Economics Bulletin

Volume 39, Issue 1

Pareto criterion and long-term perspective criterion under myopic discounting

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Abstract

Under myopic discounting, which encompasses the famous psychological and economics model, hyperbolic discounting preferences, this paper shows that efficiency by the Pareto criterion is a sufficient condition for efficiency by the long-term perspective criterion. This result provides additional justification for Pareto-improving policies under myopic discounting by showing that such policies improve normative preferences as well. As an application of our result, we provide an example of Laibson's Pareto-improving consumption tax.

The authors are grateful to the two anonymous referees and associate editor Achille Basile for their helpful comments. Kang acknowledges the research support from Nanyang Technological University (NTU) AcRF Tier-1 Grant (RG62/18).
Citation: Minwook Kang and Lei Wang, (2019) "Pareto criterion and long-term perspective criterion under myopic discounting", *Economics Bulletin*, Volume 39, Issue 1, pages 24-32
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Submitted: October 03, 2018. **Published:** January 10, 2019.

1. Introduction

Experimental and empirical evidences have shown that human preferences follow present-biased time-inconsistent discounting (Green and Myerson (2004); Frederick et al. (2002)). Economists model such preferences and apply various time-inconsistent preference models to a wide array of economic problems[1]. Modelling of present-biased time-inconsistent preferences engenders two famous criteria for comparing multi-period utility sets. The two criteria are Pareto-improving approach adopted by Laibson (1997) and Phelps and Pollak (1968), and long-term perspective criterion set by O'Donoghue and Rabin (1999).

Pareto criterion takes into account intertemporal utilities *at all periods*. On the other hand, long-term perspective criterion creates *a fictitious period 0* and focuses on the intertemporal utility at period 0. As O'Donoghue and Rabin (1999) suggested, the long-term perspective criterion is less restrictive and exhibits less present bias than Pareto criterion. Therefore, when government considers a policy, it should aim at higher normative utility, U_0, which is computed based on long-term perspective criterion (O'Donoghue and Rabin (1999)). Unfortunately, a higher normative utility does not guarantee a higher intertemporal utility at the current period. Consequently, policies that help to increase normative utility, but not intertemporal utility, may be objected by the public which aims to maximize its current intertemporal utility. In contrast, a Pareto-improving policy ensures higher intertemporal utility and hence, public consensus.

It is generally accepted that Pareto criterion is more restrictive than long-term perspective criterion (Camerer et al. (2004); O'Donoghue and Rabin (2003)). However, seldom do economists explore the mathematical connections between the two criteria[2]. In fact, contrary to common belief, Pareto criterion does not imply long-term perspective criterion in general. Take for an example, under exponential discounting, when one multi-period utility set, u, is Pareto-superior over another multi-period utility set, v, and the first period intertemporal utilities for u and v are equal, then individuals are indifferent between u and v based on long-term perspective criterion. A more extreme example would be forward-looking discounting preferences. Considering a three-period model, $(\beta_1, \beta_2, \beta_3) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{10})$, $u = (10, 12, 100)$ and $v = (17, 16, 60)$. u dominates v based on Pareto criterion, in fact the intertemporal utilities of u are higher than that of v for all three periods, but v dominates u based on long-term perspective criterion. This paper proves that under myopic discounting, which includes both quasi-hyperbolic and hyperbolic discounting, normative utility is a positive linear transformation of intertemporal utilities across all periods. Therefore, Pareto

[1] Some of the classic works on present-biased time inconsistency include Laibson's (1997) paper on undersaving and O'Donoghue and Rabin's (1999) work on self-control problems. Benabou and Tirole (2002) studied the impact of self-confidence on an individual with present-biased preference. Recent contributions touched on various economic problems. Bisin et al. (2015) showed that, banning illiquid assets can be welfare-improving; Chen, Li and Zeng (2018) modelled optimal dividend strategy of management with present bias; Li, Yan and Xiao (2014) and Yilmaz (2013) studied the contract design under quasi-hyperbolic discounting preferences.

