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The optimal tax mix with underground labor

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Abstract

The optimal mix between capital, labor and consumption taxes is derived in a model with underground labor. The Ramsey planner, which is limited by the trade-off between declared and underground labor, sets the tax rate on labor income equal to zero in order to get rid off the issue of tax evasion; so, in contrast to Coleman (2000), subsidizing labor is not optimal. This paper also points out that adding consumption taxation to the model of Correia (1992) makes the Chamley-Judd result of a zero capital tax in the long run hold even when there are restrictions on the taxation of labor; in fact, the optimal tax rate on capital income is always zero. Since consumption taxes are positive and constant for each period, I provide an alternative argument to shift the whole tax burden from income to consumption.

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1 Introduction

I construct a model in which under perfect tax enforcement, the Chamley-Judd and Coleman (2000) results hold and therefore the first-best is attainable. I then use this model to assess the optimal fiscal policy when agents can work in the underground economy.

The model used in this paper is an extension of Correia (1996) allowing government to tax consumption. In her model, there exists an untaxed factor of production; specifically, I interpret it as underground labor.

The general point of this paper is to study the role played by underground labor in the extreme and well-known results in the literature of optimal fiscal policy.

First, in contrast to the result of Coleman (2000), subsidizing labor is not optimal. I prove that the optimal labor tax is zero for $t > 0$. Therefore, there is no need of constraining labor taxes to be non-negative, as is done in Coleman (2000), Laczó and Rossi (2014), or Correia (2010); in order to get realistic tax rates.

Second, I show that the Chamley-Judd result does hold in a model with an untaxed factor of production but in which taxing consumption is endogenous. Therefore, contrary to Correia (1996), the policy-maker can shift the whole tax burden from capital either to consumption or labor, as it was the case in most previous papers.

Third, in contrast to Chari et al. (1994), there is no spike in the capital tax rate in period 1.[1] In fact, this tax rate is zero, so welfare gains of implementing the optimal tax mix do not rely on an extreme transition of capital tax rates in period 1.

Fourth, the result of a uniform and equal to zero capital tax is the same as in Chari et al. (2018). For preferences that are standard in the literature of macroeconomics, it is optimal to never distort capital accumulation when the Ramsey planner has a rich system of taxation that optimally implies no intertemporal wedges ever. This feature happens when I add the possibility of taxing consumption to the model of Correia (1996) or Chari et al. (1994) in which only income taxes are available.

In summary, the optimal tax mix is to impose a positive and constant consumption tax and no tax on capital and labor income. The Ramsey planner should set zero labor taxes in order to eliminate the distortion generated by underground labor. Also, government, in spite of there exists an untaxed factor, shifts the tax burden away from capital income because taxing consumption is possible and more efficient.

The outline of the paper is the following. Section 2 introduces the model used.

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[1]As it is common in the literature, the initial capital tax, τ_0^k, is given. Even though τ_1^k is a distortionary tax, it is a way of taxing initial capital, k_0, in the end.

In Section 3, I describe the optimal fiscal policy problem faced by the government. Optimal taxes are presented in Section 4. Finally, Section 5 concludes.

2 The model

There is continuum of identical agents that live indefinitely, which chooses how much to consume, work, and invest. The difference with respect to standard models is that they are able to work also in the underground economy, where labor taxes are not enforced.

Let us denote w_t^i and n_t^i, $i \in \{M, U\}$, the wages and hours worked in the market ($i = M$) and underground ($i = U$) labor markets. Agents can save in capital k_{t+1}, and in government bonds b_{t+1}. Therefore, the problem that agents solve is the following:

$$\max_{\{c_t, n_t^M, n_t^U, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t^M - n_t^U)$$

subject to,

$$\begin{aligned} (1+\tau_t^c)c_t + i_t + b_{t+1} &= (1-\tau_t^n) w_t^M n_t^M \\ &+ w_t^U n_t^U + (1-\tau_t^k) r_t k_t + (1+r_t^b) b_t \\ i_t &= k_{t+1} - (1-\delta)k_t \\ c_t, n_t^M, n_t^U, k_{t+1}, b_{t+1} &\geq 0. \end{aligned} \tag{1}$$

