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## Mission impossible: Buy price fails to signal information

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## Abstract

According to the well-known linkage principle, a seller benefits from revealing information relevant to bidders' payoffs in auctions. For this purpose, the seller needs a signaling device to credibly transmit her private information to bidders. One natural candidate for such a device is a buy price. In this paper, I investigate the theoretical possibility that the buy price may credibly signal the seller's private information in second-price auctions. However, I find that the buy price cannot serve as a signaling device. There exists no separating equilibrium such that a buy price signals the seller's private information.

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# 1. Introduction

As a well-known linkage principle shows, a seller benefits from revealing information relevant to bidders' payoff in auctions (Milgrom and Weber, 1982). Examples of such information include product quality and art authenticity. It is especially important for the seller in secondhand markets to credibly transmit her private information to bidders. For this purpose, the seller usually needs a signaling device. Especially in online auctions, several communication devices are available to the seller.

A reserve price is naturally considered as such a device. In fact, Cai et al. (2007) theoretically show that the reserve price serves for this purpose in second-price auctions (SPA). On the other hand, a buy price is considered as another candidate for the signaling device. For instance, Anderson et al. (2008, p. 147) point out in their empirical research that its signaling effect may improve the seller's revenue. Since the pioneering research by Budish and Takeyama (2001), the literature has clarified the conditions that the seller can increase a revenue by providing a buyout option to the bidders. To the best of my knowledge, however, there is no theoretical research that studies the signaling aspect of a buy price.

In this research, I investigate a potential use of a buy price to signal the seller's private information. As shown below, however, an increase of the buy price is costless for the seller, thus, a buy price fails to signal information.

This research is related to Khezr and Menezes (2018) that study an asking price indicated by sellers in housing markets. The asking price is similar to the buy price in a sense that both limit an upper bound of bids (i.e., counteroffers by buyers). They characterize a separating equilibrium in which the seller increases the asking price as she values the house at higher (proposition 6) in section 6. Their finding is incompatible to mine at a glance, but this inconsistency arises from the difference in selling mechanisms. The seller's private value, which is revealed by the asking price, works as the reserve price because bids below the value are not accepted. Thus, an increase in the asking price extracts fewer bids from the buyers. In other words, it is costly for the seller to increase the asking price. Moreover, the unsold house is less costly for the seller who values the house at higher; thus, the single crossing condition holds in their model as in Cai et al. (2007).

# 2. The model and result

I consider a sealed-bid SPA with a buyout option in which a seller sells a single object to $N$ bidders. The seller has private information $s \in [\underline{s}, \overline{s}] \subset \mathbb{R}_+$ which is relevant to payoffs of both the seller and bidders. The seller with $s$ values her object at $v(s)$ with $v' > 0$. Each bidder is *ex ante* identical. The bidder's valuation $u(x, s)$ consists of private value component $x \in [\underline{x}, \overline{x}] \subset \mathbb{R}_+$, which is an identical and independent draw with the distribution function $F(\cdot)$, and common value component $s$. The bidder's valuation increases with both components (i.e., $\partial u/\partial x > 0$ and $\partial u/\partial s > 0$). Note that the bidder's private signal is independent of the seller's private information.

I define $u^{-1}(\cdot|s)$ as the inverse function of $u(\cdot)$ given $s$, that is, $u^{-1}(x|s)$ is the bidder's private signal that his valuation is equal to $x$ given $s$ (i.e., $u(u^{-1}(x|s)|s) = x$). For the sake of convenience, let $G(x) = F(x)^{N-1}$ and $g(x)$ denote the distribution and density functions of the highest valuation among the $N-1$ bidders.

Following Budish and Takeyama (2001) and Reynolds and Wooders (2009), I employ a two-stage game. In the first stage, the seller chooses buy price $b$ after observing $s$. A reserve price $r$ is assumed to be exogenous. By observing $b$, the bidder forms a belief $\hat{s}$; that is, he believes that the seller's private information is $\hat{s}$. I assume that all the bidders form the same belief. After the bidders each privately learn $x$, they simultaneously decide whether to exercise a buyout option or not. If no bidder exercises a buyout option, the game proceeds to the second stage, where the bidders compete for the object in an SPA. The solution concept is a perfect Bayesian equilibrium (PBE) and I focus on a symmetric separating PBE.

As Reynolds and Wooders (2009) show, in a symmetric equilibrium, the bidder optimally employs a threshold strategy that he exercises the buyout option if and only if his private information is at or above a certain threshold. Because the bidders correctly infer $s$ in a separating equilibrium, I can apply the analysis of Reynolds and Wooders (2009), who consider independent private values, to characterize a symmetric separating PBE. The equilibrium threshold $x^*$ increases with $b$ and decreases in $\hat{s}$.