[2] Kang (2015) proved that under quasi-hyperbolic discounting preferences, Pareto improvement implies the improvement of long-term preference. However, the proof cannot be extended to general myopic discounting preferences.

criterion implies long-term perspective criterion under myopic discounting and a Pareto-improving policy guarantees both public consensus and higher normative utility.

This paper contributes in two ways. Firstly, it proposes a mathematical definition of myopic discounting which encompasses both quasi-hyperbolic and hyperbolic discounting. The former is often used by economists for its tractability (O'Donoghue and Rabin (2015)) while some psychologists argue that the latter is more consistent with human behaviour (Ainslie (2002)). By defining myopia, this paper succeeds in circumventing the controversy between the two types of discounting methods. Moreover, while relevant economics study has been primarily limited to quasi-hyperbolic discounting (O'Donoghue and Rabin (2015)), this mathematical definition of myopia expands the economics research of present bias to a more general form of discounting. Secondly, this paper provides more justification for papers using Pareto-improving criterion. Numerous papers, for example, Laibson's (1996) work on saving policies, adopt Pareto criterion[3]. Our result shows that under myopic discounting preferences, conclusions of papers that use Pareto criterion automatically hold for long-term perspective criterion.

2. Myopic discounting

Generally, there are three types of discounting: myopia which includes both quasi-hyperbolic and hyperbolic discounting; exponential which imposes constant discount rate (Frederick et al. (2002)); and forward-looking under which current preference directly depends on preferences over all future periods (Galperti and Strulovici (2014)). Among the three, myopia is the most appealing to economists and psychologists due to its consistency with social preferences and human decisions (Sellitto et al. (2010); Kirby and Herrnsteirr (1995)). This paper focuses on myopic discounting and mathematically defines it.

When $T \geq 3$, an individual's intertemporal utility at period t (U_t) is represented by:

$$U_t(\{u_\tau\}_{\tau=t}^{T}) = \sum_{i=0}^{T-t} \beta_i u_{t+i} \tag{1}$$

where $\beta_0 = 1$ and $u_t \in \mathbb{R}$ is the utility at period t.

Definition 1. *An individual is said to have myopic discounting time-inconsistent preferences when his discount rate* $(1 - \beta_i)$ *is increasing (i.e. his discount factor* β_i *is decreasing),*

$$\beta_i = \prod_{j=1}^{i} \delta_j, \ \textit{where} \ \forall j \in \{1, 2, 3, \ldots, T-1, T\}, 0 < \delta_j < 1 \ ^4 \tag{2}$$

[3] More recent examples include Bhattacharya and Lakdawalla's (2004) Pareto-improving cigarrete consumption tax, Sebastian Kodritsch's (2013) thesis on conditions for Pareto optimality in finite-horizon problems without indifferences and Garth Heutel's (2011) Pareto-improving dual policy that corrects both externality and present bias.

[4] The introduction of δ helps to simplify the proof in section 4.

and his marginal discount rate $(1 - \delta_i)$ *is non-increasing (i.e. his marginal discount factor* δ_i *is non-decreasing).*

$$\forall j \in \{1, 2, 3, \ldots, T-2, T-1\}, \delta_j \leq \delta_{j+1} \tag{3}$$

Moreover, due to the nature of present-bias, the marginal discount rate of period 1 is strictly greater than that of period 2. Or equivalently, the marginal discount factor at period 1, δ_1*, is strictly smaller than the marginal discount factor at period 2,* δ_2*.*

$$\delta_1 < \delta_2 \tag{4}$$

The definition encompasses both quasi-hyperbolic and hyperbolic discounting. Quasi-hyperbolic discounting takes the $\bar{\beta} - \bar{\delta}$ form, the discount rate at period t is $1 - \bar{\beta}\bar{\delta}^t$. Its discount rate is strictly increasing ($1 - \bar{\beta}\bar{\delta}^t < 1 - \bar{\beta}\bar{\delta}^{t+1}$) and its marginal discount rate beyond period 1 is constant at $1 - \bar{\delta}$. Its marginal discount rate of period 1 is $1 - \bar{\beta}\bar{\delta}$, which is bigger than that of period 2, $1 - \bar{\delta}$. Hyperbolic discounting takes form of $1/(1 + \bar{\delta}t)$. Its discount rate is strictly increasing as $1 - 1/(1 + \bar{\delta}t)$ is a monotonically increasing sequence. Its marginal discount rate is

$$1 - \frac{1/(1 + \bar{\delta}(t+1))}{1/(1 + \bar{\delta}t)} = 1 - \frac{1 + \bar{\delta}t}{1 + \bar{\delta}(t+1)} = \frac{\bar{\delta}}{1 + \bar{\delta}(t+1)}. \tag{5}$$

