses c_2^i and y_2^i for $i = 1, 2$ to maximize

$$\sum_i \{u(c_2^i, \bar{d}) - v(\frac{y_2^i}{\theta^i})\} n^i \quad (13)$$

subject to the resource constraint in the second period (equation (9)) and the incentive compatibility constraint:

$$u(c_2^2, \bar{d}) - v\left(\frac{y_2^2}{\theta^2}\right) \geq u(c_2^1, \bar{d}) - v\left(\frac{y_2^1}{\theta^2}\right) \quad (14)$$

Let $V_2^P(\bar{\mathbf{v}})$ denote the value function of the optimization problem in the second period. The government in the first period, therefore, chooses \bar{c}_1 , \bar{d} , \bar{s} , s_G , and \bar{y}_1 to maximize

$$\sum_i \left\{ u(\bar{c}_1, \bar{d}) - v\left(\frac{\bar{y}_1}{\theta^i}\right) \right\} n^i + \beta V_2^P(\bar{\mathbf{v}}) \quad (15)$$

subject to

$$\sum_i \{ \bar{y}_1 - \bar{c}_1 - \bar{s} - \bar{d} \} n^i - s^G = 0 \quad (16)$$

Note that because all consumers are offered a single choice $(\bar{c}_1, \bar{d}, \bar{s}, \bar{y}_1)$, the government faces no incentive compatibility constraint in the first period.

We obtain the optimal tax formula through the optimal conditions for durable goods consumption:

$$MTRD^1 = -\frac{\beta(n^2 + \phi^P)}{\frac{\partial u}{\partial \bar{c}_1}(\bar{c}_1, \bar{d})} \left\{ \frac{\partial u}{\partial \bar{d}}(c_2^2, \bar{d}) - \frac{\partial u}{\partial \bar{d}}(c_2^1, \bar{d}) \right\} \quad (17)$$

$$MTRD^2 = \frac{\beta(n^1 - \phi^P)}{\frac{\partial u}{\partial \bar{c}_1}(\bar{c}_1, \bar{d})} \left\{ \frac{\partial u}{\partial \bar{d}}(c_2^2, \bar{d}) - \frac{\partial u}{\partial \bar{d}}(c_2^1, \bar{d}) \right\} \quad (18)$$

where ϕ^P denotes the Lagrange multiplier for the incentive compatibility constraint (equation (14)). In Appendix C, we show that equation (17) is negative, whereas equation (18) is positive. These are because the marginal utility of the durable goods in the second period for type-2 consumers is lower than for type-1 consumers. These are derived from Assumption 1 and the fact that the amount of type-2 consumers' non-durable good consumption is higher than that for type-1 consumers (see equations (C.2) and (C.3) in Appendix C). These results imply that the government should design progressive taxes on durable good consumption that would supplement its optimal tax policies. These are summarized by the following proposition:

Proposition 1. *Assume durable good consumption is complementary to a non-durable good consumption. If the government cannot commit to the second-period tax policy and all types of consumers are completely pooled in the first period, then the marginal tax rate on type-1 consumers' durable good consumption should be smaller than the marginal tax rate that type-2 consumers face.*

The policy outcomes crucially depend on Assumption 1. When durable good consumption and non-durable good consumption are separable, that is, $\frac{\partial^2 u}{\partial c_t^i \partial d^i}(c_t^i, d^i) = 0$ for $i, t = 1, 2$, both equations (17) and (18) are zero. This is because the marginal utility of the durable good is not specific to the type of consumer. When durable good consumption is substitutable for non-durable good consumption, that is, $\frac{\partial^2 u}{\partial c_t^i \partial d^i}(c_t^i, d^i) < 0$ for $i, t = 1, 2$, equation (17) is positive, but equation (18) is negative. This means that the tax policies on durable good consumption should be regressive.

To examine the policy outcome of capital income taxation, combining the optimal condition for saving and non-durable good consumption in both periods yields:

$$MTRC^1 > 0, MTRC^2 < 0 \quad (19)$$

This is shown in Appendix C. In the first period, consumers choose the same level of consumption \bar{c}_1 , but the amount of type-2 consumers' consumption is larger than that of type-1 consumers' consumption (see equations (C.2) and (C.3)). Then, type-2 consumers' intertemporal marginal rate of substitution is larger than that of type-1 consumers. Applying this to equation (6), we obtain the result that tax policies on capital income should be regressive, which is consistent with Brett and Weymark (2008).

