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# A note on fiscal policy, indeterminacy, and endogenous time preference

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## Abstract

This paper presents an endogenous growth model with productive public goods and an endogenous time preference. The time preference is positively associated with consumption and negatively affected by income. Fiscal policy not only directly influences macroeconomic equilibrium and therefore the dynamic stability of macroeconomic equilibria but also indirectly influences them via the endogenous time preference. The overall effect of productive public goods provides a strong externality that generates indeterminacies of the equilibrium growth paths. This study derives the sufficient condition for the indeterminacy and clarifies the relation between fiscal policy and indeterminacy. The results show that Barro's (1990) tax rule for growth and welfare maximization, which equals the output elasticity of productive public goods, attains its purpose and stabilization of the dynamic equilibrium under certain conditions.

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# 1. Introduction

This paper clarifies the relationship between fiscal policy and indeterminacy of equilibrium growth paths in an endogenous growth model with an endogenously determined discount rate. Numerous studies have investigated the source of indeterminacy to explain endogenous business cycles. The representative sources of indeterminacy are externality effects, which generate positive feedback in a dynamic model (Benhabib and Farmer 1994, 1999). Accordingly, it is natural to associate public goods with the source of indeterminacy of equilibrium growth paths. However, the presence of public goods does not directly bring about indeterminacy. For example, Barro (1990), Futagami et al. (1993), Turnovsky and Fisher (1995), and Greiner (1998) present endogenous growth models with productive public goods or public capital.[1] In their models, the equilibrium growth paths are uniquely determined.

Alternatively, some studies have shown that indeterminacy arises by incorporating positive externality effects for both utility and production (Cazzavillan 1996; Chen 2006). Other studies have also shown that income tax financing under certain conditions generates an increasing return production function with respect to capital and labor in an equilibrium (Guo and Harrison 2008; Kamiguchi and Tamai 2011).[2]

These studies greatly contributed to the derivation of the necessary and sufficient conditions for generating indeterminacy of equilibrium growth paths under a time-invariant discount rate. However, many empirical studies have found that time preference rates varies over household income or wealth levels (Hausman 1979; Lawrance 1991; Tanaka et al. 2010). Some theoretical studies have incorporated an endogenous time preference rate, which is taken as external by agents (Palivos et al. 1997; Drugeon 1998; Shi 1999; Meng 2006).[3] In particular, Meng (2006) clarifies that the socially determined discount rate has external effects and generates indeterminacy of equilibrium growth paths.

An increase in productive public goods positively influences the marginal productivity and therefore increases aggregate income. As mentioned, empirical evidence shows that the time preference rate varies inversely with income. Then, productive public goods have two external effects: direct and indirect, through a socially determined time preference rate. Consequently, the productive public goods may bring about indeterminacy of equilibrium growth paths through self-fulfilling expectations under the socially determined discount rate. Accordingly, the relationship between indeterminacy and fiscal policy under an endogenously determined discount rate needs further investigating.

This study extends the endogenous growth model of Barro (1990) by incorporating a socially determined discount rate. We assume that the time preference rate depends on the average propensity to consume. We derive sufficient conditions for generating the indeterminacy of equilibrium growth paths and clarify the relationship between fiscal policy and indeterminacy. Previous studies have shown that the growth-maximizing income tax rate is equal to the output elasticity of public input and equivalent to the welfare-maximizing rate on the uniquely determined growth path.[4] We show that whether such tax policy stabilizes or destabilizes the economy depends on the time preference parameters.

# 2. The model

The model of this study is based on one developed by Barro (1990). In the model, time, $t$, is

[1] See Irmen and Kuehnel (2009) for an excellent survey of this literature.

[2] Fernández et al. (2004) and Lloyd-Braga et al. (2008) investigate the dynamic models with public consumption goods.

[3] Uzawa (1968) and Epstein (1987) develope models with an endogenous time preference rate taken as internal by agents.

