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### Upward-sloping labor supply, firing costs and collusion

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#### Abstract

We analyze the sustainability of collusion in a supergames framework wherein the only input is a highly qualified type of labor, with its supply being upward-sloping and the wage being sensitive to the industry input demand. Hence, when seeking to expand production, firms have to attract additional employees by offering them higher wages. We compare equilibria and social welfare in both quantity and price competitions, as well as by considering non-negligible firing costs. We prove that: the sensitivity of wages to the industry demand for labor facilitates collusion in price competition (in quantity competition, the reverse is true); in both price and quantity competitions, collusion should be welfare-enhancing when the sensitivity of wage is high enough. Moreover, the introduction of firing costs, decreasing the incentive to cut the production after a temporary rise, reduces the deviation profits making collusion easier to sustain. Our results can be extended to any context where input prices are endogenous.

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# 1 Introduction

Over the past few decades, the economics literature has thoroughly investigated the tacit collusion inherent in oligopolistic markets using the repeated framework proposed by Friedman (1971). Yet, only a few prior studies have analyzed the effect of the input pricing on the collusion sustainability in the output market, with the majority of them having focused on models of oligopsony and assumed some degree of buyer power in the wage-bargaining determination.<sup>1</sup>

In the present study, using the model set proposed by Capuano and Grassi (2018), we analyze the sustainability of collusion in a supergames framework wherein the only input is a highly qualified type of labor, with its supply being upward sloping and the wage being sensitive to the industry input demand (i.e., the industry output supply). Hence, when seeking to expand production, firms have to attract additional employees by offering them higher wages. When focusing on the sustainability of collusion, it can be seen that the sensitivity of the equilibrium wage to the industry demand for labor affects the incentives to both deviate and punish, modifying the critical discount factor. We extend the analysis by comparing equilibria and social welfare in both Cournot and Bertrand competition, as well as by considering non-negligible firing costs. We prove that the sensitivity of wages to the industry demand for labor facilitates collusion in Bertrand competition, while such sensitivity might render collusion non-sustainable in Cournot competition. Moreover, the introduction of firing costs reduces firms' incentive to decrease their output levels (i.e., to fire some employees). Within this framework, starting from the collusive outcome, a cheater firm increases its level of output during the deviation phase (hiring additional employees), while it decreases its level of output (firing employees) when the punishment phase begins. Further, to punish the cheater, the cheated firms increase their levels of output as reply to deviation. Thus, the presence of firing costs affects only the cheater's profit, thereby reducing the marginal profitability of any additional output during the deviation phase and decreasing the unilateral incentive to deviate, both in quantity and price competitions.

A key novelty of our model lies in the price-maker role played by oligopolistic firms in the input market. Typically, in a partial equilibrium model, firms are price-takers with respect to the input prices, and their strategic decisions do not affect them. According to the general equilibrium approach, even though the input prices depend on firms' decisions, firms play a price-taker role in the input market. In our model, firms act as price-makers in both the input and output markets, and the input prices belong to the firms' strategic sets.

The remainder of this paper is organized as follows. In Section 2, we describe the model and the results presented by Capuano and Grassi (2018). In Section 3, we analyze the price competition, comparing the results with the quantity competition case. Then, in Section 4, we extend our analysis by considering firing costs, while in Section 5, we exploit the welfare impact of collusion. Finally, we present our conclusions in Section 6. The proofs of the lemma and the

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<sup>1</sup>See, inter alia, Van Gompel (1995), Majumdar and Saha (1998), Bertomeu (2007), Vlassis and Varvataki (2014), and Capuano and Grassi (2018).

propositions are available in the Appendix.

## 2 The model

### 2.1 The model setting

Consider the model set proposed in Section 2 of Capuano and Grassi (2018): a market where two symmetric firms produce a homogenous good, with linear demand function  $P = 1 - q_1 - q_2$ , where  $q_1$  and  $q_2$  are the firms' outputs,  $P$  is the output price.

The labor  $l_i$ , with  $i = 1, 2$ , is the only production input and, following Horn and Wolinsky (1988), technology displays constant return to scale: input  $l_i$  produces output  $q_i = q(l_i) = l_i$  of good  $i$ . Thus, if the input price is wage ( $w$ ), the cost function is  $C_i = wq_i$ .