3.2 The complete separation case

If consumers make different choice in the first period, the government will have the information to carry out personalized lump-sum taxation in the second period. Given $\mathbf{v} = (d^1, d^2, s^1, s^2)$, the government chooses c_2^i and y_2^i for $i = 1, 2$ to maximize

$$\sum_i \{u(c_2^i, d^i) - v(\frac{y_2^i}{\theta^i})\} n^i \quad (20)$$

subject to the resource constraint in the second period (equation (9)). Let $V_2^S(\mathbf{v})$ be the value function in the second period. By this setting, the government chooses c_1^i, d^i, s^i , and y_1^i for $i = 1, 2$ to maximize

$$\sum_i \{u(c_1^i, d^i) - v(\frac{y_1^i}{\theta^i})\} n^i + \beta V_2^S(\mathbf{v}) \quad (21)$$

subject to the resource constraint in the first period (equation (8)) and the incentive compatibility constraint for a type-2 consumer:

$$U^2(c_1^2, c_2^2(\mathbf{v}), d^2, \frac{y_1^2}{\theta^2}, \frac{y_2^2(\mathbf{v})}{\theta^2}) \geq U^2(c_1^1, c_2^1(\mathbf{v}), d^1, \frac{y_1^1}{\theta^2}, \frac{y_2^1(\mathbf{v})}{\theta^2}) \quad (22)$$

where $c_2^2(\mathbf{v})$ and $y_2^2(\mathbf{v})$ are the first-best contracts for a type-2 consumer, and $c_2^1(\mathbf{v})$ and $y_2^1(\mathbf{v})$ are the first-best contracts for a type-1 consumer. Equation (22) means that the

payoff that a type-2 consumer obtains from c_1^2 and y_1^2 in the first period plus the payoff that she/he obtains from $c_2^2(\mathbf{v})$ and $y_2^2(\mathbf{v})$ need not be smaller than the payoff that she/he obtains from c_1^1 and y_1^1 in the first period plus the payoff that she/he obtains from $c_2^1(\mathbf{v})$ and $y_2^1(\mathbf{v})$.

The social optimal condition with respect to the durable good consumption of each type of consumers yields the following formula:

$$MTRD^i = -\frac{\beta\phi^S}{\lambda_1^S} \Delta_d^i, \quad i = 1, 2 \quad (23)$$

where ϕ^S denotes the Lagrange multiplier for the incentive compatibility constraint (equation (22)), and λ_1^S denotes the Lagrange multiplier for the resource constraint in the first period. The variable Δ_d^i indicates the indirect effect of type- i consumers' durable good consumption on the incentive compatibility constraint (equation (22)) through the amount of consumption and labor income in the second period. To determine the sign of Δ_d^i , we introduce the additional assumption:

Assumption 2.

$$v\left(\frac{y_t^i}{\theta^i}\right) = \frac{\kappa}{1+\nu} \left(\frac{y_t^i}{\theta^i}\right)^{\nu+1}, \quad \kappa > 0, \nu > 0$$

In Appendix D, we show that Δ_d^2 is negative under Assumption 1. We also show that Δ_d^1 is positive under Assumption 1 and 2. Intuitively, an incremental unit increase in type-1 consumers' durable good consumption can relax the incentive compatibility constraint, the government should encourage type 1 consumers' durable good consumption. On the contrary, an incremental unit increase in type-2 consumers' durable good consumption tightens the incentive compatibility constraint, thus the government should impose the positive tax rate on type-2 consumers' durable good consumption. Therefore, the progressive taxes on durable good consumption is desirable. These can be summarized by the following proposition.

Proposition 2. *Suppose that durable good consumption is complementary to non-durable good consumption, and the dis-utility function of all consumers is an iso-elastic of labor supply. If the government cannot commit to the second period tax policy and all types of consumers are completely separated in the first period, then the consumption of the durable good by type-1 consumers should be subsidized and the consumption by type-2 consumers should be taxed.*

Like Proposition 1, the policy outcomes in Proposition 2 depend on Assumption 1. If the durable good consumption and non-durable good consumption are separable, this is $\frac{\partial u^2}{\partial c_2^i \partial d^i}(c_t^i, d^i) = 0$ for $i, t = 1, 2$, then the variable Δ_d^i is zero for $i = 1, 2$. In this case, the marginal tax rate on durable good consumption for both types should be zero. If the durable good consumption is substitutable for non-durable good consumption, that

is, $\frac{\partial^2 u}{\partial c_t^i \partial d^i}(c_t^i, d^i) < 0$ for $i, t = 1, 2$, then the variable Δ_d^1 is negative and the variable Δ_d^2 is positive. This suggests that type-1 consumers should be taxed for the consumption of the durable goods, while type-2 consumers should be subsidized.