The marginal discount rate decreases with t. Hence, marginal discount rate of period 1 is strictly greater than that of period 2.

3. Research question

Given myopic discounting preferences, this paper aims to prove that Pareto criterion implies long-term perspective criterion. Equivalently, the objective of this paper is to prove that given two sets of multi-period utilities $u = \{u_\tau\}_{\tau=t}^{T}$ and $v = \{v_\tau\}_{\tau=t}^{T}$ and that u is preferred to v in terms of Pareto criterion, then u is preferred to v according to long-term perspective criterion.

Definition 2. *In terms of Pareto criterion, u is strictly preferred to v if from all periods' perspectives, the intertemporal utilities of u are greater than or equal to that of v*

$$\forall\, t \in \{1, 2, 3, \ldots, T-1, T\}, \sum_{i=0}^{T-t} \beta_i u_{t+i} \geq \sum_{i=0}^{T-t} \beta_i v_{t+i} \quad (i.e.\, U_t \geq V_t) \tag{6}$$

and from at least one period's perspective, the intertemporal utility of u is greater than that of v

$$\exists\, t \in \{1, 2, 3, \ldots, T-1, T\}, \sum_{i=0}^{T-t} \beta_i u_{t+i} > \sum_{i=0}^{T-t} \beta_i v_{t+i} \quad (i.e.\, U_t > V_t) \tag{7}$$

Definition 3. *In terms of long-term perspective criterion, u is strictly preferred to v if from the fictitious period 0's perspective, the normative utility of u is greater than that of v*[5]

$$\sum_{i=0}^{T} \beta_i u_i > \sum_{i=0}^{T} \beta_i v_i \quad (i.e.\, U_0 > V_0) \tag{8}$$

With definitions 1,2 and 3, we have the following proposition:

Proposition 1. *Under myopic discounting, Pareto-efficiency is a sufficient condition for long-term perspective efficiency.*

4. Proof of proposition 1

U_0 (the normative utility) consists of all terms in u whereas U_t (the intertemporal utilities at period t) consists of only the last $T-t+1$ terms in u , hence, given in Eq.(1) that $\beta > \mathbf{0}$, U_0 can be written as a unique linear combination of set $\{U_1, U_2, \ldots, U_{T-1}, U_T\}$. Thus, there exists a unique vector ϵ (i.e. $\{\epsilon_1, \epsilon_2, \ldots, \epsilon_{T-1}, \epsilon_T\}$) such that

$$U_0 = \sum_{i=1}^{T} \epsilon_i U_i \tag{9}$$

Lemma 4.1. *All elements in vector $\epsilon = \{\epsilon_\tau\}_{\tau=1}^{T}$ is strictly positive.*

Proof. From Eqs.(1) and (9), the unique vector ϵ can be expressed as

$$\epsilon_1 = \beta_1 \tag{10}$$

$$\forall t \in \{2, 3, \ldots, T-1, T\}, \epsilon_i = \beta_i - \sum_{j=1}^{i-1} \epsilon_j \beta_{i-j} \tag{11}$$

$\epsilon_1 = \beta_1 > 0$. The rest of the lemma can be proved by mathematical induction.
Base case:
From Eqs.(11), (10), (2) and (4), we have

$$\epsilon_2 = \beta_2 - \epsilon_1\beta_1 = \beta_2 - \beta_1\beta_1 = \delta_1\delta_2 - \delta_1\delta_1 > \delta_1\delta_1 - \delta_1\delta_1 = 0 \tag{12}$$

Inductive step:
Assume that

$$\forall i \in \{2, 3, \ldots, k-1, k\}, \forall k \geq 2, \epsilon_i > 0 \tag{13}$$

the inductive step should show that $\epsilon_{k+1} > 0$.