The labor demand curve ( $D_l$ ) is given by the sum of firms' conditioned input demands,  $l_1(q_1)$  and  $l_2(q_2)$ . The labor supply curve,  $S_l$  is linear and increasing in  $w$ .

$$D_l = l_1(q_1) + l_2(q_2) = q_1 + q_2 \quad (1)$$

$$S_l : \quad w_S = w_0 + bS_l \quad (2)$$

with  $w_0, b \geq 0$ . The elasticity of the supply curves decreases with respect to  $b$ ; when  $b = 0$  the supply curve is infinitely elastic. Thus, assuming  $w_0 = 0$ , we have the equilibrium wage  $w^*$  on the labor market such that  $D_l = S_l$ :

$$w^* = b(q_1 + q_2) \quad (3)$$

We consider the case where firms contract their employees period by period, and, in this section, we exclude firing costs.<sup>2</sup> The firm  $i$ 's profit function is:

$$\Pi_i = (1 - q_i - q_j)q_i - b(q_i + q_j)q_i \quad (4)$$

with  $i = 1, 2$  and  $j \neq i$ .  $\partial C_i / \partial q_j = bq_j > 0$  is the negative externality of the firm  $j$ 's output on the firm  $i$ 's profit. In other words, the output of firm  $j$  increases the wage in the labor market, and thus the total cost of firm  $i$ . Therefore, any increase in a firm's output causes a twofold increase in its production costs. The input demand increases, and the wage increases as well. Thus, when a firm increases its production, the higher the sensitivity of the equilibrium wage to the industry demand of labor, the higher the increase in the total cost. When firms collude in the output market, they reduce their level of production and, as a consequence, the industry input demand. In other words, the equilibrium wage reflects the production decisions of the firms.

Following Friedman (1971), let  $\Pi^{Nash}$ ,  $\Pi^{Coll}$ , and  $\Pi^d$  be respectively the one-shot payoffs in the Nash equilibrium, in case of collusion, and in case of deviation from collusion, where  $\Pi^d \geq \Pi^{Coll} \geq \Pi^{Nash}$ . In order to sustain collusion, the following incentive compatible constraint must be satisfied:

<sup>2</sup>This implies that in any period  $l_i = q_i$ .

$$\frac{\Pi^{Coll}}{1-\delta} \geq \Pi^d + \frac{\delta}{1-\delta} \Pi^{Nash} \quad (5)$$

$$\delta_i \geq \sigma^* = \frac{\Pi^d - \Pi^{Coll}}{\Pi^d - \Pi^{Nash}} \quad (6)$$

where  $\delta_i$  is the individual discount factor of firm  $i$ , measuring the weight of future profits, and  $\sigma^*$  is the critical discount factor. The higher the value of  $\sigma^*$ , the more difficult the sustainability of collusion.

## 2.2 The quantity competition

In this section we recall Capuano and Grassi (2018), where we analyzed the quantity competition case. The Cournot profits (labeled by  $CN$ ), the collusive one (labeled by  $Coll$ ), and the cheater's profit in the deviation (labeled by  $d$ ) phase are:

$$\Pi_1^{CN} = \Pi_2^{CN} = \frac{1}{9(1+b)} \quad (7)$$

$$\Pi_1^{Coll} = \Pi_2^{Coll} = \frac{1}{8(1+b)} \quad (8)$$

$$\Pi_1^d = \frac{(20b + 12b^2 + 9)}{64(b+1)^3} \quad (9)$$

Substituting equations (7), (8), and (9) in condition (6), we obtain the critical discount factor  $\delta^{CN}$ :

$$\delta^{CN} = \frac{\Pi_1^d - \Pi_1^{Coll}}{\Pi_1^d - \Pi_1^{CN}} = 9 \frac{(2b+1)^2}{52b + 44b^2 + 17} \quad (10)$$

No firm has unilateral incentives to deviate only if  $\delta_1, \delta_2 \geq \delta^{CN}$ . Moreover, the higher is the slope of the labor supply, the harder is collusion to sustain, i.e. the discount factor is increasing in  $b$ :  $\frac{\partial \delta^{CN}}{\partial b} > 0$ . In order to punish deviation, cheated firm has to increase its production. This requires to hire additional employees at a higher wage, increasing firm's cost, and thus reducing punishment profitability.