Then we turn to the policy outcome of capital income taxation. Combining the optimal conditions for savings and non-durable good consumption in both periods yields:

$$MTRC^1 = -\frac{\phi^S}{n^1} \frac{\frac{\partial u}{\partial c_1^1}(c_2^1, d^1)}{\beta \frac{\partial u}{\partial c_2^1}(c_2^1, d^1)} - \beta \phi^S \Delta_s^1 \quad (24)$$

$$MTRC^2 = \frac{\phi^S}{n^2} \frac{\frac{\partial u}{\partial c_1^2}(c_2^2, d^2)}{\beta \frac{\partial u}{\partial c_2^2}(c_2^2, d^2)} - \beta \phi^S \Delta_s^2 \quad (25)$$

where Δ_s^i are the indirect effect of type- i consumers' savings on the incentive compatibility constraint. In Appendix D, it is shown that under Assumption 1, Δ_s^i cannot be signed. If the durable good consumption and non-durable good consumption are separable, $\frac{\partial u^2}{\partial c_2^i \partial d^i}(c_t^i, d^i) = 0$ for $i, t = 1, 2$, then the variable Δ_s^i is negative for $i = 1, 2$. This means that the sign of equation (24) is ambiguous, but that of equation (25) is positive. In other words, it is optimal to tax type-2 consumers' savings at a positive rate and to tax type-1 consumers' savings at a lower, possibly negative tax rate. These policy outcomes are consistent with Brett and Weymark (2008). In addition, we show that

$$\Delta_s^1 - \Delta_s^2 \leq 0 \iff n^1 \leq n^2 \quad (26)$$

Equation (26) means that the indirect effect of type-1 consumers' savings is larger than that of type-2 consumers' savings if the number of type-1 consumers is larger than that of type-2 consumers.

4. Conclusion

This paper provides a rationale for the distortion in durable good consumption when the government cannot commit to its future tax policy. In both the complete pooling case and the complete separation case, it is optimal for high-income earners to be taxed on their durable good consumption and for low-income earners to be subsidized for their durable good consumption. These results support a real estate acquisition tax or an automobile acquisition tax. Moreover, the differential tax treatment of durable goods, such as subsidies to the poor, can be justified in the optimal tax policy.

The results of this study depend on the specification of preference, that is, Assumption 1 and 2. We show that if Assumption 1 is relaxed, the policy outcomes in

non-commitment cases are modified dramatically. In the complete separation case, Assumption 2 is necessary to determine the sign of optimal marginal tax rate assigned to low-income earners.

Savings can be considered a durable good, at least in the sense that savings affect consumers' welfare in both period. Thus, comparing the policy outcomes for durable goods consumption with those for capital income is important. In the complete pooling case, the difference of optimal tax treatment between durable good consumption and non-durable good consumption arises from the definition of the marginal tax rate function (equation (5) and (6)). It is shown that the type-1 consumers' intertemporal marginal rate of substitution is smaller than type-2 consumers'. By contrast, type-1 consumers' marginal rate of substitution between durable good consumption and non-durable good consumption is larger than type-2 consumers'. In the complete separation case, the indirect effects on the incentive compatibility constraint, Δ_d^i and Δ_s^i , are key factors in the difference in policy outcomes. We show that a marginal increase in durable good consumption of type-1 consumers relaxes the incentive compatibility constraint, but that in type-2 consumers does not. Under the condition of separability between durable goods consumption and non-durable goods consumption on consumers' utility function, a marginal increase in savings of both types tightens the same constraint.

Appendix

Appendix A

The type i consumers choose c_t^i , d^i , s^i , and l_t^i to maximize equation (2) subject to equations (3) and (4). The first order condition with respect to d^i is

$$-\frac{\partial u}{\partial c_1^i}(c_1^i, d^i)(1 + \tau'(d^i)) + \frac{\partial u}{\partial d^i}(c_1^i, d^i) + \beta \frac{\partial u}{\partial d^i}(c_2^i, d^i) = 0 \quad (\text{A.1})$$

Rearranging equation (A.1) gives equation (5). □

Appendix B

Consider the planning problem in the commitment case. The Lagrangian can be written as:

$$\begin{aligned} \mathcal{L}^C = & \sum_i U^i(\cdot)n^i + \lambda_1^C [\sum_i \{y_1^i - c_1^i - d^i - s^i\}n^i - s^G] \\ & + \lambda_2^C [\sum_i \{y_2^i + (1+r)s^i - c_2^i\}n^i + (1+r)s^G] \\ & + \phi^C [U^2(c_1^2, c_2^2, d^2, \frac{y_1^2}{\theta^2}, \frac{y_2^2}{\theta^2}) - U^2(c_1^1, c_2^1, d^1, \frac{y_1^1}{\theta^2}, \frac{y_2^1}{\theta^2})] \end{aligned} \quad (\text{B.1})$$

where $\lambda_t^C > 0$ is the Lagrange multipliers on the resource constraint in the period t and $\phi^C > 0$ is the multiplier on a type 2 consumer's incentive compatibility constraint. The first order conditions can be written as:

$$\frac{\partial \mathcal{L}^C}{\partial c_t^1} = (n^1 - \phi^C) \beta^{t-1} \frac{\partial u}{\partial c_t^1}(c_t^1, d^1) - \lambda_t^C n^1 = 0 \quad t = 1, 2 \quad (\text{B.2})$$

