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### Agglomeration under money illusion

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#### Abstract

The core-periphery model by Krugman (1991) sparked the literature of new economic geography, which studies the mechanism of spatial agglomeration of economic activities. Forslid and Ottaviano (2003) offered an analytically solvable version of the core-periphery model. Based on their results, I comment on further implications of the model besides those covered in their paper and subsequent studies. Specifically, I show that agglomeration still occurs even under complete money illusion, but with two differences: 1) agglomeration becomes less likely, that is, other things being equal, agglomeration occurs less easily, and 2) the break and the sustain points coincide.

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## 1. Introduction

Apart from the exception of urban and regional economics that deal with geographic aspects of the economy, mainstream economics lacked the idea of taking space into account. Where spatial aspects have been considered, this typically relies on Marshallian externalities to explain cities or agglomeration of economic activities. However, the mechanism then is kept as a black box. This situation motivated Krugman (1991) to provide a breakthrough by developing a spatial version of the Dixit-Stiglitz (1977) type monopolistic competition model. The model, by incorporating scale economies at individual firm level and transport costs, explains why economic activities can be geographically distributed unevenly as a consequence of market interactions; when transport cost (or more generally trade cost) is reduced under a threshold level, a cumulative process of the agglomeration of firms and worker/consumers sets in, which catastrophically changes geography such that one region becomes a “core” region. The model is also known as the core-periphery model.

Agglomeration in the core-periphery model is a result of migration towards one location of mobile workers whose decisions are affected by three forces: the market size (or market access) effect, the local competition (or market crowding) effect, and the cost-of-living effect. Other things being equal, a larger market attracts mobile workers because larger market size increases demand per firm, leading to higher wages. On the other hand, a larger market also implies a larger mass of firms competing in one location, that is, tougher local competition. This reduces local demand per firm, leading to lower wages. However, local competition helps consumers in that location because the cost of living is lower. The market size and the cost-of-living effects support agglomeration of mobile workers in one location, while the local competition effect is a dispersion force that works oppositely to discourage agglomeration. Agglomeration occurs when the two agglomeration forces outweigh the dispersion force. Importantly, trade cost between locations affects the relative strength of the three forces. In particular, when the trade cost drops below a threshold, agglomeration occurs catastrophically, creating a core-periphery geographic outcome.

The new economic geography literature sparked by Krugman (1991) can be separated into two groups. The first group builds on Krugman (1991) while maintaining the abovementioned human migration mechanism of the core-periphery model: Fujita and Krugman (1995) and Fujita, Krugman and Mori (1999) introduce continuous space to explain evolution of cities. Krugman and Livas-Elizondo (1996) and Paluzie (2001) study how openness to international trade affects domestic migration and economic concentration. Instead of the freely traded agricultural sector in Krugman’s model, Helpman (1998) introduces housing, which works as a dispersion force. Suedekum (2006) shows that the core-periphery structure can still occur in a more realistic setting by adding a non-tradable home goods sector. The second group of the new economic geography literature looks for alternative agglomeration mechanisms, that is, agglomeration caused by factors other than migration: Martin and Rogers (1995) focus on mobile capital that leads to agglomeration at low trade costs, which is now known as the footloose capital model. Krugman and Venables (1995) model agglomeration by introducing input-output linkages. Baldwin (1999) studies construction and depreciation of capital as a different source of agglomeration.

At the heart of the core-periphery model and its subsequent studies in the first group are the mobile workers who migrate to other locations seeking higher real wages, that is, wages deflated by the overall level of prices. Such migration behavior based on real returns or real wages is perfectly rational. We cannot, however, take for granted that agents always behave based on their real wages, because they may not have information on the prices in a different

location. Under such circumstances, they may respond to nominal wages rather than real wages in making migration decisions. This is known as money illusion, which has been studied at least since Fisher (1913, 1928). Later, for example, Shafir, Diamond, and Tversky (1997) provided empirical evidence for the existence of money illusion and showed that money illusion affects behavior in various situations.

It is therefore of natural interest how agglomeration in the core-periphery model is affected when money illusion exists and agents respond to nominal wages in their migration behaviors. Money illusion implies that the abovementioned cost-of-living effect does not work. However, one drawback of the core-periphery model and subsequent studies in the first group that keeps us from conducting such analysis is that they are not analytically solvable and hence rely on numerical simulations; one has to try various sets of parameter values and ‘rough out’ the results. Such analysis has become possible with the analytically solvable version of the core-periphery model by Forslid and Ottaviano (2003), which is also known as the footloose entrepreneur model. Using their result, specifically the explicit solution of the equilibrium (nominal) wages derived in their work, this paper examines how agglomeration is affected by the removal of the cost-of-living effect.