Denoting by  $\Pi_2^{-d}$  the cheated firm profit during the deviation phase, and by  $\Pi_2^{CN}$  its profit during the punishment phase, to punish deviation is profitable if and only if  $\Pi_2^{CN} \geq \Pi_2^{-d}$ ; this condition requires  $b \leq 2.9271 = \underline{b}$ . We summarize these results in the following proposition.

**Proposition 1** *Collusion is sustainable as a Subgame Perfect Nash Equilibrium if and only if:*

- both firms are sufficiently patient, i.e.  $\delta_1, \delta_2 \geq \delta^{CN}$ ;
- punishing any deviation is credible, i.e.  $b \leq \underline{b}$ .

Notice that, in the extreme case where  $b = 0$  we obtain the standard Cournot critical discount factor; i.e.,  $\delta^{CN}(b = 0) = 9/17$ .

### 3 The price competition

In this section we extend the analysis to price competition assuming that: (i) when firms collude, they fix the monopolistic price and share the market; (ii) in the deviation phase, the cheater fixes a price lower than the collusive one, serving all the market demand; and (iii) after any deviation, both firms play the Bertrand Nash reversion.

Given that firms exhibit strictly decreasing return to scale, we cannot apply the marginal-cost pricing rules to derive the pure strategy Bertrand Nash equilibrium (labeled by  $BN$ ).<sup>3</sup> However, in a  $n$ -firm homogeneous product market with convex costs, Dastidar (1995) characterizes a set of Bertrand Nash equilibria satisfying the following properties:

- (i) all firms charge the same price;
- (ii) for no firm is undercutting profitable;
- (iii) no firm gets negative profits.

When firms are symmetric the equilibrium is not unique; we look for the most severe punishment; i.e., the zero-profit one.

**Lemma 1** *The Bertrand-Nash equilibrium is a price vector  $(p_1^{BN}; p_2^{BN}) = (\frac{b}{b+1}; \frac{b}{b+1})$  such that: (i) firms' profits are zero (i.e.,  $\Pi_1(p_1^{BN}, p_2^{BN}) = \Pi_2(p_1^{BN}, p_2^{BN}) = 0$ ); (ii) there does not exist any alternative price  $p' < p_1^{BN}$  such that  $\Pi_1(p', p_2^{BN}) > \Pi_1(p_1^{BN}, p_2^{BN})$  or  $\Pi_2(p_1^{BN}, p') > \Pi_2(p_1^{BN}, p_2^{BN})$ .*

We assume that colluding firms maximize their joint profits  $\Pi_1^{Coll} + \Pi_2^{Coll}$  with respect to  $p$ .

$$\Pi_1^{Coll} + \Pi_2^{Coll} = p(1-p) - b(1-p)^2 \quad (11)$$

From the First Order Condition, we obtain:

$$p^{Coll} = \frac{1+2b}{2+2b}; \quad q_1^{Coll} = q_2^{Coll} = \frac{1}{4(b+1)}; \quad w^{Coll} = \frac{b}{2(b+1)} \quad (12)$$

$$\Pi_1^{Coll} = \Pi_2^{Coll} = \frac{1}{8(b+1)} \quad (13)$$

In the deviation phase, the cheater fixes the deviation price  $p^d$  that undercuts the competitor and makes profitable to serve all the market demand. The deviation profit is:

$$\Pi^d = (1-p^d)p^d - w^{Coll} \cdot q^{Coll} - w^d((1-p^d) - q^{Coll}) \quad (14)$$

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<sup>3</sup>See Vives (2001), Chapter 5.

where  $w^d = b((1 - p^d) + q^{Coll})$ . Maximizing (14) with respect to  $p^d$ , under the condition  $p^d < p^{Coll}$ , we obtain:

$$p^d = \frac{1 + 2b}{2b + 2} - \epsilon \cong p^{Coll} \quad \text{with } \epsilon \rightarrow 0 \quad (15)$$

The cheater offers a price infinitesimally lower than the collusive one, obtaining a deviation profit tending to the monopolistic one:

$$\Pi^d = \frac{1}{4(b + 1)} - b \left( \frac{1}{4(b + 1)} \right)^2 \quad (16)$$

Note that  $\Pi^d - \Pi^{Coll}$  is positive for any  $b \geq 0$ ; i.e., the unilateral deviation is always profitable.