$$\frac{\partial \mathcal{L}^C}{\partial c_t^2} = (n^2 + \phi^C) \beta^{t-1} \frac{\partial u}{\partial c_t^2}(c_t^2, d^2) - \lambda_t^C n^2 = 0 \quad t = 1, 2 \quad (\text{B.3})$$

$$\frac{\partial \mathcal{L}^C}{\partial d^1} = (n^1 - \phi^C) \left(\frac{\partial u}{\partial d^1}(c_1^1, d^1) + \beta \frac{\partial u}{\partial d^1}(c_2^1, d^1) \right) - \lambda_1^C n^1 = 0 \quad (\text{B.4})$$

$$\frac{\partial \mathcal{L}^C}{\partial d^2} = (n^2 + \phi^C) \left(\frac{\partial u}{\partial d^2}(c_1^2, d^2) + \beta \frac{\partial u}{\partial d^2}(c_2^2, d^1) \right) - \lambda_1^C n^2 = 0 \quad (\text{B.5})$$

$$\frac{\partial \mathcal{L}^C}{\partial s^i} = -\lambda_1^C + \lambda_2^C (1+r) = 0 \quad i = 1, 2 \quad (\text{B.6})$$

Substituting equation (B.2) into equation (B.4), equation (B.4) can be manipulated to yield equation (11) for $i = 1$. By similar operation, combining equation (B.3) and (B.5) also yield equation (11) for $i = 2$. \square

Appendix C

Consider the planning problem in complete pooling case. First, we can formulate the Lagrangian for the optimization problem in the second period as follows:

$$\begin{aligned} \mathcal{L}_2^P = \sum_i \{ u(c_2^i, \bar{d}) - v(\frac{y_2^i}{\theta^i}) \} n^i + \lambda_2^P [\sum_i \{ y_2^i + (1+r)\bar{s} - c_2^i \} n^i + (1+r)s^G] \\ + \phi^P [u(c_2^2, \bar{d}) - v(\frac{y_2^2}{\theta^2}) - u(c_2^1, \bar{d}) + v(\frac{y_2^1}{\theta^2})] \end{aligned} \quad (\text{C.1})$$

where $\lambda_2^P > 0$ is the Lagrange multipliers on the resource constraint in the second period and $\phi^P > 0$ is the multiplier on a type 2 consumer's incentive compatibility constraint in the second period. The first order conditions with respect to c_2^1 and c_2^2 can be written as:

$$\frac{\partial \mathcal{L}_2^P}{\partial c_2^1} = (n^1 - \phi^P) \beta \frac{\partial u}{\partial c_2^1}(c_2^1, \bar{d}) - \lambda_2^P n^1 = 0 \quad (\text{C.2})$$

$$\frac{\partial \mathcal{L}_2^P}{\partial c_2^2} = (n^2 + \phi^P) \beta \frac{\partial u}{\partial c_2^2}(c_2^2, \bar{d}) - \lambda_2^P n^2 = 0 \quad (\text{C.3})$$

Equations (C.2) and (C.3) imply $c_2^1 < c_2^2$. Let $V_2^P(\bar{\mathbf{v}})$ be the value function of the planning problem in the second period. By the envelop theorem, we have

$$\frac{\partial V_2^P}{\partial \bar{d}}(\bar{\mathbf{v}}) = (n^1 + \phi^P) \frac{\partial u}{\partial \bar{d}}(c_2^2, \bar{d}) + (n^2 - \phi^P) \frac{\partial u}{\partial \bar{d}}(c_2^1, \bar{d}) \quad (\text{C.4})$$

$$\frac{\partial V_2^P}{\partial \bar{s}}(\bar{\mathbf{v}}) = (1 + r) \lambda_2^P \quad (\text{C.5})$$

Based on the outcome, we turn to the optimization problem in the first period. The Lagrangian is

$$\mathcal{L}_1^P = \sum_i \{u(\bar{c}_1, \bar{d}) - v(\frac{\bar{y}_1}{\theta^i})\} n^i + \beta V_2^P(\bar{\mathbf{v}}) + \lambda_1^P [\sum_i \{\bar{y}_1 - \bar{c}_1 - \bar{s} - \bar{d}_1\} n^i - s^G] \quad (\text{C.6})$$

where $\lambda_1^P > 0$ is the Lagrange multipliers on the resource constraint in the first period. The first order condition can be written as:

$$\frac{\partial \mathcal{L}_1^P}{\partial \bar{c}_1} = \sum_i \frac{\partial u}{\partial \bar{c}_1}(\bar{c}_1, \bar{d}) n^i - \lambda_1^P \sum_i n^i = 0 \quad (\text{C.7})$$

$$\frac{\partial \mathcal{L}_1^P}{\partial \bar{d}} = \sum_i \frac{\partial u}{\partial \bar{d}}(\bar{c}_1, \bar{d}) n^i + \beta \frac{\partial V_2^P}{\partial \bar{d}}(\bar{\mathbf{v}}) - \lambda_1^P \sum_i n^i = 0 \quad (\text{C.8})$$

$$\frac{\partial \mathcal{L}_1^P}{\partial \bar{s}} = \beta \frac{\partial V_2^P}{\partial \bar{s}}(\bar{\mathbf{v}}) - \lambda_1^P \sum_i n^i = 0 \quad (\text{C.9})$$