[4] Misch et al. (2013) show that the elasticity of substitution between private capital and public input was important in determining the relationship between maximization of growth and welfare. Tamai (2013) shows similar results.

continuous. Final good $Y_t$ is produced using private capital input $K_t$ and productive public goods $G_t$. We specify the production function as

$$Y_t = A \cdot K_t^{1-\alpha} G_t^{\alpha}, \tag{1}$$

where $A > 0$ and $0 < \alpha < 1$ are constant over time.

The lifetime utility function is given as

$$V_0 = \int_0^{\infty} \frac{C_t^{1-\theta} - 1}{1-\theta} \exp\left(-\int_0^t D_s \, ds\right) dt, \tag{2}$$

where $\theta$ and $D_s$ denote the inverse of elasticity of intertemporal substitution (positive constant) and the time-varying discount rate at time $s$, respectively.

Meng (2006) mentions that individual agents' time preference is largely determined by the surrounding environment that are viewed as entirely external and cannot be controlled by individual agents. Following Shi (1999) and Meng (2006), the time preference rate is socially determined and positively associated with average consumption $\bar{C}_t$ and negatively varies with average income $\bar{Y}_t$. Specifically, we assume that the discount rate depends on average propensity to consume in the economy.[5] The rate is defined by non-linear function $F$:

$$D_t = F\left(\frac{\bar{C}_t}{\bar{Z}_t}\right), \tag{3}$$

where $\bar{Z}_t = (1-\tau)\bar{Y}_t$ and $F$ is a monotonic increasing function with respect to $\bar{C}_t/\bar{Z}_t$.

The government imposes income tax at a constant rate, $\tau$, on each household. Each household allocates its income to consumption and investment. Then, the budget equation for a representative household becomes

$$\dot{K}_t = (1-\tau)Y_t - C_t. \tag{4}$$

In equation (4), the dot above letter denotes the derivative with respect to time. The government provides productive public goods financed by income taxes. Therefore, the budget equation for the government is

$$G_t = \tau Y_t. \tag{5}$$

Using equations (1) and (5), the production function in a temporal equilibrium takes the form of

$$Y_t = A^{\frac{1}{1-\alpha}} \tau^{\frac{\alpha}{1-\alpha}} K_t. \tag{6}$$

Each household chooses its consumption stream to maximize the lifetime utility function, equation (2), subject to equations (1) and (4) and the given stream of productive public goods. Using the necessary conditions for the optimization problem, equations (1), (3), (5), and (6), we obtain

$$\frac{\dot{C}_t}{C_t} = \frac{(1-\tau)(1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}} - F\left(\frac{c_t}{(1-\tau)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}}\right)}{\theta}, \tag{7}$$

and the transversality condition. Equations (4) and (6) give

$$\frac{\dot{K}_t}{K_t} = (1-\tau)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}} - c_t, \tag{8}$$

where $c_t \equiv C_t/K_t$.

Logarithmic differentiation of $c_t$ and equations (7) and (8) provide

$$\frac{\dot{c}_t}{c_t} = \frac{(1-\alpha-\theta)\omega - F\left(\frac{c_t}{\omega}\right)}{\theta} + c_t, \tag{9}$$

where $\omega \equiv (1-\tau)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}$ is the ratio of post-tax output to private capital. The equilibrium

[5] Several studies also use the average levels in the discount rate (e.g. Schmitt-Grohé and Uribe 2002; Bian and Meng 2004; Huang et al. 2017).

dynamics of this economy is governed by equation (9). The first-order derivative of equation (9) with respect to $c_t$ is

$$\frac{d}{dc_t}\left(\frac{\dot{c}_t}{c_t}\right) = -\frac{F'\left(\frac{c_t}{\omega}\right)}{\omega} + 1. \tag{10}$$

One of the sufficient conditions for indeterminacy of equilibrium growth paths is that the sign on equation (10) becomes negative at a stationary equilibrium. Equation (10) has a negative sign if the post-tax output-capital ratio, $\omega$, is less than the marginal discount rate with respect to average propensity to consume, $F'(\cdot)$. Indeterminacy arises in such a scenario. In the next section, we derive the explicit condition for indeterminacy of the equilibrium growth paths and interpret the result.

