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# Collaboration Networks in a Hotelling Game

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## Abstract

The paper investigates the stability and efficiency of R&D collaboration in a three-firm Hotelling game. Firms are assumed to be horizontally and vertically differentiated and to provide public services where price is thus set by the regulator. We show that firm-quality effort decreases with the number of links. Nonetheless, a conflict between stability and efficiency is likely to occur. We show that the complete network is uniquely stable but efficient only for a sufficiently low level of spillover rate. As a result, an over-connection problem may arise. However, for high spillover rates, the welfare-superior networks tend to be denser provided that the horizontal differentiation is low.



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# 1 Introduction

In recent years the design of public service provision has been steadily geared to increase firm integration and to the introduction of several forms of cooperation among private providers, notably through the creation of collaboration networks. The aim of these networks is basically to reduce the production costs from sharing knowledge, to increase the efficiency of the public-service provision and ultimately to improve social welfare. Nonetheless, reaching those objectives is conditioned to appropriate incentive schemes. As a matter of fact, the price of public services is usually set by the regulator according to the cost of production. In this context firms should rely on some strategic variables other than price in order to attract the demand, notably the product quality.

In this paper we study the incentives for collaboration between vertically and horizontally differentiated firms in a Hotelling game where price is set by the regulator. The benefits from firm agreements are assumed to arise from sharing knowledge about quality improving technology. Put it differently, when firms collaborate, their individual quality-improving R&D effort pushes the quality level of their partners up.

Real examples of this case are for firms operating in the health care system and high education (see among others Motta 1992 and Brekke et al. 2006).[1] Generally, in both the cases price is set by the regulatory authority. Firms will thus resort to strategic vertical and horizontal differentiation in order to attract consumers. In particular, we explore the relationship between product differentiation and network formation in a three-firm Hotelling game with two stages. In the first stage, firms form pair-wise collaboration links to share $R\&D$ knowledge about quality-improving technology. In the second stage, firms compete in the product market choosing quality but taking prices as given.

With this tool at hand, we address two key questions. First, we investigate the network stability, specifically if the unique pair-wise stable network is the complete one; second, our analysis seeks out the conditions under which the network is welfare maximizing. Methodologically, we follow a long strand of literature dealing with three-firm networks (see Goyal and Moraga-Gonzalez (2001), Song and Vannetelbosch (2007), Correani et al. (2014), Mizuno and Okumura (2014), Correani and Di Dio (2017)) but the results might be extended to the more general and complex case with $N \geq 3$.

Recently, there has been a growing literature about strategic network formation

[1] Interesting examples of quality improving agreements and $R\&D$ networks are, for instance, in Hagedoorn 2002.

(see Goyal and Moraga González (2001), Goyal and Joshi (2003,2006), Okumura (2012) among the others), also including the case of rival firm strategic behaviour in quality improving R&D networks (see Deroian and Gannon (2006)). Specifically, Deroian and Gannon adopt a three-stage Cournot game as in Goyal and Moraga-González (2001) and, focusing on regular networks and on the second stage Nash equilibrium, they show that the profit-maximizing number of links is first decreasing than increasing relative to the inverse measure of product differentiation. Moreover, they find that $R\&D$ effort decreases with the number of partners and that quality improving networks are over-connected as compared to the social optimum.

More recently, along this stream of research, Correani and Di Dio (2017) have extended the analysis to networks of collaboration among vertically and horizontally differentiated firms which compete á la Bertrand. Their main finding is that networks are denser when firms feature low vertical differentiation but high horizontal differentiation.

Adopting the pairwise stability notion introduced by Jackson and Wolinsky (1996), we find that under quality competition and prices set by a regulator, the complete network is the unique stable network. This result departs from Correani and Di Dio (2017) where denser networks are pairwise stable only if firms feature low vertical but high horizontal differentiation. Our finding also differs from Goyal and Joshi (2003) where, under price competition, the empty network is uniquely stable. Our results shows that when firms compete on quality they can mitigate competition by forming cooperation links (i.e., complete network). This is because, in our model, links induce firms to reduce their quality making them less aggressive.