The utility function is CRRA and separable between consumption and leisure. The equilibrium conditions for agents are given by

$$\frac{U_{1,t}}{(1+\tau_t^c)} = \beta \left(1 + (1-\tau_{t+1}^k) r_{t+1} - \delta\right) \frac{U_{1,t+1}}{\left(1+\tau_{t+1}^c\right)}, \tag{2}$$

$$r_t^b = (1-\tau_t^k) r_t - \delta, \tag{3}$$

$$\frac{U_{2,t}}{U_{1,t}} = \frac{(1-\tau_t^n) w_t^M}{(1+\tau_t^c)}, \tag{4}$$

$$(1-\tau_t^n) w_t^M = w_t^U, \tag{5}$$

Following Correia (1996), Ihrig and Moe (2004), Busato and Chiarini (2004) and Orsi et al. (2014) models of underground economy; total output is $Y_t = F(k_t, n_t^M, n_t^U)$ and the first-order conditions of firms are

$$w_t^M = F_{n_t^M} \tag{6}$$

$$r_t = F_{k_t} \tag{7}$$

$$w_t^U = F_{n_t^U}. \tag{8}$$

Government raises revenues from given taxes τ_t^c, τ_t^n and τ_t^k and issue new debt b_{t+1} in order to balance budget

$$g_t - b_{t+1} = \tau_t^c c_t + \tau_t^n w_t^M n_t^M + \tau_t^k r_t k_t - (1 + r_t^b) b_t, \tag{9}$$

where government consumption, g_t, is exogenous.

The aggregate resource constraint reads

$$c_t + g_t + i_t = Y_t. \tag{10}$$

3 Ramsey equilibrium

Of course, the Chamley-Judd and Coleman (2000) results hold under perfect tax enforcement. Given that n_t^U is also taxed, there exists an optimal tax policy achieving the first-best. As in Coleman (2000), if we set $\tau_t^c = \bar{\tau}$, $\tau_t^n = -\tau_t^c$, and $\tau_t^k = 0$, $\forall t$, this tax policy eliminates the distortions affecting (2)-(5), which then become the marginal conditions that characterize the Pareto-optimal allocation for this economy.

In this section I lay out the Ramsey problem that government faces. Before going to the details of the Ramsey problem with underground labor, I show that Coleman (2000) result holds once underground consumption is also allowed; that is, underground consumption does not modify the tax shift from income to consumption and what drives this shift is that consumption tax can mimic the initial wealth expropriation or can act as lump-sum tax. Now, goods or services produced in the underground economy c^U are not taxed either. In this case, we should add

$$\frac{U_{2,t}}{U_{c^U,t}} = F_{n_t^U} \tag{11}$$

to the first-order conditions (2)-(4) and

$$c_t^U = Y_t^U \tag{12}$$

in addition to feasibility constraint (10), where Y_t^U is output produced in the underground economy. Coleman (2000) tax policy eliminates any distortion, and therefore the Pareto-optimal allocation is attainable. As in Coleman (2000), we must restrict the sign of labor taxes to be non-negative in order to get realistic values.

On the other hand, without underground consumption, I show that underground labor provides a new rationale for shifting taxes from income to consumption. The Ramsey problem amounts to find optimal taxes, $\{\tau_t^c, \tau_t^n\}_{t=0}^{\infty}$ and $\{\tau_t^k\}_{t=1}^{\infty}$, that maximize lifetime utility, subject to the private budget constraint (1), the optimality conditions (2)-(5) and the feasibility constraint (10). As usual, τ_0^k is given because it is a non-distortionary tax on given initial conditions.

I rewrite the problem in terms of quantities. The wage rates and the rental rate on capital are expressed in terms of quantities using the marginal product conditions (6)-(8). The tax rate on capital is expressed from the Euler equation (2). The tax rates on labor income and consumption can be obtained from equations (4)-(5) and read, respectively,

$$\tau_t^n = \frac{F_{n_t^M} - F_{n_t^U}}{F_{n_t^M}} \tag{13}$$

which depends on productivities of market and underground labor, and

$$\tau_t^c = \frac{U_{1,t} F_{n_t^U}}{U_{2,t}} - 1. \tag{14}$$

We have two taxes that should satisfy two intratemporal conditions. Therefore, in contrast to the model of Correia (1996) with no consumption taxation, we do not need to impose an extra constraint to the Ramsey problem.

These expressions of prices and tax rates are replaced into the agent's budget constraint (1), which is substituted forward in order to obtain the implementability constraint:

$$\sum_{t=0}^{\infty} \beta^t \left(c_t U_{1,t} - \left(n_t^M + n_t^U \right) U_{2,t} \right) = \frac{W_0 U_{2,0}}{F_{n_0^U}}, \tag{15}$$

where the term W_0 is the initial wealth, $\big(1 + (1 - \tau_0^k) F_{k_0} - \delta\big)(b_0 + k_0)$, which depends on the initial conditions. Let us use Λ to represent the Lagrange multiplier of condition (15). In this paper, (15) is a sufficient condition to guarantee the existence of a competitive equilibrium.