[5]Without loss of generality, it is assumed that $u_0 = v_0 = 0$. See O'Donoghue and Rabin(1999).

From Eqs.(11) and (2),

$$
\begin{aligned}
\epsilon_{k+1} &= \beta_{k+1} - \sum_{j=1}^{k} \epsilon_j \beta_{k+1-j} = \beta_k \delta_{k+1} - \sum_{j=1}^{k} \epsilon_j \beta_{k+1-j} \\
&= (\epsilon_k + \sum_{i=1}^{k-1} \epsilon_i \beta_{k-i}) \delta_{k+1} - \sum_{j=1}^{k} \epsilon_j \beta_{k+1-j} \\
&= \epsilon_k (\delta_{k+1} - \delta_1) + \sum_{i=1}^{k-1} \epsilon_i (\beta_{k-i} \delta_{k+1} - \beta_{k+1-i})
\end{aligned}
\tag{14}
$$

Given Eqs.(4) and (3), $\delta_1 < \delta_2 \leq \delta_{k+1}$. With the assumption of the inductive step,

$$\epsilon_k (\delta_{k+1} - \delta_1) > 0 \tag{15}$$

Moreover, given Eqs.(2) and (3), $\forall i \in \{1, 2, 3, \ldots, k-1\}$

$$\frac{\beta_{k-i} \delta_{k+1}}{\beta_{k+1-i}} = \frac{\delta_{k+1}}{\delta_{k+1-i}} \geq 1 \tag{16}$$

Eq.(16) implies that $\beta_{k-i} \delta_{k+1} \geq \beta_{k+1-i}$.
Along with the assumption of the inductive step,

$$\sum_{i=1}^{k-1} \epsilon_i (\beta_{k-i} \delta_{k+1} - \beta_{k+1-i}) \geq 0 \tag{17}$$

Combining Eqs.(15) and (17), $\epsilon_{k+1} > 0$. □

Given Eqs.(6), (7), (9) and that vector ϵ is strictly greater than $\mathbf{0}$, Eq.(8) holds true.

5. Application

The main result of this paper can be applied to any Pareto-improving policies under myopic discounting preferences. In this section, we present a simple example of how our result is applied to Laibson-style Pareto-improving revenue-neutral consumption tax policies. In a three-period consumption-savings model, we assume that the period-utility is $u_t(c_t) = ln(c_t)$, the gross interest rate is 100%, the income in period 1 is 100 and there is no income in period 2 and 3. In period 2, given the consumer's savings in period 1 (k_1), the sophisticated consumer solve the following utility maximization problem:

$$\max_{k_2|k_1} u(c_2) + \beta_1 u(c_3) \tag{18}$$

subject to

$$
\begin{aligned}
(1 + t_2)\, c_2 + k_2 &= k_1 + \theta_2. \\
c_3 &= k_2
\end{aligned}
\tag{19}
$$

where c_2, k_2, t_2 and θ_2 represent consumption, savings, consumption-tax, and lump-sum subsidy in period 2 respectively. We substitute the constraints of *Eq.* (19) into the objective function of *Eq.* (18) and obtain the following equality from the first order condition with respect to k_2

$$\beta_1(k_1 + \theta_2 - k_2) = k_2 \tag{20}$$

Under revenue-neutral policy, we have $\theta_2 = t_2 c_2^*$, where c_2^* represents the equilibrium consumption at period 2. Substitute this revenue-neutral condition into the first constraint in *Eq.* (19), we get $c_2^* = k_1 - k_2$. Hence, $\theta_2 = t_2(k_1 - k_2)$. Substituting this equality into *Eq.* (20), we have

$$k_2 = \frac{\beta_1 + \beta_1 t_2}{1 + \beta_1 + \beta_1 t_2} k_1 \tag{21}$$

In period 1, the consumer determines the savings in period 1 (k_1) from the following maximization problem