# 3. Dynamic stability and fiscal policy

This section investigates the existence of a stationary equilibrium and its stability and clarifies the relationship between dynamic stability and fiscal policy. To clarify the effect of non-linearity in $F$ on equilibrium dynamics and the sufficient condition for indeterminacy, we should adopt a specific form that can evaluate a non-linearity (i.e., degree of externality), because it is a key to generate indeterminacy.

The discount function is assumed to take the form of a logistic equation:[6]

$$F(x_t) = \frac{\sigma}{1 + \left(\frac{\sigma}{\rho} - 1\right)\exp(-\beta x_t)}, \tag{11}$$

where $\sigma = (1+\kappa)\rho$ and $x_t = c_t/\omega$. Because $F(0) = \rho$ and $F(\infty) = \sigma$, $\rho$ is the lower bound of the discount rate, and $\sigma$ is the upper bound ($\rho > 0$ and $\sigma > 0$). $\kappa$ denotes a coefficient related to the ratio of upper bound to lower bound ($\kappa > 0$). $\beta$ is the intensity of the non-linearity ($\beta > 0$). If $\beta$ is small, it is likely to be linear function.

Tanaka et al. (2010) use a similar form of logistic equation as the probability of choosing immediate reward over the delayed reward in *t* days to estimate the parameters of time preferences. On our paper, equation (11) is interpreted as a weight function and gives the time preference rate as the ratio of the upper bound with respect to $x_t$. Furthermore, this specific function enables us to investigate the effect of non-linearity on dynamic properties.

From equation (11), regarding the existence of a stationary equilibrium, we derive the following proposition:

**Proposition 1.** *Suppose that the time preference rate takes the form of equation (11). If* $(1-\alpha)\omega > \sigma$ *and* $(\theta + \alpha - 1)\omega + \rho > 0$, *there exists at least one stationary equilibrium.*

(Proof) The value of $c$ at the stationary equilibrium satisfies equation (9) with $\dot{c} = 0$:

$$c = \frac{(\theta + \alpha - 1)\omega + \frac{\sigma}{1 + \kappa \exp\left(-\beta \frac{c}{\omega}\right)}}{\theta}.$$

The sufficient condition for a positive balanced growth rate is $0 < c/\omega < 1$. Let us denote the average propensity to consume as $x$. Then, the above equation becomes

$$x = \frac{(\theta + \alpha - 1)\omega + \frac{\sigma}{1 + \kappa \exp(-\beta x)}}{\theta\omega}. \tag{12}$$

We define $R(x)$ as the right-hand side of equation (12). When $(\theta + \alpha - 1)\omega + \rho > 0$, we

[6] Meng (2006) adopts a linear discounting function, whereas we adopted equation (11), which exhibits non-linearity. However, there is no difference in the functional property if the parameter values are appropriately chosen. Therefore, our formulation covers Meng's model.

have

$$R(0) = \frac{\theta + \alpha - 1}{\theta} + \frac{\rho}{\theta\omega} > 0 \text{ and } R(1) = \frac{\theta + \alpha - 1}{\theta} + \frac{\sigma e^{\beta}}{[e^{\beta} + \kappa]\theta\omega} > R(0).$$

If $1 > R(1)$, then $1 > R(1) > R(0)$ holds. Applying the intermediate value theorem, there exists at least one stationary equilibrium when $1 > R(1)$, or equivalently,

$$(1 - \alpha)\omega > \frac{\sigma e^{\beta}}{e^{\beta} + \kappa}.$$

If $(1 - \alpha)\omega > \sigma$, we arrive at

$$(1 - \alpha)\omega > \sigma > \frac{\sigma e^{\beta}}{e^{\beta} + \kappa}.$$

Therefore, there exists at least one stationary equilibrium if both $(1 - \alpha)\omega > \sigma$ and $(\theta + \alpha - 1)\omega + \rho > 0$ hold. *Q.E.D.*

The first condition requires that the balanced growth rate with a maximum discount rate is positive. The second condition ensures that the utility function is bounded under a minimum discount rate. Both conditions are standard in economic growth theory, and they ensure the existence of a stationary equilibrium. There is the possibility of at most three stationary equilibria (Figure 1).