However, social welfare is maximised by the complete network only if the rate of spillover is sufficiently low. As a result, networks of alliances could be over-connected and a conflict between stability and efficiency is likely to occur. Also in this case our results depart from those obtained in models where price is a strategic variable and the complete network is the unique socially efficient network (see, for instance, Okumura 2012). Notably, the main source of this difference is that when knowledge spillovers are high, linked firms reduce their quality, lowering consumer surplus and thus social welfare. Finally, as in Deroian and Gannon (2006) we find that firm quality effort decreases with the number of partners.

Results concerning network stability are presented analytically while those related to welfare are obtained through numerical simulations. However, in order to assess the robustness of the numerical results against changes in the values of the key parameters, we carry out some sensitivity checks.

The remainder of the paper is organized as follows. In Section 2 we set out the model. In Section 3 we present the main results. Section 4 concludes.

## 2 The model setup

The model is basically a two-stage Hotelling game. In the first stage firms might form pair-wise collaboration links. In the second stage firms compete by setting their quality improving $R\&D$ effort $e_i$. Price $p_i$ is exogenously set by the regulator so that we can write $p_i = p_j = p > 0$. Each firm incurs a constant marginal cost for production, $c$. Obviously, to guarantee positive marginal firms' profit it must be $p > c$. Without loss of generality, we will assume $c = 0$.

Specifically, we consider a set of three oligopolistic firms, $N = \{1, 2, 3\}$, which are vertically and horizontally differentiated and let $I_i \in [0, 1]$ $i \in N$, denotes the "location"of firm (product) $i$ on the $[0, 1]$ interval (the Hotelling line). We assume $I_1 < I_2 < I_3$. The quality of the product $i$ is described by a number $\theta_i = e_i + l \sum_{i \neq j} s_{ij} e_j$ where $e_i \geq 0$ is the quality improving $R\&D$ effort of firm $i$, $l \in [0, 1]$ is the exogenous rate of spillover and $s_{ij} \in \{0, 1\}$ is a binary variable representing the pair-wise relationship between firms $i$ and $j$. When firms $i$ and $j$ form a link then $s_{ij} = 1$; on the contrary, if they are unlinked, $s_{ij} = 0$. According to this formulation, the collaboration agreements help to increase the quality of the product. According to Deroian and Gannon (2006), the effort of every firm exclusively spills over the corresponding partners and then there are no spillovers from outside the industry.
For the sake of simplicity, link formation is assumed to be costless. Moreover, we do not analyse firm strategic location (i.e. we treat each $I_i$ $i \in N$ as exogenous). This approach is justified by reasons of tractability but also by the consideration that the choice of the location is, in general, more of a long-term decision than the choice of the product quality (Brekke et al., 2006). All the possible network structures are depicted in Figure 1 where $g_{ij}^{pc}$ is the partially connected network in which only firms $i$ and $j$ are linked; $g_i^s$ is for the star network where every firm $j \neq i$ is connected to the hub firm $i$. Finally, $g^c$ and $g^e$ are, respectively, the complete network (all firms are linked) and the empty network (no link among firms).

Firms are assumed to charge the same price $p$ for the product and to choose quality effort $e_i$ at the second stage. Turning to the demand side, consumers are assumed to be distributed uniformly on $[0, 1]$. The utility of a consumer located in $z \in [0, 1]$ for one unit of good $i$ with quality $\theta_i$ is:

$$u_i = r + \theta_i - \tau (z - I_i)^2 - p, \tag{1}$$

where, as usual, $r$ is the willingness to pay, $\tau$ measures the unit transportation cost and $p$ the exogenous price of product. The transportation cost parameter $\tau$ measures