$$\max_{k_1} u(c_1) + \beta_1 u(c_2) + \beta_2 u(c_3) \tag{22}$$

subject to *Eq.* (19) and *Eq.* (21) with additional budget constraint

$$(1 + t_1)\, c_1 + k_1 = 100 + \theta_1. \tag{23}$$

where c_1, k_1, t_1 and θ_1 represent consumption, savings, consumption-tax, and lump-sum subsidy in period 1 respectively. Substitute all the constraints into *Eq.* (22), we obtain following equality from the first order condition with respect to k_1

$$(\beta_1 + \beta_2)(100 - \theta_1 - k_1) = k_1 \tag{24}$$

Similar to period 2, the revenue-neutral condition requires $\theta_1 = t_1 c_1^*$, where c_1^* is the equilibrium consumption at period 1. Then, $c_1^* = 100 - k_1$ can be derived from the budget constraint in *Eq.* (23). The subsidy at period 1 can be expressed as $\theta_1 = t_1(100 - k_1)$. Thus, the saving at period 1 can be computed as

$$k_1 = \frac{100(\beta_1 + \beta_2)(1 + t_1)}{(\beta_1 + \beta_2)(1 + t_1) + 1} \tag{25}$$

Laibson (1996) showed that there are infinite number of Pareto-improving taxes (t_1, t_2) for any time-inconsistent preferences. In table 1, one pair of Pareto-improving taxes are listed for the following cases: quasi-hyperbolic discounting, hyperbolic discounting, myopic discounting that is neither quasi-hyperbolic nor hyperbolic and non-myopic discounting. Further testing is carried out by looping through different tax rates with a step-size of 0.1%. The result confirms that under myopic discounting, all the tested Pareto improving consumption taxes also increase normative utility. Under non-myopic discounting, Pareto-improving tax does not guarantee a higher normative utility as demonstrated by the last case in the table.

Table 1: Change in three periods' inter-temporal utilities, U_1, U_2 and U_3, and normative utility, U_0, under quasi-hyperbolic discounting $(1, \frac{1}{3}, \frac{1}{6}, \frac{1}{12})$, hyperbolic discounting $(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4})$, myopic discounting that is neither quasi-hyperbolic nor hyperbolic $(1, \frac{3}{4}, \frac{3}{5}, \frac{6}{11})$ and non-myopic discounting $(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{10})$

$\beta_0, \beta_1, \beta_2, \beta_3$	Consumption tax	c_1, c_2, c_3	U_1	U_2	U_3	U_0
$1, \frac{1}{3}, \frac{1}{6}, \frac{1}{12}$	No	66.666667, 25.000000, 8.333333	5.626041	3.925630	2.120264	2.113070
	Yes ($\tau_1 = 5\%, \tau_2 = 5\%$)	65.573770, 25.500911, 8.925319	5.627563	3.968345	2.188892	2.116585
$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	No	54.545455, 30.303030, 15.151515	6.610692	4.770298	2.718101	3.816125
	Yes ($\tau_1 = 10\%, \tau_2 = 10\%$)	52.173913, 30.855540, 16.970547	6.613067	4.845056	2.831479	3.828267
$1, \frac{3}{4}, \frac{3}{5}, \frac{6}{11}$	No	42.553191, 32.826748, 24.620061	8.291325	5.893915	3.203562	6.655210
	Yes ($\tau_1 = 5\%, \tau_2 = 6\%$)	41.365047, 32.665712, 25.969241	8.291328	5.929011	3.256913	6.660121
$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{10}$	No	54.545455, 30.303030, 15.151515	6.610692	4.770298	2.718101	3.408410
	Yes ($\tau_1 = 6\%, \tau_2 = 10\%$)	53.097345, 30.259777, 16.642878	6.614364	4.815811	2.811982	3.403868

6. Conclusion

By defining myopic discounting which encompasses both quasi-hyperbolic and hyperbolic discounting, this paper shows that a welfare-improving policy for intertemporal utilities across all periods also improves normative utility at fictitious period 0. Equivalently, Pareto criterion implies long-term perspective criterion under myopic discounting.

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