In the deviation phase, the cheated firm fixes the collusive price  $p^{Coll}$  and does not sell any unit of the good. Hence, due to the production costs already incurred, its profit is negative during the deviation phase, while it increases to zero in the punishment phase. This means that the cheated firm finds always profitable starting the Bertrand Nash reversion. From equations (13),(14) and Lemma 1, we obtain:

**Proposition 2** *Collusion is sustainable as a Subgame Perfect Nash Equilibrium if and only if both firms are sufficiently patient, i.e.  $\delta_1, \delta_2 \geq \delta^{BN} = \frac{b+2}{3b+4}$ .*

We remark that the higher is the slope of the labor supply, the easier is collusion to sustain; i.e., the discount factor is decreasing in  $b$  ( $\frac{\partial \delta^{BN}}{\partial b} = -\frac{2}{(3b+4)} < 0$ ).

The comparison between the critical discount factors  $\delta^{BN}$  and  $\delta^{CN}$  confirms that collusion is easier to sustain under Bertrand than under Cournot competition. Indeed, in quantity competition, deviating the cheater produces additional quantity, and the cheated firm continues producing and selling the collusive output. The latter obtains minor, but positive, profits. However, punishing reduces cheated's profit and may not be profitable. On the contrary, in price competition, the cheater undercuts the rival and cleans the market. The cheated firm has a negative profit (i.e., it produces but does not sell). Hence, in this case, the cheated firm has always a positive incentive to punish, since punishing increases its profit to zero. In other words, in a tighter input market the sustainability of collusion increases (decreases) under price (quantity) competition.

Furthermore, the impact of  $b$  on the critical discount factors  $\delta^{BN}$  and  $\delta^{CN}$  is different. In quantity competition, the critical discount factor is increasing in  $b$ : when  $b$  is higher than the threshold value  $\underline{b}$ , the critical discount factor is affected by a discontinuity jump, reaching its maximum value ( $\delta^{CN} = 1$ ) for any  $b > \underline{b}$ . This means that, when the equilibrium wage is sensitive enough to the labor demand, collusion is never sustainable. On the contrary, in price competition,  $b$  negatively affects the critical discount factor; i.e., an increase in the rigidity of the labour supply plays a pro-collusive role.

This interesting result depends, in general, on the different nature of the two competitions,<sup>4</sup> in particular, on the different magnitude of the effect of the parameter  $b$  on the deviation profits. Consider the incentive compatible constraint described in equation 5. Since collusive profits are the same in both the competitions, the trend of  $\delta$  depends only on the impact of  $b$  on the deviation and Nash profits. In Bertrand competition quantity deviation is higher (since the cheater firm undercuts the rival); thus, the impact of  $b$  on deviation profit is higher; at the same time, the Nash profits are unaffected by  $b$  ( $\pi^{BN} = 0$ ), thus the second term of the constraint 5 is more sensitive to  $b$  in Bertrand than in Cournot, and the slope of  $\delta$  changes sign.

Figure 1 illustrates the critical discount factors in quantity and price competitions as a function of  $b$ .

## 4 Firing costs

In this section, we assume that: (i) firms contract employees for  $t > 1$  periods; (ii) firms have to compensate an employee with a money transfer  $f > 0$  (lump sum), when the latter is fired.<sup>5</sup>

In this framework, there is only one case where a firm reduces production, i.e. when the cheater reduces its output in the punishment phase. Indeed, deviating from collusion, the cheater increases its production (hiring new employees); when punishment starts it has to decrease it (firing employees), and firing costs negatively affect its profit. On the contrary, the cheated firm increases its output after a deviation; hence, firing costs do not affect the profitability of starting the punishment phase. The impact of firing cost is described by the following proposition:

**Proposition 3** *Firing costs reduce unilateral incentives to deviate in quantity and price competitions.*

Proposition 3 states that firing costs unambiguously reduce the marginal profit during the deviation phase, and the cheater's profit during the punishment phase. As a consequence, the critical discount factor is decreasing with respect to  $f$  ( $\frac{\partial \delta^{CN}}{\partial f} < 0$  and  $\frac{\partial \delta^{BN}}{\partial f} < 0$ ). This means that firing costs are a pro-collusive factor.

## 5 Social welfare and collusion

Collusion typically causes allocative inefficiency in the market, since firms jointly reduce their outputs. In our model, a lower level of the industry output is

<sup>4</sup>In the case of Cournot competition goods are strategical substitutes, in the case of Bertrand competition prices are strategical complements.

