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Progressivity of burden-sharing in a Lindahl Equilibrium: a unifying criterion

Wolfgang Buchholz
University of Regensburg and CESifo Munich

Dirk Rübbelke
TU Bergakademie Freiberg

Abstract

In this paper, we show that progressivity (regressivity) of burden sharing in a Lindahl equilibrium is a direct consequence of gross complementarity (substitutability) between the private and the public good when the public good is taken as the numéraire. We then link these novel basic conditions to the conditions for progressivity (regressivity) that have been presented in the literature so far.

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Contact: Wolfgang Buchholz - wolfgang.buchholz@wiwi.uni-regensburg.de, Dirk Rübbelke - dirk.ruebbelke@vwl.tu-freiberg.de.
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1. Motivation

Fair burden-sharing among agents is a precondition for a successful cooperation on public good provision. One fairness principle that has already been prominent in the classical theory of public finance for a long time (see, e.g. Musgrave 1959) is the *benefit principle*, which means that an agent's cost/"tax" share in financing a public good should depend on her (marginal) willingness to pay for it. In this note, we consider the Lindahl equilibrium that is entailed by the benefit principle in a standard public good economy and explore the conditions under which a progressive pattern of public good contributions results in this equilibrium, so that the relative shares the agents spend for the public good in the Lindahl equilibrium are increasing with their incomes. Then burden-sharing complies with an apparent normative postulate flowing from the *ability-to-pay principle*.[1] The objective of this note is to provide a fairly simple general criterion, which helps to interpret the specific criteria for progressive burden-sharing in the Lindahl equilibrium that have been obtained before in the literature (see Aaron and McGuire 1970, Kovenock and Sadka 1981, and Snow and Warren 1983) from an uniform perspective.

The structure of this paper is as follows: After presenting the framework of the analysis in Section 2, Section 3 shows a simple proof that progressivity (and inversely regressivity) of burden-sharing is a direct consequence of gross complementarity (substitutability) between the public and the private good, if the public good serves as the numéraire and the price of the private good varies. In Section 4, we describe how this unifying criterion can be related to the more intricate criteria known from the literature so far. Section 5 concludes.

2. The Framework

There are n agents with the same preferences $u(x_i, G)$, where x_i is agent i's private consumption and G is public good supply. The utility function $u(x_i, G)$ has the standard properties, i.e. it is twice continuously differentiable, strictly monotone increasing in both variables, and strictly quasi-concave. Moreover, both goods are assumed to be strictly non-inferior ("normal"), which means that the demand for both goods is increasing when income grows while the relative price between both goods remains the same. Then, in an x_i-G-diagram the expansion

[1]Bringing the benefit and the ability-to-pay principle in line might also be of some empirical relevance in the context of climate change policy where burden-sharing according to Lindahlian precepts has been suggested (see, e.g., Uzawa, 2003, and Groot and Swart, 2018) while, at the same time, equitable burden-sharing according to the ability-to-pay principle is called for.

paths, which connect all the points at which the indifference curves have the same slope (being equal to the relative price of both goods), are running north-east. Clearly, the normality property does not depend on whether we use the private or the public good as the numéraire.

At any point (x_i, G) the marginal rate of substitution between the public and the private good, i.e. the marginal willingness to pay for the public good or the "shadow price" of the public good, is

$$m(x_i, G) = \frac{\partial u / \partial G}{\partial u / \partial x_i}(x_i, G). \tag{1}$$

In an x_i-G-diagram, $\frac{1}{m(x_i, G)}$ then is the slope of an indifference curve at point (x_i, G).