Let $z$ denote the maximum valuation among $N-1$ bidders. Given $x^*$, $\hat{s}$, and $s$, by choosing $b$, the seller obtains the expected payoff of

$$V(b, \hat{s}, s) = \underbrace{v(s)F\left(u^{-1}(r|\hat{s})\right)^N}_{\text{(A)}} + \underbrace{b[1 - F(x^*)^N]}_{\text{(B)}} + \underbrace{N\int_{u^{-1}(r|\hat{s})}^{\overline{x}} \int_{u^{-1}(r|\hat{s})}^{\min\{x, x^*\}} u(z, \hat{s})g(z)f(x)dzdx}_{\text{(C)}}. \quad (1)$$

Note that $u^{-1}(r|s)$ is the bidder's private signal that his valuation is equal to reserve price $r$ given $s$. As Eq. 1 represents, the expected payoff consists of three components: (A) the consumption utility when the object remains unsold; (B) the revenue when the object is sold at the buy price; and (C) the revenue when the object is sold through an SPA. Importantly, the terms (B) and (C) in equation (1), which represent the expected revenue, do not depend on the seller's private information $s$ but solely depend on the bidders' belief $\hat{s}$. As I will discuss later, this fact is essential to understand why the buy price cannot signal the seller's private information.

Let me write $ER(\hat{s}) = (B) + (C)$. I am now ready for stating the finding of this research.

**Theorem.**
*There exists no separating PBE that a buy price signals the seller's private information.*

*Proof.* We prove the statement by contradiction. Let $b(s)$ denote an equilibrium buy price strategy. In a separating equilibrium, for $s$ and $s'$ with $s > s'$,
$V(b(s), s, s) = v(s)F(u^{-1}(r|s)) + ER(s) \geq V(b(s'), s', s) = v(s)F(u^{-1}(r|s')) + ER(s')$,
$V(b(s'), s', s') = v(s')F(u^{-1}(r|s')) + ER(s') \geq V(b(s), s, s') = v(s')F(u^{-1}(r|s)) + ER(s)$.
By adding these equations, I obtain

$$[v(s) - v(s')][F(u^{-1}(r|s)) - F(u^{-1}(r|s'))] \geq 0.$$

Because $v(s) > v(s')$, $F(u^{-1}(r|s)) \geq F(u^{-1}(r|s'))$ should hold. However, $s > s'$ yields $F(u^{-1}(r|s)) < F(u^{-1}(r|s'))$ because of $\partial u / \partial s > 0$. *Q.E.D.*

This theorem implies that the buy price fails to become a signaling device. The underlying reason behind the finding is that the seller incurs no cost for increasing a buy price because the object is successfully traded even though the bidders do not exercise the buyout option. A high buy price extracts high bids from those bidders who believe that a high buy price implies a high-valued object. Thus, the seller has an incentive to manipulate the bidders' belief by increasing a buy price *independent of the seller's private information*. This seller's incentive is captured by the fact that the expected revenue $ER(\hat{s})$ is independent of $s$. This is the important difference between the reserve and buy prices. Increasing a reserve price is costly to the seller because higher reserve prices yield lower probability of a successful sale.

## 3. Concluding remark

It would be costly for the seller to reduce the buy price. Clearly, all the bidders exercise the buyout option for the buy price of zero, and thus the expected revenue collapses to zero. Does this observation suggest the existence of a signaling equilibrium in which the seller with better information posts a lower buy price? The answer is no. The key point is that, given a buy price $b$, it is more costly for a seller with a better information to sell her object at the buy price. Thus, if bidders exercise the buyout option for buy price $b$ with a positive probability, lower-type sellers could benefit from posting the same price $b$.

Although this research employs the independent private values environment, the result might apply to a general setting (i.e., interdependent values), because it is essentially costless for the seller to increase the buy price.

In online auctions, it is observed that sellers tend to indicate a very high buy price (see, e.g., Lucking-Reiley, 2000, p. 245). The literature has suggested rationale behind this phenomenon: Buyers can benefit from exercising the buyout option when they are risk averse (Budish and Takeyama, 2001; Reynolds and Wooders, 2009) or impatient (Mathews, 2004). Empirical research, however, reports a substantial volume of deviations from theoretical predictions (e.g., Ivanova-Stenzel and Kröger, 2008). This requires the other rational explanations instead of signaling. We await the complete answer to come from future research.

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