We now consider the stability of the stationary equilibrium. Figure 2 illustrates the dynamics of $x$, governed by[7]

$$\frac{\dot{x}_t}{x_t} = \frac{(1 - \alpha - \theta)\omega - \dfrac{\sigma}{1 + (\sigma/\rho - 1)\exp(-\beta x_t)}}{\theta} + \omega x_t. \tag{13}$$

The points at $x^L$, $x^M$, and $x^H$ in Figure 2 respectively correspond to points A, B, and C in Figure 1. As illustrated in Figure 2, the slope of equation (13) is positive at the stationary equilibrium with both $x^L$ and $x^H$. These equilibria are sources. At the stationary equilibrium with $x^M$, the slope of equation (13) is negative; it is a sink. Therefore, the existence of $x^M$ within a feasible range is sufficient for the indeterminacy of the equilibrium growth path. Formally, we establish the following proposition:

**Proposition 2.** *The equilibrium growth path is indeterminate if*

$$\kappa > 1 \textit{ and } \max\left(\log\kappa, \frac{4\theta\omega}{\sigma}\right) < \beta \approx \lambda \log\kappa,$$

*where*

$$\lambda \equiv \left[\frac{2\theta\omega}{2\omega(\theta + \alpha - 1) + \sigma}\right] > 1.$$

(Proof) An inflection point is defined as a point of a curve at which a change in the direction of curvature occurs. Thus, the inflection point satisfies $F'' = 0$. The level of $x$ at the inflection point is $x^F = \log\kappa/\beta$. If $\kappa < 1$, then $x^F \leq 0$ holds. Then, the stationary equilibrium is unique and corresponds to the point, $x^H$. The indeterminacy of the equilibrium growth path does not occur. Therefore, $\kappa > 1$ is necessary to trigger the indeterminacy of equilibrium growth path.

At the inflection point, the $R$ function satisfies

$$R\left(\frac{\log\kappa}{\beta}\right) = \frac{\theta + \alpha - 1}{\theta} + \frac{\sigma}{2\theta\omega} \text{ and } R'\left(\frac{\log\kappa}{\beta}\right) = \frac{\beta\sigma}{4\theta\omega}.$$

[7] Using the definition of $x$ and $\omega$, equation (9) can be written as equation (13).