The partial derivatives of $m(x_i, G)$ w.r.t. the first and the second variable are denoted by $m_1(x_i, G) = \frac{\partial m(x_i, G)}{\partial x_i}$ and $m_2(x_i, G) = \frac{\partial m(x_i, G)}{\partial G}$, respectively. Normality as defined above implies that $m_1(x_i, G) > 0$ and $m_2(x_i, G) < 0$, which means that in the x_i-G-diagram the indifference curves become flatter when x_i is increased while G is kept constant, and that they become steeper when G is increased while x_i is fixed. This clearly follows because given normality, expansion paths are running north-east and indifference curves are convex (and could also be easily shown by a mathematical proof). The initial endowment of agent i measured in units of the private good is denoted by w_i. Agents are ranked according to their income levels, i.e. $w_1 \leq \ldots \leq w_n$.

The public good is produced by a summation technology for which the marginal rate of transformation mrt between the public and the private good is identical for all agents and normalized to one. Thus, an allocation $(x_1, \ldots, x_n, G)$ is feasible, if letting $g_i := w_i - x_i$ we have

$$G = \sum_{i=1}^{n} g_i \qquad \text{or} \qquad G + \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} w_i. \tag{2}$$

Given some allocation $(x_1, \ldots, x_n, G)$, agent i's cost share for public good provision is denoted by $p_i = \frac{g_i}{G}$, so that (2) gives $x_i + p_i G = w_i$.

A feasible allocation $(\tilde{x}_1,...,\tilde{x}_n,\tilde{G})$ is said to satisfy the benefit principle if $\tilde{p}_i = m(\tilde{x}_i,\tilde{G})$ and thus $\tilde{g}_i = m(\tilde{x}_i,\tilde{G})\tilde{G} = \tilde{p}_i\tilde{G}$ holds for each agent $i = 1,...,n$, i.e., public good supply $\tilde{G}$, when evaluated by its shadow price $mrs\ (\tilde{x}_i,\tilde{G})$, represents the equivalent to i's public good contribution $\tilde{g}_i$. Given $\tilde{p}_i = m(\tilde{x}_i,\tilde{G})$ as her personal public good price, agent i as a price-taker would choose the public good level $\tilde{G}$, i.e. $\tilde{G}$ maximizes $u(w_i - \tilde{p}_i G, G)$. A feasible allocation $(\tilde{x}_1,...,\tilde{x}_n,\tilde{G})$, which satisfies the benefit principle, hence is the Lindahl equilibrium in which all agents, being confronted with their individual Lindahl prices $\tilde{p}_i$, would demand the same level of the public good. Given normality, $w_k > w_j$ implies $\tilde{p}_k > \tilde{p}_j$ for the Lindahl prices (since otherwise agent k would demand more of the public good than agent j) and also $\tilde{x}_k < \tilde{x}_j$ for private consumption (since, again given normality, $\tilde{x}_k \geq \tilde{x}_j$ would imply $\tilde{p}_k = m(\tilde{x}_k,\tilde{G}) \leq m(\tilde{x}_j,\tilde{G}) = \tilde{p}_j$).

3. The Basic Progressivity Criterion

Let $G^{MG}(\omega,q)$ and $x^{MG}(\omega,q)$ be Marshallian demand functions for the public and the private good, respectively, when, different from the usual approaches, the public good is taken as the numéraire so that income ω and the private good price q are measured in units of the public good. These Marshallian demand functions result from maximizing utility $u(x,G)$ under the budget constraint $qx + G = \omega$. Normality then implies that $G^{MG}(\omega,q)$ and $x^{MG}(\omega,q)$ are increasing in ω and that $x^{MG}(\omega,q)$ is falling in q. If public good demand $G^{MG}(\omega,q)$ is falling (rising) in q, so that an increase of the private good price has a negative (positive) cross-price effect on public good demand, we label the public and the private good as *gross* x*-price complements* (*gross* x*-price substitutes*).

Now let us fix any level of public good supply $\bar{G} > 0$ and consider the auxiliary function $\Phi(x,\bar{G}) = \dfrac{x}{m(x,\bar{G})}$, which depends on the level of private consumption x. As a starting point for our analysis of progressivity (regressivity) conditions, we have the following result:

Lemma: The function $\Phi(x,\bar{G})$ is decreasing (increasing) in x for all $\bar{G} > 0$ if and only if the public and the private good are gross x-price complements (substitutes).