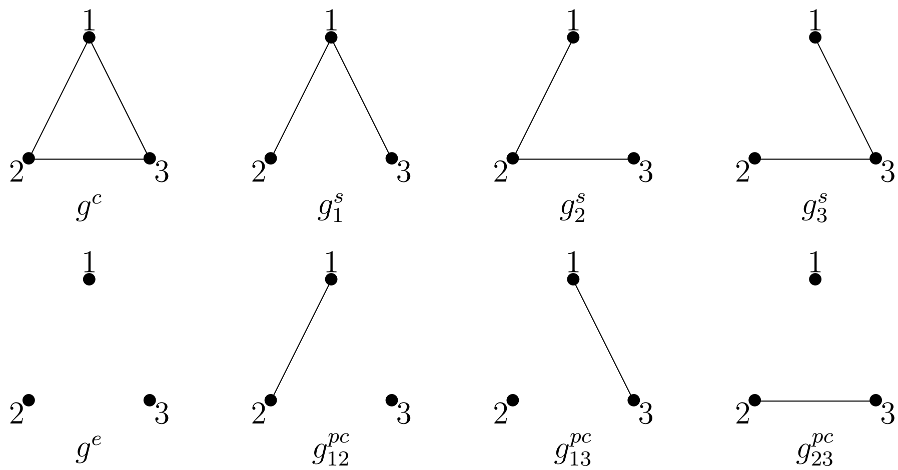


Figure 1: Network structures with three firms

the substitutability between any given pair of products. If $\tau$ is small, then products are scarcely horizontally differentiated. Besides, we assume that $r$ is always large enough for the whole market to be covered, even if $\theta_i$ is very small.
The general formulation of the indifferent consumer between firm $i$ and $j$ is:

$$z_{ij} = \frac{\theta_i - \theta_j}{2\tau (I_j - I_i)} + \frac{I_j + I_i}{2}, \tag{2}$$

which allows us to define the demand functions for firms 1, 2 and 3 as $D_1 = z_{12}$, $D_2 = z_{23} - z_{12}$ and $D_3 = 1 - z_{23}$.

According to Deroian and Gannon (2006), we assume a quadratic cost function of quality effort $e_i$ given by $\gamma e_i^2/2$, $\gamma > 0$, so that we can write the second stage profit function of firm $i$ as

$$\pi_i = pD_i - \gamma \frac{e_i^2}{2}, \quad i \in N. \tag{3}$$

Firstly, by backward induction we identify the second stage Nash equilibrium strategies profile of the game $E = \{e_i^*\}_{i=1}^3$ and thus the set of firms' equilibrium profits $\{\pi_i^*\}_{i=1}^3$. Then we will extend the analysis to the issue of network stability according to the following definition from Jackson and Wolinsky (1996):

**Definition** - *Let us define $\pi_i^*(g)$ the maximum profits obtained by firm $i$ when* *network $g$ is formed. A network $g$ is pair-wise stable if and only if for any pair of firms $i, j$:*

1. *if* $s_{ij} = 1$ *then* $\pi_i^*(g) \geq \pi_i^*(g - s_{ij})$ *and* $\pi_j^*(g) \geq \pi_j^*(g - s_{ij})$;
2. *if* $s_{ij} = 0$ *and* $\pi_i^*(g + s_{ij}) > \pi_i^*(g)$ *then* $\pi_j^*(g + s_{ij}) < \pi_j^*(g)$. $\square$

The network $g + s_{ij}$ is obtained by replacing $s_{ij} = 0$ in network $g$ by $s_{ij} = 1$. By the same token, the network $g - s_{ij}$ is obtained by severing an existing link between firms $i$ and $j$.

In other words, a network $g$ is pair-wise stable if no pair of firms has an incentive to form a new link and no firm has an incentive to unilaterally severe an existing link.[2]

It is worth noticing that the Jackson and Wolinsky definition may be equivalently replaced by a condition on firm profits expressed in differential terms. Specifically, if $\frac{\partial \pi_i^*}{\partial s_{ij}} \geq 0$ and $\frac{\partial \pi_j^*}{\partial s_{ij}} \geq 0$ jointly hold for every $s_{ij} \in \{0, 1\}$, then firm $i$ and $j$ will form a quality-improving agreement. To be sure, this definition requires both that $s_{ij}$ is a continuous variable and that the profit function is monotone with respect to $s_{ij}$ for any network structure. This is the case for firms 1 and 3 but not for firm 2. This is why for the case related to firm 2 we will use the original Jackson and Wolinsky definition.