Previewing the results, it will be shown that agglomeration still occurs even under complete money illusion, but with two differences. One difference is that agglomeration becomes less likely, that is, other things being equal, agglomeration occurs less easily, and the other, more technical difference, is that the break point and the sustain point coincide under money illusion. The result that agglomeration occurs less easily implies that money illusion can be considered a dispersion force. Indeed, it is natural and rational for mobile workers to make both consumption and migration decisions based on real wages. However, migration based on real wages is not necessary for agglomeration to occur.

The result that agglomeration becomes less likely can be understood intuitively by recalling the three forces at work. Under complete money illusion, agents do not take into account the overall prices in their migration decisions but rather their decisions are totally based on the nominal wages. Therefore, money illusion cancels the cost-of-living effect, and agglomeration depends only on the market size effect and the local competition effect. The agglomeration force is weakened; with only one agglomeration force, agglomeration occurs less easily.

In the next section, I review the short-run result by Forslid and Ottaviano (2003). Based on that result, in section 3, I analyze migration under money illusion, which is compared with the standard result by Forslid and Ottaviano (2003). Section 4 summarizes.

## **2. Review of the short-run result by Forslid and Ottaviano (2003)**

### **2.1 Assumptions**

There are two regions in the economy, region 1 and region 2. The population of the economy consists of skilled workers and unskilled workers. The total amounts of skilled workers and unskilled workers are denoted by  $H$  and  $L$ , respectively.<sup>1</sup> The skilled workers are mobile between the two regions, and the amounts of skilled workers in region 1 and region 2 are denoted by  $H_1$  and  $H_2$ , respectively. Hence,  $H_1 + H_2 = H$ . (Hereafter, subscripts are used to denote the regions.) The unskilled workers, in contrast, are not mobile, and they are assumed to be evenly distributed between the two regions. That is, each region has  $L/2$  unskilled workers.

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<sup>1</sup> I follow the notation by Forslid and Ottaviano (2003) in this section.

The same population also comprises the consumers. There are two types of goods to consume: differentiated good  $X$  (manufactured good) and homogeneous good  $A$  (agricultural good). All consumers have the same preferences, which are specified by the following two-tier structure. The upper tier utility function is

$$U_i = X_i^\mu A_i^{1-\mu}, \quad (1)$$

and, following Dixit and Stiglitz (1977), the lower tier component is given by

$$X_i \equiv \left( \int_{s \in N} d_i(s)^{(\sigma-1)/\sigma} ds \right)^{\sigma/(\sigma-1)}, \quad (2)$$

where  $\mu \in (0, 1]$  is a constant,  $X_i$  is consumption of good  $X$  (specified as a composite of all the differentiated varieties of good  $X$ ),  $A_i$  is consumption of good  $A$ ,  $N$  is the mass of varieties of good  $X$ ,  $d_i(s)$  is the consumption of variety  $s$  of good  $X$ , and  $\sigma$  is both the elasticity of demand of any variety and the elasticity of substitution between any two varieties of good  $X$ .

The firms in the  $X$  sector are monopolistically competitive and employ both skilled and unskilled workers. In order to produce  $x(s)$  units of variety  $s$ , a firm incurs a fixed input requirement of  $\alpha$  units of skilled labor and a marginal input requirement of  $\beta x$  units of unskilled labor. Then the total production cost of a firm is given by

$$TC_i(s) = w_i \alpha + w_i^L \beta x_i(s), \quad (3)$$

where  $w_i$  is the skilled wage and  $w_i^L$  is the unskilled wage.

In sector  $A$ , firms produce a homogeneous good under perfect competition, employing only unskilled labor. Production technology is constant returns to scale such that one unit of output requires one unit of unskilled workers. The homogeneous good produced in sector  $A$  is freely traded between the two regions. Good  $X$ , however, requires trade costs, as specified in the next subsection.