Proof: Assume that x increases from x' to x'' and let $q' = \dfrac{1}{m(x', \bar{G})}$, $q'' = \dfrac{1}{m(x'', \bar{G})}$, $\omega' = q'x' + \bar{G}$ and $\omega'' = q''x'' + \bar{G}$ (see Figure 1).

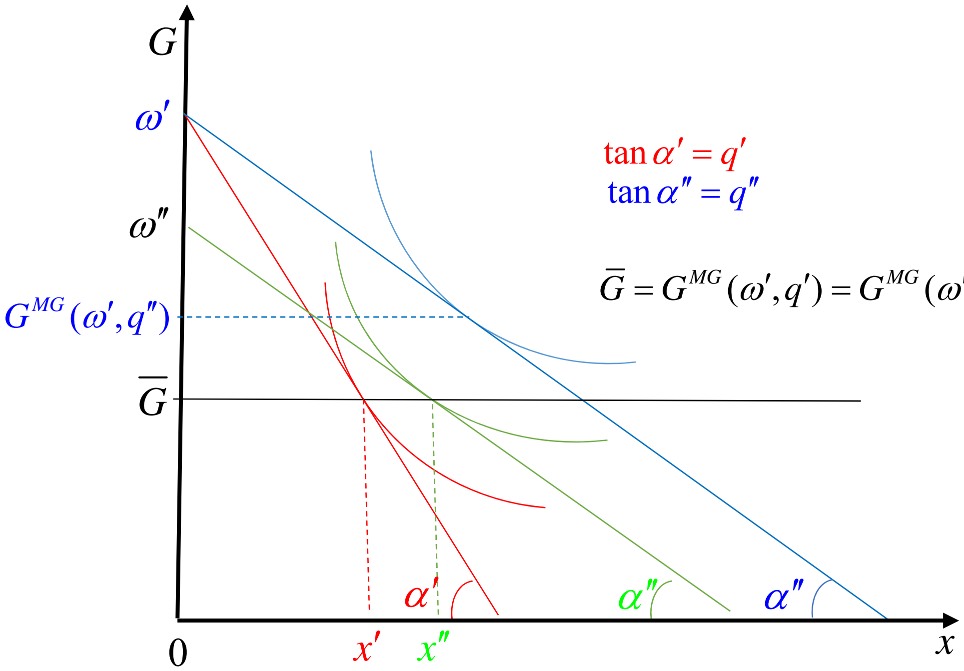

Figure 1

The if-part: As $q'' < q'$, gross x-price complementarity gives

$$G^{MG}(\omega', q'') > G^{MG}(\omega', q') = \bar{G} \tag{3}$$

Since $G^{MG}(\omega'', q'') = \bar{G}$ has to hold, $\omega'' < \omega'$ is required by normality, which implies $\Phi(x'', \bar{G}) = q''x'' = \omega'' - \bar{G} < \omega' - \bar{G} = q'x' = \Phi(x', \bar{G})$.

The only-if-part: Assume to the contrary that the public and the private good are not gross x-price complements. Then there exist some ω' and virtual private good prices q' and q'' with $q'' < q'$ so that in (3) the reverse inequality holds when $\bar{G} := G^{MG}(\omega', q')$. For the re-adjustment to the public good demand $\bar{G}$, $\omega'' > \omega'$ is required, which gives $\Phi(x'', \bar{G}) > \Phi(x', \bar{G})$ for

$x' = \frac{\omega' - \overline{G}}{q'}$ and $x'' = \frac{\omega' - \overline{G}}{q''}$. Hence, the function $\Phi(x, \overline{G})$ cannot be decreasing everywhere. The case of gross x -price substitutability is treated in an analogous way. QED

We now apply the Lemma to formulate the basic unifying criterion for progressivity (regressivity), which is at the core of our paper.