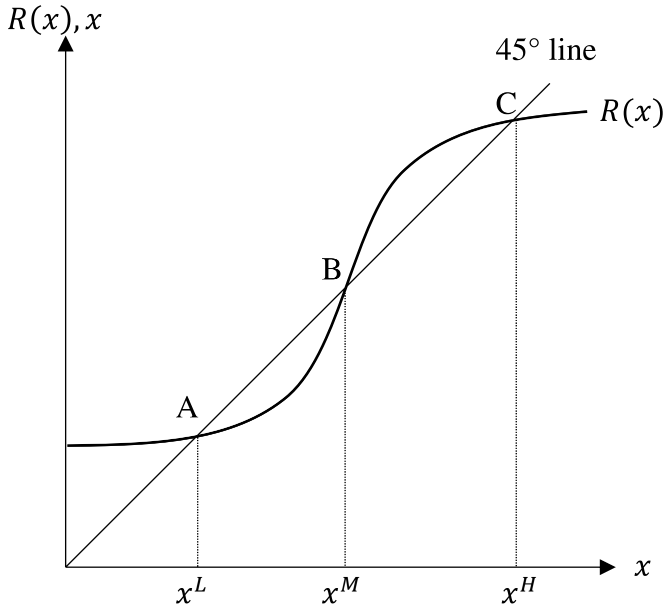


**Figure 1. Multiple stationary equilibria**

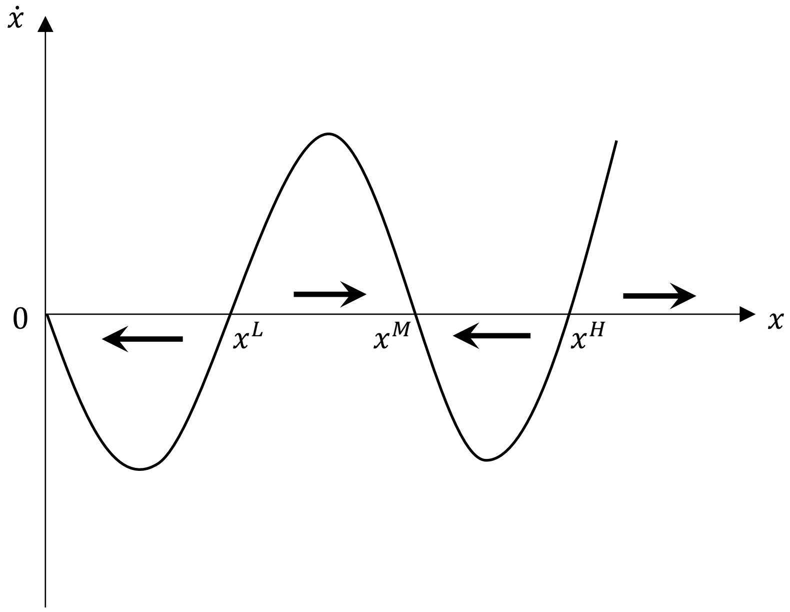


**Figure 2. Dynamic stability of the steady state**

The inflection point of $F$ is the fixed point, $R$, if and only if

$$\frac{\log \kappa}{\beta} = \frac{\theta + \alpha - 1}{\theta} + \frac{\sigma}{2\theta\omega}.$$

Note that, from the assumption, we have the following relation:

$$(1-\alpha)\omega > \frac{\sigma}{2} \Rightarrow \frac{\theta + \alpha - 1}{\theta} + \frac{\sigma}{2\theta\omega} < 1.$$

Therefore, the inflection point of $F$ is a feasible fixed point of $R$ if $\beta > \log \kappa$. The necessary and sufficient conditions that $x^F$ is the stationary equilibrium is

$$\frac{1}{\beta} = \frac{1}{\log \kappa}\left[\frac{\theta + \alpha - 1}{\theta} + \frac{\sigma}{2\theta\omega}\right] \Leftrightarrow \beta = \lambda \log \kappa.$$

The sufficient condition for $x^F = x^M$ is $R'(x^F) > 1$ (i.e., $\beta > 4\theta\omega$). If $\beta$ is sufficiently close to $\lambda \log \kappa$, the $R$ curve crosses the 45° line around $x^F$. Consequently, $x^M$ exists around $x^F$ if $\kappa > 1$ and $\max(\log \kappa, 4\theta\omega) < \beta \approx \lambda \log \kappa$. *Q.E.D.*

Equation (3) implies that households with high average propensities to consume are not patient compared with households with low average propensities to consume. In other words, as households become wealthier, they become more patient, whereas higher-consumption households tend to be impatient. Thus, people tend to concentrate on current consumption. The former provides a positive externality and the latter a negative one for households. The positive externality generates positive feedback; an increase in income makes people more patient. Therefore, they tend to prefer future consumption. Because it decreases current consumption and increases savings, capital accumulation will be enhanced. Consequently, the income increase is self-fulfilling.

In particular, under the specific form of equation (11), this positive feedback effect becomes strong when the slope of $R$ in Figure 1 is steeper than the 45° line. Recall that Figure 2 relates to Figure 1; points A and C are sources, and point B is a sink. In Proposition 2, the first condition, $\kappa > 1$, requires that a gap between the lower and upper bound of discount rate is sufficiently large to cause the indeterminacy. If the gap is small (large), non-linearity of $F$ will be potentially weakened (strengthened). The second condition assures that there exists the stationary equilibrium at which the gradient is larger than unity.