<sup>5</sup>Notice that in a seminal contribution Dixit (1980) shows that increasing production requires always investment in capacity; i.e., additional costs. In our model, additional costs occur only when the increase in production is temporary, followed by a reduction in the Nash reversion phase.

associated with lower levels of labor demand and wage, decreasing the industry costs; i.e., the marginal cost is lower when firms collude. In other words, there exists a trade-off between allocative efficiency and cost reduction. We prove that, when the sensitivity of wage to the input demand is high enough, collusion may enhance welfare.

The social welfare in the good market, measured as the sum of consumers surplus and profits, in the cases of quantity competition, price competition, and collusion are:

$$W^{CN} = \frac{(2q^{CN})^2}{2} + 2\Pi^{CN} = \frac{2}{9} \frac{b+2}{(b+1)^2} \quad (17)$$

$$W^{BN} = \frac{(2q^{BN})^2}{2} + 2\Pi^{BN} = \frac{1}{2(b+1)^2} \quad (18)$$

$$W^{Coll} = \frac{(2q^{Coll})^2}{2} + 2\Pi^{Coll} = \frac{1}{8} \frac{2b+3}{(b+1)^2} \quad (19)$$

The comparison of equations (17), (18), and (19), leads us to the following propositions:

**Proposition 4** *In the case of quantity competition, collusion is welfare enhancing when  $b \in [5/2, \underline{b}]$ .*

**Proposition 5** *In the case of price competition, collusion is welfare enhancing when  $b \geq 1/2$ .*

Both in quantity and price competitions, increasing the level of output causes an increase in the marginal cost. When the slope of the input supply is high enough collusion, reducing industry total costs, enhances social welfare; in other words, the reduction in total costs more than compensate the allocative inefficiency. In price competition, collusion can be sustainable at any level of  $b$ , when firms are sufficiently patient. Then, for values of  $b$  high enough, collusion is sustainable and welfare enhancing. On the contrary, in quantity competition, an increase of  $b$  has two opposite effects: on the one hand, increasing  $b$  the difference between collusive and non-cooperative welfare increases (in favor of collusive equilibrium); on the other hand, increasing  $b$  the punishment is not credible and collusion is not sustainable. Thus, only for intermediate values of  $b$ , collusion is sustainable and welfare enhancing.

Notice that the results on welfare crucially depend on the increasing marginal costs, due to the assumption that wages rise in output.<sup>6</sup>

## 6 Conclusion

Empirical evidence suggests that, in the presence of an oligopsony or in the case of monopsonistic competition in the labor market, firms try to coordinate

<sup>6</sup>Similarly, in the well-known contribution by De Fraja and Delbono (1989), the increasing marginal cost assumption causes an increase in welfare when total output decreases, and it is a critical argument for privatization.



themselves in terms of setting wages and reducing their production costs.<sup>7</sup> However, although firms do not collude in the labor market, the wage sensitivity to the output supply affects firms' profitability with regard to their ability to expand their levels of production, and it also plays a crucial role in the collusion sustainability in the output market.

In this paper, we have analyzed firms' incentive to collude when the labor supply curve is upward sloping and when expanding production requires the offering of higher wages. In the collusive scheme, both deviation and punishment require an increase in production. When increasing production proves too costly, deviation from collusion may not be profitable. Moreover, collusion sustainability requires the cheated firm to punish the cheater firm as a reply to any deviation; thus, punishing may be not profitable. In this case, the net impact of the sensitivity of the supply function with respect to wages on the critical discount factor is ambiguous. We have proved that, in the case of quantity competition, the greater the extent to which wages are sensitive to the industry demand for labor, the harder it is to sustain collusion. In the case of price competition, the opposite is true.

Furthermore, we have shown that firing costs are univocally pro-collusive, reducing the firm's incentives to decrease its level of production and rendering deviation less profitable (in both quantity and price competitions).

Finally, analyzing social welfare, we have proved that, in both price and quantity competitions, collusion can be socially preferable to a non-cooperative outcome.

Extensions of our analysis may exploit the role of firing cost in firms versus unions bargaining processes.

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<sup>7</sup>See Gonzaga et al. (2013) for recent examples of firms aimed to collusively reduce their labor cost.