Proposition 1: If the public and the private good are gross x -price complements (substitutes), burden-sharing in a Lindahl equilibrium $(\tilde{x}_1, ..., \tilde{x}_n, \tilde{G})$ is progressive (regressive), i.e. the contribution-income ratio $\frac{\tilde{g}_i}{w_i}$ is increasing (decreasing) in w_i.

Proof: For each agent i it is implied by $\tilde{p}_i = m(\tilde{x}_i, \tilde{G})$ that

$$\frac{\tilde{g}_i}{w_i} = \frac{\tilde{p}_i \tilde{G}}{\tilde{x}_i + \tilde{p}_i \tilde{G}} = \frac{\tilde{G}}{\Phi(\tilde{x}_i, \tilde{G}) + \tilde{G}}. \tag{4}$$

Therefore, the assertion is an immediate consequence of the Lemma since private good consumption in a Lindahl equilibrium $\tilde{x}_i$ is increasing in income w_i. QED

4. Comparison with Previous Criteria

It is now straightforward to see that the conditions for progressivity (regressivity) of burden-sharing in the Lindahl equilibrium, which already exist in the literature, boil down to conditions on x -price complementarity (substitutability). This relates the general criterion presented in Proposition 2 to previous work, as to the condition provided by Kovenock and Sadka (1981, p. 97).[2]

Proposition 2: The public and the private good are gross x -price complements (substitutes) if

$$\frac{m_1(x, G)x}{m(x, G)} > 1 \qquad (< 1) \tag{5}$$

holds for the elasticity of the marginal willingness to pay for the public good $m(x, G)$ w.r.t. private consumption x.

[2] This condition also plays a central role in Ebert and Tillmann (2007) who investigate the progressivity issue in a more general setting in which public good supply is exogenously given and a budget surplus may arise.

Proof: The assertion follows from the Lemma since $\frac{\partial\Phi(x,\bar{G})}{\partial x} = \frac{m(x,G) - m_1(x,G)x}{m(x,G)^2}$. QED

To infer another type of conditions for x-price complementarity (substitutability) let $G^{Mx}(w,p)$ and $x^{Mx}(w,p)$ be the usual Marshallian demand functions for the public and the private good, respectively, given the private good endowment w and the public good price p measured in units of the private good. Furthermore, let G_1^{Mx} and G_2^{Mx} denote the first derivatives of G^{Mx} w.r.t. the income w and the public good price p, respectively. Then we directly obtain the condition provided by Snow and Warren (1983, p. 321) (see also Lambert 2001, p. 177, and Lambert 2012, p. 487).

Proposition 3: The private and the public good are gross x-price complements (substitutes) if for the income and price elasticity of Marshallian public good demand we have

$$-\frac{G_2^{Mx}(w,p)p}{G^{Mx}(w,p)} < \frac{G_1^{Mx}(w,p)w}{G^{Mx}(w,p)} \qquad (>). \tag{6}$$

Proof: Since $G^{MG}(\omega,q) = G^{Mx}(\frac{\omega}{q},\frac{1}{q})$ for $q = \frac{1}{p}$ (see also Figure 1), the public and the private good are x-price complements (substitutes) if

$$\frac{dG^{MG}(\omega,q)}{dq} = \frac{dG^{Mx}(\frac{\omega}{q},\frac{1}{q})}{dq} = -\frac{1}{q^2}\left(G_1^{Mx}(\frac{\omega}{q},\frac{1}{q})\omega + G_2^{Mx}(\frac{\omega}{q},\frac{1}{q})\right) < 0 \quad (>0). \tag{7}$$

Substituting $p = \frac{1}{q}$ and $w = p\omega = \frac{\omega}{q}$ in (7) implies $-pG_1^{Mx}(w,p)w - p^2 G_2^{Mx}(w,p) < 0 \quad (>)$. Dividing this by $pG^{Mx}(w,p)$ then gives the assertion. QED

The previous findings on progressivity of burden-sharing in a Lindahl equilibrium directly follow by combining Proposition 2 and Proposition 3 with Proposition 1.