Combining (C.2), (C.3), (C.4), (C.6) and (C.7) yields equations (17) and (18). Similarly, combining (C.2), (C.3), (C.5), (C.7), and (C.9) yields equation (19). \square

Appendix D

Consider the planning problem in the complete separation case. The same as in the complete pooling case, we formulate the Lagrangian for the optimization problem in the second period:

$$\mathcal{L}_2^S = \sum_i \{u(c_2^i, d^i) - v(\frac{y_2^i}{\theta^i})\} n^i + \lambda_2^S [\sum_i \{y_2^i + (1 + r) s^i - c_2^i\} n^i + (1 + r) s^G] \quad (\text{D.1})$$

where $\lambda_2^S > 0$ is the Lagrange multipliers on the resource constraint in the second period. The first order conditions corresponding to this problem are:

$$\frac{\partial \mathcal{L}_2^S}{\partial c_2^i} = \frac{\partial u}{\partial c_2^i}(c_2^i, d^i) - \lambda_2^S = 0 \quad i = 1, 2 \quad (\text{D.2})$$

$$\frac{\partial \mathcal{L}_2^S}{\partial y_2^i} = -\frac{v'(\frac{y_2^i}{\theta^i})}{\theta^i} + \lambda_2^S = 0 \quad i = 1, 2 \quad (\text{D.3})$$

$$\frac{\partial \mathcal{L}_2^S}{\partial \lambda_2^S} = \sum_i \{y_2^i + (1+r)s^i - c_2^i\}n^i + (1+r)s^G = 0 \quad (\text{D.4})$$

Let $V_2^S(\mathbf{v})$ be the value function of the planning problem in the second period. By the envelop theorem, we have

$$\frac{\partial V_2^S}{\partial d^i}(\mathbf{v}) = \frac{\partial u}{\partial d^i}(c_2^i, d^i)n^i \quad i = 1, 2 \quad (\text{D.5})$$

$$\frac{\partial V_2^S}{\partial s^i}(\mathbf{v}) = (1+r)n^i\lambda_2^S \quad i = 1, 2 \quad (\text{D.6})$$

The Lagrangian function in the second period can be written as:

$$\begin{aligned} \mathcal{L}_1^S = & \sum_i \{u(c_1^i, d^i) - v(\frac{y_1^i}{\theta^i})\}n^i + \beta V_2^S(\mathbf{v}) + \lambda_1^S [\sum_i \{y_1^i - c_1^i - s^i - d^i\}n^i - s^G] \quad (\text{D.7}) \\ & + \phi^S [U^2(c_1^2, c_2^2(\mathbf{v}), d^2, \frac{y_1^2}{\theta^2}, \frac{y_2^2(\mathbf{v})}{\theta^2}) - U^2(c_1^1, c_2^1(\mathbf{v}), d^1, \frac{y_1^1}{\theta^2}, \frac{y_2^1(\mathbf{v})}{\theta^2})] \end{aligned}$$

where $\lambda_1^S > 0$ is the Lagrange multipliers on the resource constraint in the first period and $\phi_1^2 > 0$ is the multipliers on a type 2 consumer's incentive compatibility constraint. The first order condition with respect to c_2^i , d^i , and s^i for all i can be written as:

$$\frac{\partial \mathcal{L}_1^S}{\partial c_1^1} = (n^1 - \phi^S) \frac{\partial u}{\partial c_1^1}(c_1^1, d^1) - \lambda_1^S n^1 = 0 \quad (\text{D.8})$$

$$\frac{\partial \mathcal{L}_1^S}{\partial c_1^2} = (n^2 + \phi^S) \frac{\partial u}{\partial c_1^2}(c_1^2, d^2) - \lambda_1^S n^2 = 0 \quad (\text{D.9})$$

$$\frac{\partial \mathcal{L}_1^S}{\partial d^1} = (n^1 - \phi^S) \left\{ \frac{\partial u}{\partial d^1}(c_1^1, d^1) + \beta \frac{\partial u}{\partial d^1}(c_2^1, d^1) \right\} - \lambda_1^S n^1 + \beta \phi^S \Delta_d^1 = 0 \quad (\text{D.10})$$