Therefore, the Ramsey problem can be formulated as

$$
\begin{aligned}
\max_{\{c_t, n_t^M, n_t^U, k_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \Big\{ U\left(c_t, 1 - n_t^M - n_t^U\right) \\
& + \Lambda\left(c_t U_{1,t} - \left(n_t^M + n_t^U\right) U_{2,t}\right) \\
& - \lambda_t \left(c_t + g_t + k_{t+1} - (1-\delta)k_t - F(k_t, n_t^U, n_t^M)\right) \Big\} \\
& - \Lambda \, \frac{W_0 U_{2,0}}{F_{n_0^U}},
\end{aligned} \tag{16}
$$

where $\Lambda > 0$ is the multiplier of the implementability constraint, and $\lambda_t > 0; \forall t$.

The equations that characterize the Ramsey equilibrium are the feasibility constraint (10), the implementability constraint (15), and the first-order conditions to (16).

I will use the notation $U_{ij,t}$ for the second derivatives of the utility with respect to argument j and the notation $F_{i_t j_t}$ for the second derivative of the production function with respect to variable j.

The first-order conditions of (16) with respect to c_t and k_{t+1}, $\forall t$, are

$$U_{1,t} + \Lambda\left(U_{1,t} + c_t U_{11,t}\right) = \lambda_t \ , \tag{17}$$

$$\lambda_t = \beta \lambda_{t+1}(1 + F_{k_{t+1}} - \delta). \tag{18}$$

The first-order conditions of (16) with respect to n_t^M and n_t^U, for $t > 0$, are:

$$U_{2,t} + \Lambda\left(U_{2,t} - \left(n_t^M + n_t^U\right) U_{22,t}\right) = \lambda_t F_{n_t^M}, \tag{19}$$

$$U_{2,t} + \Lambda\left(U_{2,t} - \left(n_t^M + n_t^U\right) U_{22,t}\right) = \lambda_t F_{n_t^U}. \tag{20}$$

For $t = 0$ the first-order conditions with respect to n_0^M and n_0^U are:

$$
\begin{gathered}
U_{2,0} + \Lambda\left(U_{2,0} - \left(n_0^M + n_0^U\right) U_{22,0}\right) \\
+\Lambda(k_0 + b_0) \frac{\left[-U_{22,0}(1+(1-\tau_0^k)F_{k_0}-\delta)+U_{2,0}(1-\tau_0^k)F_{k_0 n_0^M}\right] F_{n_0^U} - U_{2,0}(1+(1-\tau_0^k)F_{k_0}-\delta)F_{n_0^U n_0^M}}{F_{n_0^U}^2} = \lambda_0 F_{n_0^M},
\end{gathered} \tag{21}
$$

$$
\begin{gathered}
U_{2,0} + \Lambda\left(U_{2,0} - \left(n_0^M + n_0^U\right) U_{22,0}\right) \\
+\Lambda(k_0 + b_0) \frac{\left[-U_{22,0}(1+(1-\tau_0^k)F_{k_0}-\delta)+U_{2,0}(1-\tau_0^k)F_{k_0 n_0^U}\right] F_{n_0^U} - U_{2,0}(1+(1-\tau_0^k)F_{k_0}-\delta)F_{n_0^U n_0^U}}{F_{n_0^U}^2} = \lambda_0 F_{n_0^U}.
\end{gathered} \tag{22}
$$

4 Optimal taxes

In this section I show how the optimal tax mix looks in a model with underground labor. Specifically, Propositions 1-3 describe the properties of the dynamic optimal tax rates.

Proposition 1. In the presence of underground labor, the optimal tax rate on labor income is set to zero for $t > 0$.

Proof. The MRS of the Ramsey equilibrium between declared and underground labors, which is the combination of (19)-(20), reads $F_{n_t^M} = F_{n_t^U}$, $t > 0$. Given this equality, we can verify that $\tau_t^n = 0$, $t > 0$, from the expression of the labor tax (13).□

Therefore, in contrast to Coleman (2000), it is not optimal to subsidize labor for $t > 0$, which implies that we do not need to constrain the Ramsey problem in order to avoid negative and unrealistic labor income tax rates. However, as in Coleman (2000) when τ_t^n is restricted to be non-negative,[2] we find the same result (no tax on labor) but for another reason. Government does not tax labor income in order to eliminate the distortion giving incentives to participate in the underground economy. With this policy, underground labor become as attractive as declared labor.

In the initial period, the value of τ_0^n depends on the value of $F_{k_0 n_0^i}$ and $F_{n_0^U n_0^i}$ because n_0^i and n_t^i (for $t \geq 1$), $i \in \{M, U\}$, do not enter symmetrically in the implementability constraint.

It has been pointed out by the literature that in this kind of fiscal policy problems, the optimal policy is to tax capital heavily in the first periods, and then decrease this tax rate to zero. It is worth making some comments about the optimal steady-state tax rate on capital, see Proposition 2.