The borderline case between progressivity and regressivity of burden-sharing is given when agents' expenses for the public good are proportional to their incomes. According to the reasoning leading to Proposition 1, this outcome results if the functions $\Phi(x,\bar{G})$ are constant in x, which is clearly satisfied if the underlying utility function is of the special type $u(x_i,G) = h(G)x_i$. Hence, our main result also reflects the criterion Cornes and Sandler (1996, pp. 203-204) have presented for proportional burden-sharing in Lindahl equilibria.

Based on Proposition 3 we can now, in addition, provide a result that relates x-price complementarity (substitutability) with the more familiar notion of G-price complementarity (substitutability), which refers to the situation where the private good serves as the numéraire and the price of the public good changes. The private and the public good are called *gross G-price complements* (*substitutes*), if Marshallian demand for the private good $x^{Mx}(w,p)$ is decreasing (increasing) in p, i.e. if $\frac{dx^{Mx}(w,p)}{dp} = \frac{d(w - pG^{Mx}(w,p))}{dp} = -(pG_2^{Mx}(w,p) + G^{Mx}(w,p)) < 0$ (>0) holds, which is equivalent to

$$-\frac{G_2^{Mx}(w,p)p}{G^{Mx}(w,p)} < 1 \qquad (>1). \tag{8}$$

Combining (6) and (8) thus leads to a further sufficient condition for x-complementarity (substitutability) and thus for progressiveness of burden-sharing in the Lindahl equilibrium.

Corollary: The private and the public good are gross x-price complements (substitutes), if the two goods are G-price complements (substitutes) and the income elasticity of Marshallian public good demand is larger (smaller) than one.

5. Conclusion

Our paper provides a unifying condition for progressiveness (regressiveness) of burden-sharing in a Lindahl equilibrium that gives a novel interpretation for the conditions existing so far. Yet, the results of the paper go far beyond the public good case. So we indicate (see the Corollary) how it is possible to conclude from the sign of one cross-price elasticity to the sign of the other cross-price elasticity and thus to interlink the two versions of gross complementarity (substitutability) between two goods in a general household model.

References

Aaron, H., and M. McGuire (1970) "Public Goods and Income Distribution" *Econometrica*, **38**, 907-920.

Cornes, R. and T. Sandler (1996) *The Theory of Externalities, Public Goods, and Club Goods*, 2nd Ed., Cambridge University Press: Cambridge UK.

Ebert, U. and G. Tillmann (2007) "Distribution Neutral Provision of Public Goods" *Social Choice and Welfare* **7**, 107-124

Groet, L. and J. Swart (2018) "Climate Change Control: The Lindahl Solution" *Mitigation and Adaptation Strategies for Global Change* **23**, 757-782.

Kovenock, D. and E. Sadka (1981) "Progression under the Benefit Approach to Taxation" *Economics Letters* **8**, 95-99.

Lambert, P.J. (2001) *The Distribution and Redistribution of Income*, 3rd Ed., Manchester University Press: Manchester UK.

Lambert, P.J. (2012) "Redistributional Effects of Progressive Income Taxes" in *Handbook of Income Inequality Measurement* by J. Silber, Ed., Springer: Berlin et al., 485-510.

Musgrave, R. (1959) *The Theory of Public Finance*, McGraw-Hill: New York.

Samuelson, P.A. (1974) "Complementarity: An Essay on the 40th Anniversary of the Hicks-Allen Revolution in Demand Theory" *Journal of Economic Literature* **12**, 1255-1289.

Snow, A. and R.S. Warren Jr. (1983) "Tax Progression in Lindahl Equilibrium" *Economics* Letters **12**, 319-326.

Uzawa, H. (2003) *Economic Theory and Global Warming*, Cambridge University Press: Cambridge UK.