$$\frac{\partial \mathcal{L}_1^S}{\partial d^2} = (n^2 + \phi^S) \left\{ \frac{\partial u}{\partial d^2}(c_1^2, d^2) + \beta \frac{\partial u}{\partial d^2}(c_2^2, d^2) \right\} - \lambda_1^S n^2 + \beta \phi^S \Delta_d^2 = 0 \quad (\text{D.11})$$

$$\frac{\partial \mathcal{L}_1^S}{\partial s^i} = -\lambda_1^S n^i + \beta \lambda_2^S n^i + \beta \phi^S \Delta_s^i, \quad i = 1, 2 \quad (\text{D.12})$$

where

$$\Delta_d^i \equiv \frac{\partial u}{\partial c_2^2}(c_2^2, d^2) \frac{dc_2^2}{dd^i} - \frac{v'(\frac{y_2^2}{\theta^2})}{\theta^2} \frac{dy_2^2}{dd^i} - \frac{\partial u}{\partial c_2^1}(c_2^1, d^1) \frac{dc_2^1}{dd^i} + \frac{v'(\frac{y_2^1}{\theta^2})}{\theta^2} \frac{dy_2^1}{dd^i}, \quad i = 1, 2$$

$$\Delta_s^i \equiv \frac{\partial u}{\partial c_2^2}(c_2^2, d^2) \frac{dc_2^2}{ds^i} - \frac{v'(\frac{y_2^2}{\theta^2})}{\theta^2} \frac{dy_2^2}{ds^i} - \frac{\partial u}{\partial c_2^1}(c_2^1, d^1) \frac{dc_2^1}{ds^i} + \frac{v'(\frac{y_2^1}{\theta^2})}{\theta^2} \frac{dy_2^1}{ds^i}, \quad i = 1, 2$$

The bordered Hessian associated with equations from (C.2) to (C.4):

$$A \equiv \begin{pmatrix} \frac{\partial^2 u}{\partial c_2^1 \partial c_2^1}(c_2^1, d^1) n^1 & 0 & 0 & 0 & -n^1 \\ 0 & \frac{\partial^2 u}{\partial c_2^2 \partial c_2^2}(c_2^2, d^2) n^2 & 0 & 0 & -n^2 \\ 0 & 0 & -\frac{v''(\frac{y_2^1}{\theta^2})}{\theta^1 \theta^1} n^1 & 0 & n^1 \\ 0 & 0 & 0 & -\frac{v''(\frac{y_2^2}{\theta^2})}{\theta^2 \theta^2} n^2 & n^2 \\ -n^1 & -n^2 & n^1 & n^2 & 0 \end{pmatrix} \quad (\text{D.13})$$

Its determinant is

$$|A| = \left(\frac{n^1 n^2}{\theta^1 \theta^2} \right)^2 \sum_{i \neq j} \left[n^i \frac{\partial^2 u}{\partial c_2^j \partial c_2^j}(c_2^j, d^j) v''\left(\frac{y_2^j}{\theta^j}\right) \left\{ \frac{\partial^2 u}{\partial c_2^i \partial c_2^i}(c_2^i, d^i) - v''\left(\frac{y_2^i}{\theta^i}\right) \right\} \right] > 0 \quad (\text{D.14})$$

By the strict concavity of $u(\cdot)$ and the strict convexity of $v(\cdot)$, $|A|$ is positive. By the Cramer's rule, we obtain for $i = 1, 2$ and $i \neq j$:

$$\frac{dc_2^i}{dd^i} = \frac{\frac{\partial^2 u}{\partial c_2^i \partial d^i}(c_2^i, d^i)}{|A|} \left(\frac{n^1 n^2}{\theta^1 \theta^2} \right)^2 \left[\frac{\partial^2 u}{\partial c_2^i \partial c_2^i}(c_2^i, d^i) \sum_i \{ n^i \theta^i v''\left(\frac{y_2^i}{\theta^i}\right) \} - n^j v''\left(\frac{y_2^i}{\theta^i}\right) v''\left(\frac{y_2^j}{\theta^j}\right) \right] < 0 \quad (\text{D.15})$$

$$\frac{dc_2^j}{dd^i} = \frac{\frac{\partial^2 u}{\partial c_2^i \partial d^i}(c_2^i, d^i)}{|A|} \left(\frac{n^1 n^2}{\theta^1 \theta^2} \right)^2 n^i v''\left(\frac{y_2^i}{\theta^i}\right) v''\left(\frac{y_2^j}{\theta^j}\right) > 0 \quad (\text{D.16})$$

$$\frac{dy_2^i}{dd^i} = \frac{\frac{\partial^2 u}{\partial c_2^i \partial d^i}(c_2^i, d^i)}{|A|} \left(\frac{n^1 n^2}{\theta^1 \theta^2} \right)^2 n^i \frac{\partial^2 u}{\partial c_2^j \partial c_2^j}(c_2^j, d^j) v''\left(\frac{y_2^j}{\theta^j}\right) < 0 \quad (\text{D.17})$$