If $x^M$ is feasible, the equilibrium growth path is indeterminate. However, the stationary equilibrium, $x^M$, can be structurally unstable in the sense of being transitory. For instance, the partial differentiation of $R$ with respect to $\omega$ yields

$$\frac{\partial R}{\partial \omega} = -\frac{\sigma}{[1 + \kappa \exp(-\beta x)]\theta\omega^2} < 0.$$

A rise in $\omega$ moves the locus of $R$ downward. Because fiscal policy influences the value of $\omega$, a slight change in fiscal policy possibly removes $x^M$, and the economy will jump to a new stationary equilibrium and a uniquely determined equilibrium growth path. Therefore, a change in fiscal policy might eliminate (cause) the multiple stationary equilibrium and increase (decrease) the possibility of a uniquely determined equilibrium growth path.

The effect of a change in tax rate on net output-capital ratio is

$$\frac{d\omega}{d\tau} = \frac{(\alpha - \tau)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}-1}}{1-\alpha} \gtreqless 0 \Leftrightarrow \tau \lesseqgtr \alpha. \qquad (14)$$

Using equation (14), $dR/d\omega < 0$ and geometrical analysis, regarding the relation between tax rate and indeterminacy, the following proposition holds:

**Proposition 3.** *Suppose that the economy with $\tau < \alpha$ is initially on the stationary equilibrium as a sink. A rise in the income tax rate could possibly resolve the indeterminacy of equilibrium growth path.*

Proposition 3 implies that an economic policy of enhancing productivity contributes to the efficacy of a stabilization policy. Hence, an excess increase in productivity may reduce the economic growth rate and social welfare under an endogenous discount rate. To clarify this issue, we consider the compatibility of the growth-maximizing tax policy and economic stabilization. With a constant time preference rate, Barro (1990) demonstrated that the tax policy, $\tau = \alpha$, maximized the equilibrium growth rate and social welfare. Hereafter, we refer to $\tau = \alpha$ as the Barro tax rule.