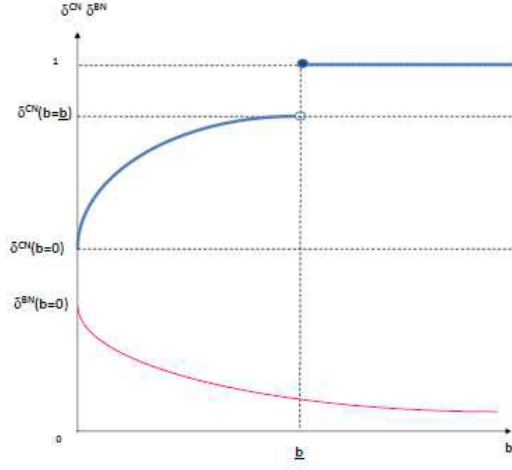


Figure 1: The critical discount factors in quantity competition (above) and price competition (below).

## Appendix

### Proof of Proposition 1:

See Capuano and Grassi (2018).

#### Proof of Lemma 1:

i) Assume  $p_i = p_j = p$ . Symmetric duopolistic profits are equal to zero if and only if:  $p = \frac{b}{b+1} = p^{BN}$  :  $\Pi_i(p, p) = p \frac{(1-p)}{2} - b(1-p) \frac{(1-p)}{2} = 0$ , where  $p^{Coll} - p^{BN} = \frac{1}{2+2b} > 0$ .

ii) There exists an unilaterally-profitable deviation if and only if  $p' < p^{BN}$ . Thus, if  $p_1 = p^{BN}$ , and  $p_2 = p'$ , we have  $\Pi_2(p^{BN}, p') = (1-p')p' - b(1-p^{BN}) \frac{(1-p^{BN})}{2} - b \left( \frac{(1-p^{BN})}{2} + (1-p') \right) \left( (1-p') - \frac{(1-p^{BN})}{2} \right)$ . Then,  $\forall p' \geq \frac{1+2b}{2+2b} - \frac{1}{2\sqrt{(b+1)^3}}$ ,  $\Pi_2(p^{BN}, p') \geq \Pi_2(p^{BN}, p^{BN})$ .

However,  $\forall b > 0$ ,  $p' - p^{BN} = \frac{1}{2} \left( (b+1)^{-1} - (b+1)^{-3/2} \right) > 0$ .

#### Proof of Proposition 2:

Substituting equations (13), (14) and the Bertrand profits in condition (6), we obtain the critical discount factor  $\delta^{BN} = \frac{\Pi_1^d - \Pi_1^{Coll}}{\Pi_1^d - \Pi_1^{BN}} = \frac{b+2}{3b+4}$ , where  $\forall b \in \mathbb{R}^+$ ,  $\delta^{BN} < 1$ .

### Proof of Proposition 3:

In quantity competition,  $E\Pi^d = \Pi^d(q^d, q^{Coll}) + \delta \left( -f(q^d - q^{CN}) + \frac{\Pi^{CN}}{1-\delta} \right)$ . The optimal deviation output is such that  $q^d = \arg \max_q E\Pi^d : \frac{\partial \Pi^d(q^d, q^{Coll})}{\partial q^d} - \delta f = 0$ . Since  $\Pi^d(q^d, q^{Coll})$  is concave with respect to  $q^d$ , then  $\frac{\partial q^d}{\partial f} < 0$ . Moreover, since  $q^d(f = 0) \neq q^d(f > 0)$ , then  $\Pi^d(f > 0) < \Pi^d(f = 0)$  and  $E\Pi^d(f > 0) < E\Pi^d(f = 0)$ . The same argument can be applied to price competition.

**Proof of Proposition 4:**

$W^{Coll} - W^{CN} = \frac{1}{8} \frac{2b+3}{(b+1)^2} - \frac{2}{9} \frac{b+2}{(b+1)^2} = \frac{1}{72} \frac{2b-5}{(b+1)^2} \geq 0$  if and only if  $b \geq \frac{5}{2}$ . However, by Proposition 1, collusion is sustainable only if  $b \leq 2.9271 = \underline{b}$ . As  $\underline{b} > 5/2$ , when  $b \in [5/2, \underline{b}]$  collusion is sustainable and  $W^{Coll} - W^{CN} > 0$ .

**Proof of Proposition 5:**

$W^{Coll} - W^{BN} = \frac{1}{8} \frac{2b+3}{(b+1)^2} - \frac{1}{2(b+1)^2} = \frac{1}{8} \frac{2b-1}{(b+1)^2} \geq 0$  if and only if  $b \geq \frac{1}{2}$ .

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