$$\frac{dy_2^j}{dd^i} = \frac{\frac{\partial^2 u}{\partial c_2^i \partial d^i}(c_2^i, d^i)}{|A|} \left(\frac{n^1 n^2}{\theta^1 \theta^2}\right)^2 n^i \frac{\partial^2 u}{\partial c_2^i \partial c_2^i}(c_2^i, d^i) v''\left(\frac{y_2^i}{\theta^i}\right) < 0 \quad (\text{D.18})$$

$$\frac{dc_2^i}{ds^i} = \left(\frac{n^1 n^2}{\theta^1 \theta^2}\right)^2 \frac{n^i(1+r)}{|A|} \frac{\partial^2 u}{\partial c_2^j \partial c_2^j}(c_2^j, d^j) v''\left(\frac{y_2^i}{\theta^i}\right) v''\left(\frac{y_2^j}{\theta^j}\right) < 0 \quad (\text{D.19})$$

$$\frac{dc_2^j}{ds^i} = \left(\frac{n^1 n^2}{\theta^1 \theta^2}\right)^2 \frac{n^i(1+r)}{|A|} \frac{\partial^2 u}{\partial c_2^i \partial c_2^i}(c_2^i, d^i) v''\left(\frac{y_2^i}{\theta^i}\right) v''\left(\frac{y_2^j}{\theta^j}\right) < 0 \quad (\text{D.20})$$

$$\frac{dy_2^i}{ds^i} = \left(\frac{n^1 n^2}{\theta^1 \theta^2}\right)^2 \frac{n^i(1+r)}{|A|} \frac{\partial^2 u}{\partial c_2^i \partial c_2^i}(c_2^i, d^i) \frac{\partial^2 u}{\partial c_2^j \partial c_2^j}(c_2^j, d^j) v''\left(\frac{y_2^i}{\theta^i}\right) > 0 \quad (\text{D.21})$$

$$\frac{dy_2^j}{ds^i} = \left(\frac{n^1 n^2}{\theta^1 \theta^2}\right)^2 \frac{n^i(1+r)}{|A|} \frac{\partial^2 u}{\partial c_2^i \partial c_2^i}(c_2^i, d^i) \frac{\partial^2 u}{\partial c_2^j \partial c_2^j}(c_2^j, d^j) v''\left(\frac{y_2^j}{\theta^j}\right) > 0 \quad (\text{D.22})$$

The variable Δ_d^i can be rewritten as follows:

$$\Delta_d^i = \lambda_2^S \left(\frac{dc_2^2}{dd^i} - \frac{dy_2^2}{dd^i} - \frac{dc_2^1}{dd^i} \right) + \frac{v'\left(\frac{y_2^1}{\theta^2}\right)}{\theta^2} \frac{dy_2^i}{dd^i} \quad (\text{D.23})$$

For $i = 1$, substituting (D.15) to (D.18) into equation (D.23) yields:

$$\begin{aligned} \Delta_d^1 &= \left(\frac{n^1 n^2}{\theta^1 \theta^2}\right)^2 \frac{\frac{\partial^2 u}{\partial c_2^1 \partial d^1}(c_2^1, d^1)}{|A|} [(\theta^2)^2 \frac{\partial^2 u}{\partial c_2^2 \partial c_2^2}(c_2^2, d^2) \mu_d \\ &\quad + \lambda_2^S v''\left(\frac{y_2^2}{\theta^2}\right) \{v''\left(\frac{y_2^1}{\theta^1}\right) - n^1 (\theta^1)^2 \frac{\partial^2 u}{\partial c_2^2 \partial c_2^2}(c_2^2, d^2)\}] \end{aligned} \quad (\text{D.24})$$

where

$$\mu_d \equiv -\lambda_2^S v''\left(\frac{y_2^1}{\theta^1}\right) + n^1 \frac{v'\left(\frac{y_2^1}{\theta^2}\right)}{\theta^2} v''\left(\frac{y_2^2}{\theta^2}\right)$$

Equation (D.3) implies $v'\left(\frac{y_2^1}{\theta^1}\right) < v'\left(\frac{y_2^2}{\theta^2}\right)$. Since $v(\cdot)$ is a convex function, we obtain $\frac{y_2^1}{\theta^1} < \frac{y_2^2}{\theta^2}$. Moreover, under Assumption 2, the variable μ_d can be rewritten as follows:

$$\mu_d = \kappa^2 \nu \left(\frac{1}{\theta^2}\right)^2 \left(\frac{y_2^2}{\theta^2}\right)^{\nu-1} \left[-\frac{y_2^2}{y_2^1 \theta^1} \left(\frac{y_2^1}{\theta^1}\right) \left(\frac{1}{\theta^1}\right)^2 + \left(\frac{y_2^1}{\theta^2}\right)^\nu \left(\frac{1}{\theta^2}\right)^2 \right] < 0 \quad (\text{D.25})$$

Because $u(\cdot)$ is concave function, the bracket of equation (D.24) is positive. By Assumption 1, Δ_d^1 is positive.