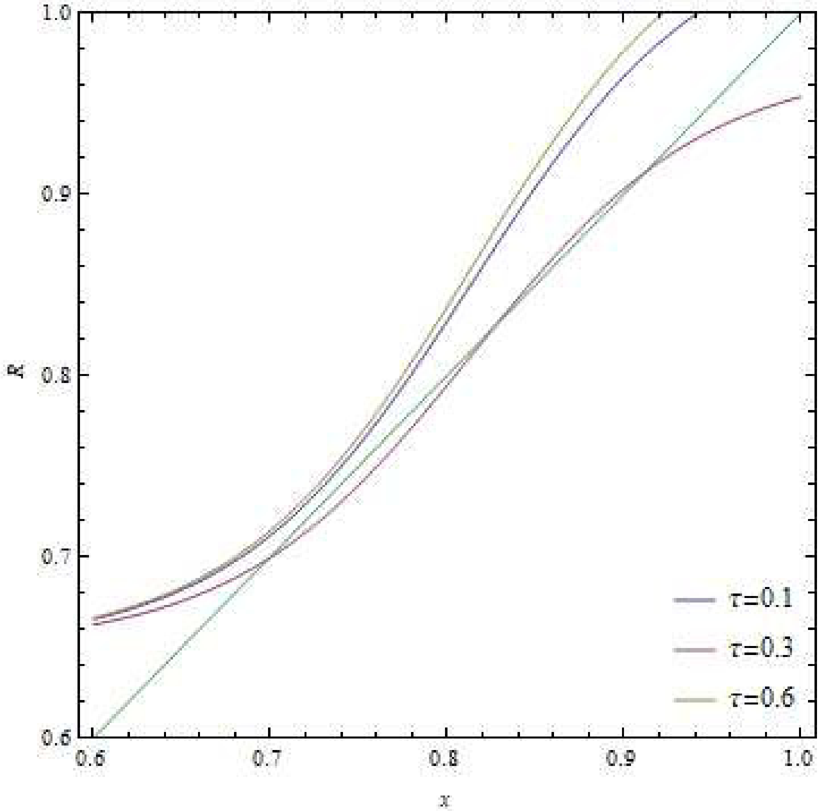


**Figure 3. The loci of** $R$ **and stationary equilibria if** $\beta = 15$

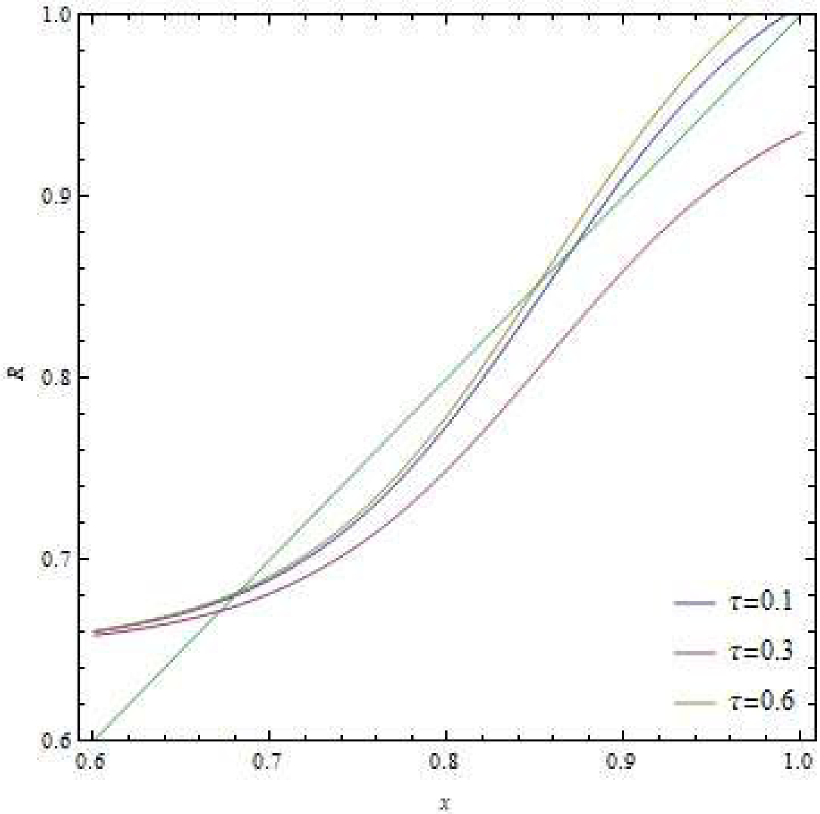


**Figure 4. The loci of** $R$ **and stationary equilibria if** $\beta = 14.25$

At a steady state, the effect of a change in the tax rate on the balanced growth rate is

$$\frac{d\gamma}{d\tau} = (1-x)\frac{d\omega}{d\tau} - \omega\frac{dx}{d\tau} = \left[1 - x - \omega\frac{dx}{d\omega}\right]\frac{d\omega}{d\tau}, \tag{15}$$

where the effect of $\omega$ on $x$ is given by

$$\frac{dx}{d\omega} = -\frac{\sigma}{\omega}\frac{[1+\kappa\exp(-\beta x)]}{[1+\kappa\exp(-\beta x)]^2\theta\omega - \beta\sigma\kappa\exp(-\beta x)}. \tag{16}$$

The sign on $dx/d\omega$ is important for holding the Barro tax rule, $\tau = \alpha$, as the growth-maximizing tax rate. If the denominator on the right-hand side of equation (16) is positive, $dx/d\omega < 0$ holds. Then, the term in square brackets on the right-hand side of equation (15) is positive. For instance

$$\theta \geq \phi\beta \Leftrightarrow \frac{dx}{d\omega} < 0, \tag{17}$$

where

$$\phi \equiv \frac{\kappa\exp(-\beta x_t)}{1+\kappa\exp(-\beta x_t)}\frac{D}{\omega} \in (0,1).$$