## 2.2 Consumer and firm behaviors

In the above setting, consumers' utility maximization leads to demand by residents in region  $i$  for each variety  $s$  of good  $X$  produced in region  $j$ ,  $d_{ji}(s)$ , being

$$d_{ji}(s) = \frac{p_{ji}(s)^{-\sigma}}{P_i^{1-\sigma}} \mu Y_i, \quad i, j = \{1, 2\} \quad (4)$$

where  $Y_i$  is aggregate income in region  $i$ ,  $p_{ii}(s)$  is the price of variety  $s$  in region  $i$  produced in region  $i$ , and  $p_{ji}(s)$  is the price of variety  $s$  in region  $i$  produced in region  $j$ .  $P_i$  is called the price index of good  $X$  in region  $i$ , which is

$$P_i = \left[ \int_{s \in n_i} p_{ii}(s)^{1-\sigma} ds + \int_{s \in n_j} p_{ji}(s)^{1-\sigma} ds \right]^{1/(1-\sigma)}, \quad (5)$$

where  $n_i$  and  $n_j$  are the masses of the firms. Thus, consumers maximize utility (1) subject to the following budget constraint:

$$\int_{s \in n_i} p_{ii}(s) d_{ii}(s) ds + \int_{s \in n_j} p_{ji}(s) d_{ji}(s) ds + p_i^A A_i = Y_i, \quad (6)$$

where  $p_i^A$  is the price of the agricultural good.

On the supply side, setting the unskilled wage ( $w_i^L$ ) equal to one, profit of a typical manufacturing firm located in region  $i$  is

$$\Pi_i = p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta[d_{ii}(s) + \tau d_{ij}(s)] - \alpha w_i. \quad (7)$$

Here,  $\tau$  represents the ‘iceberg’ trade cost, meaning that  $\tau \in (1, +\infty]$  units must be shipped for one unit of the manufactured good to reach the other region. Profit maximization by the firms leads to their setting prices so that marginal revenues equal marginal costs, that is,

$$p_{ii}(s) = \beta\sigma/(\sigma-1) \text{ and } p_{ij}(s) = \tau\beta\sigma/(\sigma-1). \quad (8)$$

Substituting (8) into (5), the price index becomes

$$P_i = \frac{\beta\sigma}{\sigma-1} [n_i + \phi n_j]^{1/(1-\sigma)}, \quad (9)$$

where  $\phi \equiv \tau^{1-\sigma} \in (0, 1]$  is called the freeness of trade. The larger (smaller)  $\phi$  is, the lower (higher) are trade costs.

The sector A assumptions imply that the unit production cost for a firm in sector A equals the unskilled wage  $w_i^L$ . Then perfect competition leads to marginal cost pricing so that  $p_i^A = w_i^L = 1$ .

### 2.3 Equilibrium

Equilibrium is defined as the goods and factor markets clearing (i.e., supply equals demand) under free entry. Because, in the short run, the amount of skilled workers is given as  $H_i$ , in equilibrium, the mass of firms in the  $X$  sector in each region, which is the variety of  $X$  goods produced in each region, is given by

$$n_i = H_i/\alpha. \quad (10)$$

As is well explained by Forslid and Ottaviano (2003, p.233), free entry in their model implies that skilled wage is bid up until it equals the operating profit and therefore no pure profits are earned ( $\Pi_i = 0$ ). Hence,

$$w_i = \frac{1}{\alpha} \{p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta[d_{ii}(s) + \tau d_{ij}(s)]\}. \quad (11)$$

Substituting (8) into (11), this means

$$w_i = \frac{\beta x_i}{\alpha(\sigma-1)}. \quad (12)$$

The goods market clearing condition is

$$x_i = \frac{\sigma-1}{\beta\sigma} \left( \frac{\mu Y_i}{n_i + \phi n_j} + \frac{\phi \mu Y_j}{\phi n_i + n_j} \right), \quad (13)$$

where

$$Y_i = L/2 + w_i H_i. \quad (14)$$

The factor market clearing condition for unskilled workers is

$$L = \beta x_i n_i + \beta x_j n_j + (1-\mu)(Y_i + Y_j). \quad (15)$$

Using (10) and (12), (13) becomes

$$w_i = \frac{\mu}{\sigma} \left( \frac{Y_i}{H_i + \phi H_j} + \frac{\phi Y_j}{\phi H_i + H_j} \right). \quad (16)$$

Finally, substituting (14) into (16) and rearranging, a system of two linear equations in  $w_1$  and  $w_2$ , which are equilibrium skilled wages, is obtained as

$$w_i = \frac{(\mu/\sigma) L}{1 - (\mu/\sigma)} \frac{2\phi H_i + \{1 - (\mu/\sigma) + [1 + (\mu/\sigma)]\phi^2\} H_j}{2 \phi (H_i^2 + H_j^2) + \{1 - (\mu/\sigma) + [1 + (\mu/\sigma)]\phi^2\} H_i H_j}, \quad (17)$$

and it can be confirmed that (17) satisfies (15).