For $i = 2$, substituting (D.15) to (D.18) into Δ_d^2 yields:

$$\begin{aligned} \Delta_d^2 = & \left(\frac{n^1 n^2}{\theta^1 \theta^2}\right)^2 \frac{\frac{\partial^2 u}{\partial c_2^1 \partial d^1}(c_2^1, d^1)}{|A|} v''\left(\frac{y_2^1}{\theta^2}\right) \left[+n^2 \frac{v'\left(\frac{y_2^1}{\theta^2}\right)}{\theta^2} \frac{\partial^2 u}{\partial c_2^1 \partial c_2^1}(c_2^1, d^1) \right. \\ & \left. + \lambda_2^S n^1 \left\{ -2v''\left(\frac{y_2^1}{\theta^1}\right) + (\theta^1)^2 \frac{\partial^2 u}{\partial c_2^1 \partial c_2^1}(c_2^2, d^2) \right\} \right] \end{aligned} \quad (\text{D.26})$$

Then, equation (D.26) is negative due to Assumption 1 and the concavity of sub-utility function $u(\cdot)$.

Similarly, the variable Δ_s^i can be rewritten as follows:

$$\begin{aligned} \Delta_s^i = & n^i (n^1 n^2)^2 (1+r) \left[\frac{\partial^2 u}{\partial c_2^1 \partial c_2^1}(c_2^1, d^2) \frac{\partial^2 u}{\partial c_2^2 \partial c_2^2}(c_2^2, d^2) \mu_s + \right. \\ & \left. + \lambda_2^S v''\left(\frac{y_2^1}{\theta^1}\right) v''\left(\frac{y_2^2}{\theta^2}\right) \left\{ \frac{\partial^2 u}{\partial c_2^1 \partial c_2^1}(c_2^1, d^1) - \frac{\partial^2 u}{\partial c_2^2 \partial c_2^2}(c_2^2, d^2) \right\} \right] \quad i = 1, 2 \end{aligned} \quad (\text{D.27})$$

where

$$\mu_s \equiv -\lambda_2^S v''\left(\frac{y_2^1}{\theta^1}\right) \left(\frac{1}{\theta^1}\right)^2 + \frac{v'\left(\frac{y_2^1}{\theta^2}\right)''}{v} \left(\frac{y_2^2}{\theta^2}\right) \left(\frac{1}{\theta^2}\right)^2$$

Moreover, under Assumption 2, the variable μ_d can be rewritten as follows:

$$\mu_s = \kappa^2 \nu \left(\frac{1}{\theta^2}\right)^2 \left(\frac{y_2^2}{\theta^2}\right)^{\nu-1} \left[-\frac{y_2^2}{\theta^1} \left(\frac{y_2^1}{\theta^1}\right) \left(\frac{1}{\theta^1}\right)^4 + \left(\frac{y_2^1}{\theta^2}\right)^\nu \left(\frac{1}{\theta^2}\right)^4 \right] < 0 \quad (\text{D.28})$$

Because $u(\cdot)$ is concave function, the first term in the bracket of equation (D.27) is negative. If the durable good consumption and non-durable good consumption are not separable, that is, $\frac{\partial^2 u}{\partial c_t^i \partial d^i}(c_t^i, d^i) \neq 0$, the second term in the bracket of equation (D.27) is ambiguous. Then, Δ_s^i cannot be signed. If the durable good consumption and non-durable good consumption are not separable, the bracket of equation (D.27) is vanished. Therefore, Δ_s^i is negative. Taking the difference between Δ_s^1 and Δ_s^2 are

$$\Delta_s^1 - \Delta_s^2 = -\frac{(n^1 - n^2)(1+r)}{\theta^2} \left(\frac{n^1 n^2}{\theta^1 \theta^2}\right)^2 \left\{ \frac{\partial^2 u}{\partial c_2^2 \partial c_2^2}(c_2^2, d^2) \right\}^2 \left\{ \frac{v''\left(\frac{y_2^1}{\theta^1}\right)}{(\theta^2)^2} - \frac{v''\left(\frac{y_2^2}{\theta^2}\right)}{(\theta^1)^2} \right\} \left\{ v'\left(\frac{y_2^2}{\theta^2}\right) - v'\left(\frac{y_2^1}{\theta^2}\right) \right\} \quad (\text{D.29})$$

By the strict convexity of $v(\cdot)$ and the first order conditions with respect to y_2^i (equation (D.3)), we obtain equation (26):

$$\Delta_s^1 - \Delta_s^2 \geq 0 \iff n^1 - n^2 \geq 0 \quad (\text{D.30})$$

□

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