Also we will study the social welfare implications of quality-improving alliances. As usual, we express the social welfare related to a network $g$, as the sum of maximized firms' profits and consumers surplus as follows:

$$W(g) = \sum_{i=1}^{3} \left( \pi_i^* + CS_i \right), \tag{4}$$

where the aggregate consumers surplus is:

$$\sum_{i=1}^{3} CS_i = \int_0^{z_{12}} u_1(z)dz + \int_{z_{12}}^{z_{23}} u_2(z)dz + \int_{z_{23}}^{1} u_3(z)dz. \tag{5}$$

# 3 Results

In this section we draw the main results. Specifically, we analyse both the optimal quality effort and the network stability in Section 3.1. The relationship between

[2] As Goyal and Moraga Gonzàlez (2001) point out, this definition of stability is quite weak and should be seen as a necessary condition for strategic stability.

network formation and social welfare is studied in Section 3.2.

Deriving firm $i$'s profit function (3) with respect to $e_i$ we obtain the following optimal $R\&D$ efforts:

$$e_1^* = \frac{1 - ls_{12}}{2\gamma\tau(I_2 - I_1)}p, \tag{6}$$

$$e_2^* = \left[\frac{1 - ls_{23}}{2\gamma\tau(I_3 - I_2)} + \frac{1 - ls_{12}}{2\gamma\tau(I_2 - I_1)}\right]p, \tag{7}$$

$$e_3^* = \frac{1 - ls_{23}}{2\gamma\tau(I_3 - I_2)}p. \tag{8}$$

Let us observe that forming cooperation links induces firms to lower their own quality effort. The second order necessary condition is always satisfied for any firm $i \in N$ because $\frac{\partial^2 \pi}{\partial e_i^2} < 0$ for each $e_i$ and thus the strategies profile $E = (e_1^*, e_2^*, e_3^*)$ is the second stage Nash equilibrium of the game. Substituting optimal $R\&D$ efforts into (3) we obtain the equilibrium firms' profits that allow us to characterize the stable collaboration networks under exogenous prices.

**Proposition 1:** *In a three-firm Hotelling game with fixed prices and endogenous quality-improving R&D investment, the complete network $g^c$ is the unique pair-wise stable network.*

**Proof**:
To show that the unique pair-wise stable network is the complete one, we first prove the following three claims.

- ***Claim 1***: Firm 1 increases its profits if it is linked with firm 2 independently of the network structure. Indeed, differentiating $\pi_1^*$ with respect to $s_{12}$ we obtain

$$\frac{\partial \pi_1^*}{\partial s_{12}} = p\frac{1}{2\tau(I_2 - I_1)}\frac{\partial(\theta_1^* - \theta_2^*)}{\partial s_{12}} - \gamma e_1^* \frac{\partial e_1^*}{\partial s_{12}}, \tag{9}$$

where $\frac{\partial(\theta_1^* - \theta_2^*)}{\partial s_{12}} = le_3^* \geq 0$ and $\frac{\partial e_1}{\partial s_{12}} \leq 0$. This implies that $\pi_1^*$ is a monotone increasing function in $s_{12}$ for any $s_{13} \in \{0, 1\}$ and $s_{23} \in \{0, 1\}$.

- ***Claim 2***: Firm 3 increases its profits if it is linked with firm 2 independently of the network structure.
Differentiating $\pi_3^*$ with respect to $s_{23}$ we obtain

$$\frac{\partial \pi_3^*}{\partial s_{23}} = -p\frac{1}{2\tau\left(I_3 - I_2\right)}\frac{\partial(\theta_2^* - \theta_3^*)}{\partial s_{23}} - \gamma e_3^* \frac{\partial e_3^*}{\partial s_{23}}, \tag{10}$$

where $\frac{\partial(\theta_2^* - \theta_3^*)}{\partial s_{23}} = -le_1^* < 0$ and $\frac{\partial e_3}{\partial s_{23}} < 0$. This implies that $\pi_3^*$ is a monotone increasing function in $s_{23}$ for any $s_{12} \in \{0, 1\}$ and $s_{13} \in \{0, 1\}$; in other words, firm 3 increases its profits if it is linked with firm 2 independently of the network structure.