### 3. Migration and agglomeration under money illusion

While consumption behavior is derived from utility maximization subject to prices of goods and consumers' budgets, I depart from Forslid and Ottaviano (2003) and other studies that examine human migration in the new economic geography literature by assuming here that there is money illusion in making migration decisions. Specifically, defining  $h \equiv H_1/H$  as the share of skilled workers that reside in region 1, I rewrite the Marshallian adjustment by Forslid and Ottaviano (2003, p.234) under the assumption that migration follows the *nominal* wage differential:

$$\dot{h} \equiv dh/dt = \begin{cases} w_1 - w_2 & \text{if } 0 < h < 1 \\ \min\{0, w_1 - w_2\} & \text{if } h = 1 \\ \max\{0, w_1 - w_2\} & \text{if } h = 0 \end{cases}. \quad (18)$$

It then becomes necessary to focus on  $w_1 - w_2$ . Using the equilibrium wages of the skilled workers obtained in (17),

$$w_1 - w_2 = \frac{\psi(2\phi - \kappa) L}{H} \frac{2\kappa - 1}{2\phi + (2\phi - \kappa)(h^2 - h)}, \quad (19)$$

where  $\psi \equiv \frac{(\mu/\sigma)L}{1 - \mu/\sigma} > 0$  and  $\kappa \equiv 1 - \mu/\sigma + (1 + \mu/\sigma)\phi^2 > 0$ .

Then we have

$$\frac{d(w_1 - w_2)}{dh} = \frac{\psi(2\phi - \kappa) L}{H} \frac{\left\{ \frac{[(1-h)^2 + h^2]\kappa + 4(1-h)h\phi}{[\phi + (2\phi - \kappa)(h^2 - h)]^2} \right\}}{2}. \quad (20)$$

Inspecting (20), because the final term inside the large bracket is positive,  $d(w_1 - w_2)/dh$  is positive if  $2\phi - \kappa$  is positive, which implies

$$\phi > \frac{1 - (\mu/\sigma)}{1 + (\mu/\sigma)}. \quad (21)$$

Forslid and Ottaviano (2003, p.234) denote this threshold level  $\phi_w$ , which they obtain by differentiating  $w_1/w_2$  with respect to  $h$ . However, they steer away from  $\phi_w$  as they move on to analyze migration based on real wages. If there is money illusion and the skilled workers decide migration based on nominal wages, as I assume here, then  $\phi_w$  becomes the point of reference.

If freeness of trade ( $\phi$ ) exceeds  $\phi_w$ , then  $d(w_1 - w_2)/dh > 0$ , so the symmetric equilibrium in which the skilled workers are evenly distributed between the two regions becomes unstable and catastrophic agglomeration in one region occurs; geographic symmetry breaks beyond  $\phi_w$ . Hence, under money illusion,  $\phi_w$  becomes the 'break point' in the terminology of new economic geography.

In contrast, if  $\phi$  subceeds  $\phi_w$ , then  $d(w_1 - w_2)/dh < 0$ , so agglomeration no longer becomes sustainable and the symmetric equilibrium becomes stable. In the sense that agglomeration is no longer sustainable with  $\phi$  under  $\phi_w$ , using the new economic geography

terminology,  $\phi_w$  is the ‘sustain point’. The result for the present money illusion case is illustrated by the bifurcation diagram in figure 1.

This result can be interpreted as follows. When  $\phi < \phi_w$ , setting up a profitable new firm in, say, region 1, so that it can hire skilled workers from region 2 by offering higher (nominal) wages is not possible. This is because, at high trade costs, the two region’s markets are close to isolation, and setting up a new firm in one region simply toughens local competition while relaxing competition in the other region. Tougher local competition implies a lower local price index, which leads to lower demand per firm, lower operating profits, and correspondingly lower skilled wages.

In contrast, when  $\phi > \phi_w$ , that is, when trade is freer than the threshold, the two regional markets are quite integrated. Therefore, how many firms there are in the home market no longer matters much; setting up a new firm in region 1 does not increase local competition relative to region 2 as much. Instead, bringing in skilled workers from region 2 to region 1 increases the market size of region 1, whereby setting up a new firm in region 1 becomes profitable; the increased market size leads to larger demand per firm, higher operating profits, and correspondingly higher skilled wages.