Proposition 2. In the presence of underground labor, the optimal tax rate on capital income is equal to zero for all t.

Proof. First, it is easy to show that

$$\frac{U_{1,t+1} + \Lambda\left(U_{1,t+1} + c_{t+1}U_{11,t+1}\right)}{U_{1,t} + \Lambda\left(U_{1,t} + c_t U_{11,t}\right)} = \frac{U_{1,t+1}}{U_{1,t}}, \forall t, \tag{23}$$

so that substituting (17) into (18), the latter reads

$$U_{1,t} = \beta(1 + F_{k_{t+1}} - \delta)U_{1,t+1}, \forall t. \tag{24}$$

[2]See also Correia (2010) and Laczó and Rossi (2014).

Then, by comparing the Euler equation of the competitive equilibrium (2) with (24), we can conclude that $\tau_t^k = 0$, $t \geq 1$, and $\tau_t^c = \tau_{t+1}^c$, $t \geq 0$.□

Equation (24) coincides with the Pareto-optimum Euler equation, so it implies what Chari et al. (2018) call *no intertemporal distortions ever*; namely, the Ramsey solution has no intertemporal wedges for all t. Therefore, the reason why capital taxation should never be taxed is the same as in Chari et al. (2018); specifically, consumption taxation enrich the tax systems of Correia (1996) and Chari et al. (1994). Since we use the same standard preferences, this new instrument optimally eliminates any intertemporal wedge or distortion, even at $t = 1$,[3] so optimal capital tax rates should be equal to zero for all t.

Hence, in contrast to Correia (1996), the possibility of taxing consumption makes the Chamley-Judd result hold even when there exist restrictions on the taxation of production factors.

Since c_0 and c_t (for $t \geq 1$) enter symmetrically in the implementability constraint, $\tau_1^k = 0$. So, contrary to Chari et al. (1994), it is not optimal to heavily tax capital income in period 1. This is important because implies that the welfare gain of implementing the optimal tax mix does not rely on an extreme transition of tax rates in period 1 with respect to the actual values of the *status quo*.

Finally, Proposition 3 describes the optimal dynamic tax rate on consumption.

Proposition 3. In the presence of underground labor, the optimal tax rate on consumption is positive and constant.

Proof. Proposition 1 and 2 implies that the government budget is

$$g_t + r_t^b b_t = \tau_t^c c_t + (b_{t+1} - b_t)\,,\ t > 0, \tag{25}$$

so $\tau_t^c > 0$, otherwise debt would be explosive, i.e., $(b_{t+1} - b_t) > 0$; and therefore the implementability constraint would not be satisfied. From Proposition 2 we know that $\tau_t^c = \tau_{t+1}^c$, $t \geq 0$. Hence, we can conclude that the consumption tax rate is positive and constant for $t \geq 0$.□

According to Propositions 1-3, it is optimal to tax only consumption. This result is the same as the one obtained in several models without underground economy in which labor, capital, and consumption taxes are chosen optimally, e.g., Coleman (2000), Correia (2010), and Laczó and Rossi (2014). However, I provide a different argument to show that income should not be taxed.

[3] Contrary to our case, in the model of Chari et al.(2018) capital is taxed in period 1 because τ_0^c appears in the implementability constraint.

5 Conclusions

In this paper, we have studied the design of the optimal tax mix in the presence of an underground sector where labor tax is not enforced.

I have shown that it is optimal to cut taxes susceptible to be evaded, in this case labor taxes. I formally prove that the optimal labor tax is zero for $t > 0$. Therefore, subsidizing labor is not optimal and there is no need of constraining labor taxes to be non-negative, as is done in Coleman (2000), Laczó and Rossi (2014), or Correia (2010); in order to get realistic tax rates.

Once optimal policy addresses the issue of underground labor, this paper points out that the possibility of taxing consumption makes the Chamley-Judd result of a zero capital tax in the long run hold even when there are restrictions on the taxation of labor; in fact, the optimal tax rate on capital income is always zero. Hence, I show that the findings of Chari et al. (2018) hold in the standard model with underground labor.

Given that consumption taxes are positive for each period, I provide an alternative argument to shift the whole tax burden from income to consumption.

All these findings show that it is very important that government takes underground economy into account when designing the optimal tax system. The take-home message is that government should reduce the tax burden on labor so as to discourage the participation in the underground economy.

Finally, this paper computes the optimal tax rates given a level of tax enforcement, which in reality is a combination of monitoring and surcharges. Hence, an interesting point for future research would be the study of which is the optimal level of enforcement when government has to spend resources in order to increase the probability of detection. Specifically, we could analyze the trade-off between tax and enforcement rates.

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