- ***Claim 3***: Firms 1 and 3 are always linked.
Let us observe that $\pi_1^*$ and $\pi_3^*$ are both monotonically increasing functions in $s_{13}$ for any $s_{23} \in \{0, 1\}$ and $s_{12} \in \{0, 1\}$, i.e.

$$\frac{\partial \pi_1^*}{\partial s_{13}} = \frac{lp^2\left(ls_{23} - 1\right)}{4\gamma\left(I_2 - I_1\right)\left(I_2 - I_3\right)\tau^2} \geq 0, \tag{11}$$

$$\frac{\partial \pi_3^*}{\partial s_{13}} = \frac{lp^2\left(ls_{12} - 1\right)}{4\gamma\left(I_1 - I_2\right)\left(I_3 - I_2\right)\tau^2} \geq 0. \tag{12}$$

Conditions (11) and (12) hold because $(I_2 - I_3) \leq 0$, $(ls_{23} - 1) \leq 0$, $(I_1 - I_2) \leq 0$ and $(ls_{12} - 1) \leq 0$. In other words, for every network $g$ where firms 1 and 3 are not linked, we have $\pi_1^*(g) \leq \pi_1^*(g + s_{13})$ $\pi_3^*(g) \leq \pi_3^*(g + s_{13})$ with $s_{13} = 1$.

According to the Claim 3, it follows that all networks with $s_{13} = 0$, namely $g^e, g_2^s, g_{12}^{pc}, g_{13}^{pc}$, are not pair-wise stable.
Consequently, there are only four networks candidates to be pairwise stable: the complete network $g^c$, the partially connected network $g_{13}^{pc}$ and two star networks, $g_3^s$ and $g_1^s$. In what follows we will scrutinise these cases.

**Partially connected network $g_{13}^{pc}$:** we show that the partially connected network $g_{13}^{pc}$ is not pair-wise stable because both firms 1 and 2 have the incentive to form a link, i.e. $s_{12} = 1$. From claim 1 it follows that firm 1 wants to cooperate with firm 2 because $\pi_1^*\left(g_1^s\right) \geq \pi_1^*\left(g_{13}^{pc}\right)$ independently of the network structure.
Thus, we still need to verify if the optimal strategy of firm 2 is to cooperate with firm 1 or not. Given that $\pi_2^*$ is not a monotonic function of $s_{12}$ we have to directly compare $\pi_2^*(g_{13}^{pc})$ with $\pi_2^*(g_1^s)$:

$$\pi_2^*(g_{13}^{pc}) = p\left[\frac{2p(1-l)}{4\tau^2\gamma(I_3 - I_2)(I_2 - I_1)} + \frac{I_3 - I_1}{2}\right] - \frac{\gamma p^2}{2}\left[\frac{1}{2\gamma\tau(I_3 - I_2)} + \frac{1}{2\gamma\tau(I_2 - I_1)}\right]^2 \tag{13}$$

$$\pi_2^*(g_1^s) = p\left[\frac{p(2-3l)}{4\tau^2\gamma(I_3-I_2)(I_2-I_1)} + \frac{I_3-I_1}{2}\right] - \frac{\gamma p^2}{2}\left[\frac{1}{2\gamma\tau(I_3-I_2)} + \frac{1-l}{2\gamma\tau(I_2-I_1)}\right]^2. \tag{14}$$

Comparing (13) with (14) it is possible to show that $\pi_2^*(g_1^s) > \pi_2^*(g_{13}^{pc})$ for any admissible set of parameters $(\gamma, p, l, \tau, I_1, I_2, I_3)$.[3]

Therefore, $g_{13}^{pc}$ is not pair-wise stable because firms 2 and 1 will form a link. Following the same procedure it is possible to show that $\pi_2^*(g_3^s) > \pi_2^*(g_{13}^{pc})$ for any admissible set of parameters $(\gamma, p, l, \tau, I_1, I_2, I_3)$, confirming that also firm 3 will form a link with firm 2 so as to render the network $g_{13}^{pc}$ unstable.[4]

**Star networks $g_1^s$ and $g_3^s$:** In this case star networks are not pair-wise stable since $\pi_2^*(g^c) > \pi_2(g_1^s)$ and $\pi_2^*(g^c) > \pi_2(g_3^s)$ for any admissible set of parameters $(\gamma, p, l, \tau, I_1, I_2, I_3)$.
The intuition behind this result is that in absence of price competition, firms can increase their demand only by increasing their own quality effort. However, quality is costly, especially if firms are scarcely horizontally differentiated. Then, link creation allows firms to relax quality competition and increase profits. This effect is noticeable for the internal firm, which directly competes with the two corner firms.