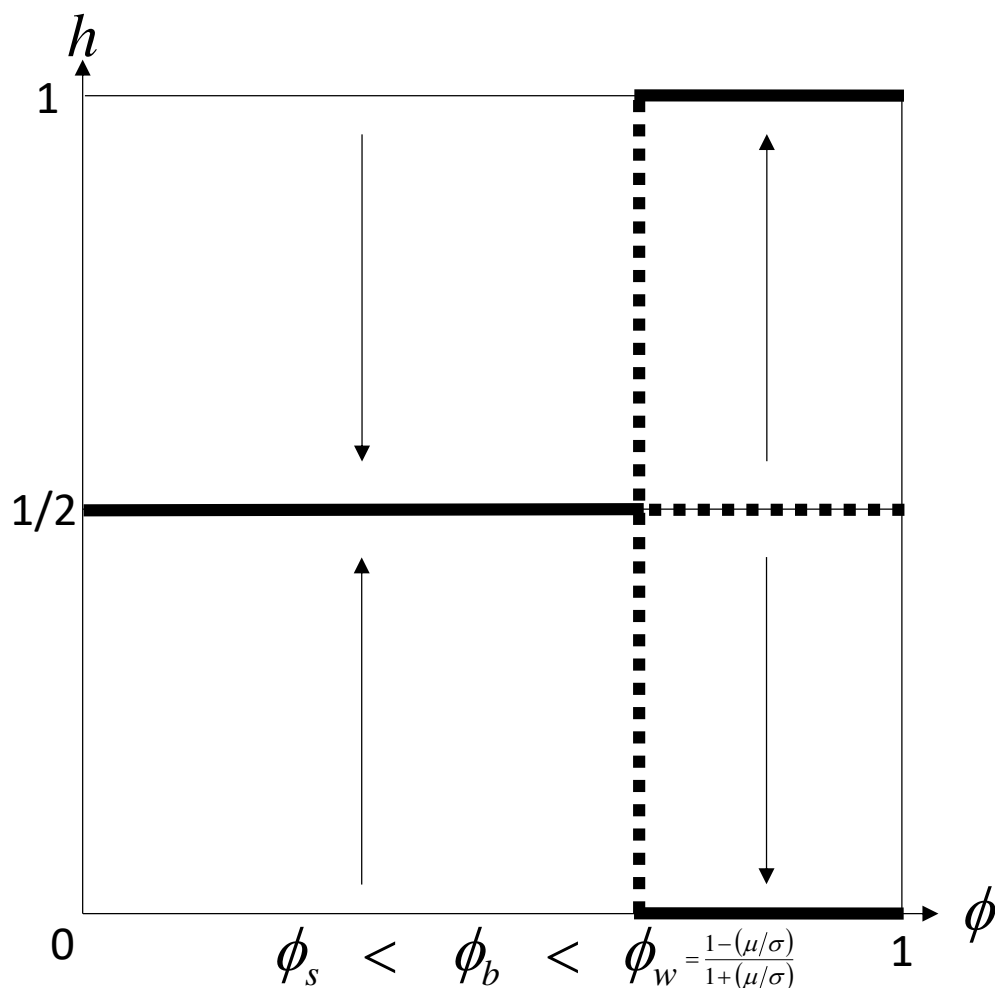
How does the result compare to the core-periphery model? First, please recall that  $\phi_w < 1$ , which becomes important in the money illusion case.  $\phi_w < 1$  implies that there is always a range of  $\phi$  within which the agglomeration equilibrium becomes stable; agglomeration still occurs under money illusion when  $\phi_w < \phi < 1$ .

Second, as obtained by Forslid and Ottaviano (2003, pp.236-237),  $\phi_s < \phi_b < \phi_w$ . This means that  $\phi_w$  is located to the ‘right’ of the sustain point ( $\phi_s$ ) and the break point ( $\phi_b$ ) in the bifurcation diagram of the core-periphery model.<sup>2</sup> That is, other things being equal, agglomeration under money illusion occurs under higher freeness of trade (or lower trade costs) compared to the standard result by Forslid and Ottaviano (2003). In this sense, money illusion makes agglomeration more difficult. The result that agglomeration becomes less likely can be understood intuitively by recalling the three forces at work. Under complete money illusion, agents do not take into account the overall prices in their migration decisions but rather their decisions are totally based on the nominal wages. Therefore, money illusion cancels the cost-of-living effect, and agglomeration depends only on the market size effect and the local competition effect. The agglomeration force is weakened; with only one agglomeration force, agglomeration occurs less easily.

Third, Krugman (1991) and Forslid and Ottaviano (2003) find in their core-periphery models ranges of  $\phi$  where both the symmetric and the agglomeration equilibria are stable, known as the ‘overlap’. In contrast, because the only threshold that appears in the present money illusion setting is  $\phi_w$ , there is no overlap; the break and the sustain points coincide under money illusion.

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<sup>2</sup> See Baldwin et al. (2002, p.33 and p.101) on the bifurcation diagrams for the core-periphery models by Krugman (1991) and Forslid and Ottaviano (2003), respectively.



**Figure 1: Bifurcation under money illusion**

#### 4. Summary

The core-periphery model by Krugman (1991) explained geographical dispersion and agglomeration of economic activities generated by human migration. The underlying mechanism that generates agglomeration in the core-periphery model relies on three forces: the market size, cost-of-living, and local competition effects. Based on numerical simulations, Krugman (1991) demonstrated that transport cost plays a key role in determining the relative strength of these forces. Specifically, when transport cost is reduced under a threshold level, the combined market size and the cost-of-living effect outweighs the local competition effect.

In the core-periphery model, mobile agents make migration decisions based on real wages, that is, considering both nominal wages and prices or living costs of different locations. Subsequent studies of agglomeration generated by human migration in the new economic geography literature followed this real-wage-based migration.

Using the analytical solution of the core-periphery model obtained by Forslid and Ottaviano (2003), I examined in this paper what would happen to agglomeration if there were money illusion and the mobile agents only considered nominal wages in making their migration decisions.



It was shown that agglomeration still occurs without the cost-of-living effect. That is, even if the mobile workers were under complete money illusion in making their migration decisions only by comparing nominal wages of different locations, agglomeration would still occur when trade is freer (or trade cost is lower) than a threshold. The threshold freeness of trade (the threshold trade cost), however, is higher (lower) than that of the standard case by Forslid and Ottaviano (2003), implying that agglomeration becomes more difficult. This is because the cost-of-living effect, which was one of the two agglomeration forces in the core-periphery model, no longer works under money illusion.

## References

- Baldwin, Richard E. (1999) "Agglomeration and endogenous capital" *European Economic Review* **43**, 253-280.
- Baldwin, Richard E., Rikard Forslid, Philippe Martin, Gianmarco I. P. Ottaviano, and Frederic Robert-Nicoud (2003) *Economic Geography and Public Policy*, Princeton, MA: Princeton University Press.
- Dixit, Avinash K. and Joseph E. Stiglitz (1977) "Monopolistic competition and optimum product diversity" *American Economic Review* **67**, 297-308.
- Fisher, Irving (1913). "A Remedy for the Rising Cost of Living: Standardizing the Dollar" *American Economic Review* **3**, 20-28.
- Fisher, Irving (1928) *The Money Illusion*, New York: Adelphi.
- Forslid, Rikard and Gianmarco I.P. Ottaviano (2003) "An analytically solvable core-periphery model" *Journal of Economic Geography* **3**, 229-240.
- Fujita, Masahisa and Paul Krugman (1995) "When is the economy monocentric?: von Thunen and Chamberlin unified" *Regional Science and Urban economics* **25**, 505-528.
- Fujita, Masahisa, Paul Krugman and Tomoya Mori (1999) "On the evolution of hierarchical urban systems" *European Economic Review* **43**, 209-251.
- Helpman, Elhanan. (1998) "The size of regions", Chapter 2 in David Pines, Efraim Sadka, and Itzhak Zilcha eds., *Topics in Public Economics*, Cambridge: Cambridge University Press, pp. 33-54.
- Krugman, Paul (1991) "Increasing returns and economic geography" *Journal of Political Economy* **99**, 483-499.
- Krugman, Paul and Raul Livas-Elizondo (1996) "Trade policy and the Third World metropolis", *Journal of Development Economics* **49**, 137-150.
- Krugman, Paul and Anthony J. Venables (1995) "Globalization and the inequality of Nations", *Quarterly Journal of Economics* **60**, 857-880.
- Martin, Philippe and Carol Ann Rogers (1995) "Industrial location and public infrastructure" *Journal of International Economics* **39**, 335-351.
- Paluzie, Elisenda (2001) "Trade policy and regional inequalities" *Papers in Regional Science* **80**, 67-85.
- Shafir, Eldar, Peter Diamond and Amos Tversky (1997) "Money illusion" *Quarterly Journal of Economics* **112**, 341-374.
- Suedekum, Jens (2006) "Agglomeration and regional costs of living" *Journal of Regional Science* **46**, 529-543.