**Complete network $g^c$:** we show that the complete network is pair-wise stable because firm 2 has no incentive to sever its links with firms 1 and 3. From Claims 1 and 2 we know that firms 1 and 3 have the incentive to remain linked with firm 2, independently of the network structure. Thus, it suffices to check whether firm 2 increases its profits maintaining the cooperative links with both firms 1 and 3. In this respect, let us consider profits of firm 2 in the case of complete network:

$$\pi_2^*(g^c) = p\left[\frac{p2(1-l)^2}{4\tau^2\gamma(I_3-I_2)(I_2-I_1)} + \frac{I_3-I_1}{2}\right] - \frac{\gamma p^2}{2}\left[\frac{1-l}{2\gamma\tau(I_3-I_2)} + \frac{1-l}{2\gamma\tau(I_2-I_1)}\right]^2, \tag{15}$$

and compare it with profit $\pi_2(g_1^s)$, reported in expression (14), and with the profit obtained in the case of star network $g_3^s$:

$$\pi_2(g_3^s) = p\left[\frac{p(2-3l)}{4\tau^2\gamma(I_3-I_2)(I_2-I_1)} + \frac{I_3-I_1}{2}\right] - \frac{\gamma p^2}{2}\left[\frac{1-l}{2\gamma\tau(I_3-I_2)} + \frac{1}{2\gamma\tau(I_2-I_1)}\right]^2. \tag{16}$$

[3] Comparing $\pi_2^*(g_1^s)$ with $\pi_2^*(g_{13}^{pc})$ we obtain that $\pi_2^*(g_1^s) > \pi_2^*(g_{13}^{pc})$ if $l < 2$ which always holds because $l \in [0, 1]$.

[4] We have that $\pi_2^*(g_3^s) > \pi_2^*(g_{13}^{pc})$ if $l > 0$ which always holds because $l \in [0, 1]$.

As before, it is easy to show that $\pi_2^*(g^c) > \pi_2(g_1^s)$ and $\pi_2^*(g^c) > \pi_2(g_3^s)$ for any admissible set of parameters $(\gamma, p, l, \tau, I_1, I_2, I_3)$, confirming that the complete network is pair-wise stable.[5] ■

It is worth noticing that, in the presence of fixed price, link formation is not dependent on the location of the firms. Put it different, the exogeneity of prices forces firms to lower competition by forming dense networks of collaboration. In this way firms are able to reduce their quality costs rather than differentiating their goods. This result departs from Correani and Di Dio (2017).
An interesting case may come up when the quality difference between two firms is very large. As an example, this is the case in which firms 2 and 1 are not horizontally differentiated, i.e. $I_2 \approx I_1$, such that $z_{12} < 0$. In this case firm 1 does not invest in quality, $e_1 = 0$ and the market becomes a duopoly formed by firm 2 and 3. These firms will have the same quality level $e_2^* = e_3^* = \frac{1-ls_{23}}{2\gamma\tau(I_3-I_2)}p$ with profits $\pi_2^* = p\frac{I_3+I_2}{2} - \frac{\gamma}{2}\left(e_2^*\right)^2$ and $\pi_3^* = p\left(1 - \frac{I_3+I_2}{2}\right) - \frac{\gamma}{2}\left(e_3^*\right)^2$. Since those profit functions are increasing in $s_{23}$, we conclude that firms 2 and 3 will form a stable cooperative link independently of their own location.

## 3.1 Welfare analysis

In this section we investigate the relationship between the network structure and social welfare in order to check if the unique pair-wise stable network, namely the complete one, is also welfare maximising. In order to assess the effects of network formation on the social welfare $W$ we carry out some numerical simulations where parameters are set to capture different degrees of horizontal differentiation and spillovers. However, in order to seek out how the setting of these parameters may affect the results we further perform a sensitivity analysis.

For simplicity, we set parameters such that $u_i > 0$ (all consumers purchase) and $\pi_i > 0$ (all firms produce) hold for any set of firms' locations $\{I_i\}_{i=1}^3$ with $I_1 < I_2 < I_3$, i.e $r = 12$, $p = 1$, $\tau = 10$, $c = 0$ and $\gamma = 15$. Then, we study the effect of network formation following two symmetric cases to simplify the analysis. In the first case we assume $I_3 - I_2 = I_2 - I_1 = \Delta_I$ and $I_2 = 0.5$, with $\Delta_I$ ranging from 0 (agglomeration) to 0.5 (maximum horizontal differentiation). In the second case $I_1 = 0$ and $I_3 = 1$ with firm 2's location $I_2$ moving from 0.1 to 0.9.

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[5] We obtain that $\pi_2^*(g^c) > \pi_2(g_1^s)$ and $\pi_2^*(g^c) > \pi_2(g_3^s)$ if, respectively, $l\,(2I_3 - 3I_2 + I_1) > -2(I_2 - I_1)$ and $l\,(3I_2 - 2I_1 - I_3) > -2(I_3 - I_2)$ which are always verified for any set of firm locations $\{I_i\}_{i=1}^3$ and $l \in [0, 1]$.

For each of the above cases we study the network architectures which maximise social welfare.

| $\Delta_I$ | $l = 0.1$ | $l = 0.5$ | $l = 0.9$ | $l = 1$ |
|---|---|---|---|---|
| 0.05 | $g^c$ | $g^c$ | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ |
| 0.1 | $g^c$ | $g^c$ | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ |
| 0.2 | $g^c$ | $g^c \wedge g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ |
| 0.3 | $g^c$ | $g_3^s \wedge g_1^s$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ |
| 0.4 | $g^c$ | $g_3^s \wedge g_1^s$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ |
| 0.5 | $g^c$ | $g_{13}^{pc} \wedge g_3^s \wedge g_1^s$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ |

Table 1: Welfare maximising network structures for different levels of spillovers and horizontal differentiation. $g_3^s \wedge g_1^s$ means that both networks $g_3^s$ and $g_1^s$ maximise social welfare.

| $I_2$ | $l = 0.1$ | $l = 0.5$ | $l = 0.9$ | $l = 1$ |
|---|---|---|---|---|
| 0.1 | $g^c$ | $g^c$ | $g_3^s$ | $g_3^s$ |
| 0.3 | $g^c$ | $g^c$ | $g_3^s$ | $g_3^s$ |
| 0.5 | $g^c$ | $g_{13}^{pc} \wedge g_3^s$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ |
| 0.7 | $g^c$ | $g_1^s$ | $g_1^s$ | $g_1^s$ |
| 0.9 | $g^c$ | $g^c$ | $g_1^s$ | $g_1^s$ |

Table 2: Welfare maximising network structures for different levels of spillovers and central firm's location.

In Tables 1 and 2 we report network structures maximising social welfare, according to different levels of spillover rate and horizontal differentiation. From them it is possible to draw the following result:

**Result 1:** *Social welfare is maximised by the complete network if the rate of spillover* ($l$) *is sufficiently low.*

The intuition behind this result can be related to the effect of spillover rate on quality investment. When $l$ is high, we observe a reduction of quality investments $\{e_i^*\}_{i=1}^3$ (see expressions (6), (7) and (8)) driving down the level of consumer surplus. This negative effect can be mitigated by reducing the number of connections among firms.

On the other hand, a high spillover rate has a stronger negative effect on the quality investments $\{e_i^*\}_{i=1}^3$, and thus on welfare, as the distance between two adjacent firms decreases (low horizontal differentiation). As a consequence, severing a cooperation link between two undifferentiated firms is welfare improving if $l$ is high. This intuition is summarized in the following result:

**Result 2:** *If the spillover rate is sufficiently high and firms are less horizontally differentiated (i.e.* $\Delta_I \to 0$; $I_2 \to I_1$ *or* $I_2 \to I_3$*) then social welfare is maximized by denser networks.*

## 3.2 Sensitivity analysis

We now carry out a series of checks (sensitivity analysis) to assess the robustness of the previous results against changes in the values of some key parameters, namely the transportation costs $\tau$, the marginal cost of quality $\gamma$ and the exogenous price $p$. Results are summarised in Tables 3 and 4.

Table 3 shows how the solution might change for different degrees of spillovers and horizontal differentiation relative to the benchmark case. Also, it shows how the solution may change for different values of $\tau$,$\gamma$ and $p$, taking as given both the level of spillovers and horizontal differentiation. In Table 4 the same analysis is carried out for different degrees of spillovers and central firm location. Also in this case, it shows how the solution may change for different values of $\tau$,$\gamma$ and $p$, taking as given both the level of spillovers and firm location.

We can notice that for higher $\tau$, welfare is maximised by less dense networks.

| $\Delta_I$ | $l$ | Benchmark | $\tau = 5$ | $\tau = 20$ | $\gamma = 8$ | $\gamma = 30$ | $p = 0.5$ | $p = 2$ |
|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ |
| 0.1 | 0.9 | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ | $g_3^s \wedge g_1^s$ |
| 0.3 | 0.1 | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ |
| 0.3 | 0.9 | $g_{13}^{pc}$ | $g_3^s \wedge g_1^s$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ | $g_3^s \wedge g_1^s$ |
| 0.5 | 0.1 | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ |
| 0.5 | 0.9 | $g_{13}^{pc}$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ | $g_{13}^{pc}$ |

Table 3: Sensitivity - Welfare maximising network structures for different levels of spillovers and horizontal differentiation.

| $I_2$ | $l$ | Benchmark | $\tau = 5$ | $\tau = 20$ | $\gamma = 8$ | $\gamma = 30$ | $p = 0.5$ | $p = 2$ |
|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ |
| 0.1 | 0.9 | $g_3^s$ | $g_3^s$ | $g_3^s$ | $g_3^s$ | $g_3^s$ | $g_3^s$ | $g_3^s$ |
| 0.9 | 0.1 | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ | $g^c$ |
| 0.9 | 0.9 | $g_1^s$ | $g_1^s$ | $g_1^s$ | $g_1^s$ | $g_1^s$ | $g_1^s$ | $g_1^s$ |

Table 4: Sensitivity - Welfare maximising network structures for different levels of spillovers and central firm location.

This is due to the relationship between the quality effort $e_i$ and the transportation cost $\tau$. Indeed, a decreasing $\tau$ reduces product differentiation, inducing more quality competition among firms. This, in turn, yields a reduction of both firm profits and social welfare because of the higher cost, which can be mitigated by forming more links. This is why in Tables 3 and 4 social welfare is maximised by more (less) dense networks for lower (higher) values of $\tau$. The same conclusions apply to price. A lower price implies less dense welfare maximising networks relative to the benchmark. Indeed a decreasing price reduces quality effort and profits; thus, the only way to keep higher levels of welfare is to reduce the number of links.

Our sensitivity analysis thus confirm the robustness of the results, namely that the complete network maximises social welfare when the spillover rate is sufficiently small (Result 1, see Table 3). By the same token, denser networks are welfare maximising if the spillover rate is high and firm are scarcely differentiated (Result 2, see Table 4). We thus conclude that for high quality spillovers it might arise a conflict between stability and welfare; in other words, when firms are horizontally differentiated and the spillover rate is sufficiently high, networks of alliances could be over-connected as compared to the social optimum.

# 4 Concluding remarks

This paper provides insights concerning the relationship between quality-improving alliances and product differentiation in a Hotelling game where prices are set by a regulator. A collaboration link is interpreted as a technological partnership which helps increase product quality of firms. We find the following results. First, firm-quality effort decreases with the number of links. Second, we find that the complete network is uniquely stable but efficient only for a sufficiently low level of spillovers rate (over-connection issue). Third, when the rate of spillovers is high, social welfare is maximised by the complete network if firms are not differentiated. Nonetheless, the model is quite stylised in a number of respects and therefore it could be fruitfully enriched along various directions. A natural extension would be to enlarge the set of strategic variables including location choice or assuming endogenous spillover rate (absorptive capacity). Along similar lines, one further option would be to shift the analysis from quality improving to cost-reducing alliances.

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