The sign on $d\gamma/d\tau$ coincides with the sign on $d\omega/d\tau$. From equations (2), (11), (15)–(17), we obtain

$$\operatorname{sgn}\frac{dV_0}{d\tau} = \operatorname{sgn}\frac{d\omega}{d\tau}. \tag{18}$$

Considering equations (14)–(18) and Propositions 2 and 3, the following proposition holds:

**Proposition 4.** *Suppose that the economy with $\tau \neq \alpha$ is initially on the stationary equilibrium as a sink. If $\theta \geq \phi\beta$, the Barro tax rule could possibly actualize stable balanced growth and maximize the equilibrium growth rate and social welfare.*

Proposition 4 implies that the government can actualize multiple tasks, such as stable high growth and welfare using the Barro tax rule if the stationary equilibrium with $x^M$ is *structurally* unstable. However, it is necessary to investigate compatibility between stabilization and growth- and welfare maximization within realistic parameter values.

*Example 1*. For holding Proposition 2, we provide a numerical analysis when the parameters are set to $\alpha = 0.3$, $A = 0.5$, $\theta = 2$, $\sigma = 0.1$, $\kappa = 2 \times 10^5$, and $\beta = 15$.[8] The analysis investigates three equilibria with different tax rates: $\tau = 0.1$, $\tau = 0.3$, and $\tau = 0.6$. Under these parameters, $\max(\log\kappa, 4\theta\omega) < \beta$ holds, and the values of $\lambda\log\kappa$ are respectively, 14.350, 15.050, and 14.203 for $\tau = 0.1$, $\tau = 0.3$, and $\tau = 0.6$. $\beta \approx \lambda\log\kappa$ seems to be satisfied if $\tau = 0.3$, but not if $\tau = 0.1$ and $\tau = 0.6$. The conditions in Proposition 4 are unsatisfied for all three cases.

Figure 3 illustrates the $R$ curves (solid sigmoid curve) and 45° line derived from the calibrated results. As in Figure 1, there are three stationary equilibria if $\tau = 0.3$. The stationary equilibrium with the smallest and largest values of $x$ are sources, whereas the stationary equilibrium with the middle value of $x$ is a sink. Therefore, Barro tax rule destabilizes the economy, and Propositions 3 and 4 do not hold (see Figure 3).

*Example 2*. We now set $\beta = 14.25$ and other parameter values from Example 1. Then, $\max(\log\kappa, 4\theta\omega) < \beta$ holds for $\tau = 0.1$, $\tau = 0.3$, and $\tau = 0.6$. The values of $\lambda\log\kappa$ are identical to those of Example 1. $\beta \approx \lambda\log\kappa$ seem to be satisfied if $\tau = 0.6$ and $\tau = 0.3$ The conditions in Proposition 4 are satisfied for each of $\tau = 0.1$, $\tau = 0.3$, and $\tau = 0.6$.

The loci of the $R$ curves are illustrated in Figure 4. For $\beta = 14.25$, the stationary

[8] The value of $\alpha$ has been widely used since Barro (1990). The value of $\theta$ is based on Ogaki and Reinhart (1998). We chose the values of $\kappa$ and $\sigma$ to keep the time preference rate in the range $(0,1]$ and covered the range between 0.01 and 0.03, which was usually assumed by previous studies. Finally, we set $A$ to be the realistic values of growth rate.

equilibrium is uniquely determined if $\tau = 0.3$. In contrast, the indeterminacy of equilibrium growth paths occurs for $\tau = 0.1$ or $\tau = 0.6$. A change in the tax rate from $\tau = 0.1$ or $\tau = 0.6$ to $\tau = 0.3$ eliminates endogenous economic fluctuation. The equilibrium growth rate and social welfare are maximized at $\tau = 0.3$.

Examples 1 and 2 show that the possibility of generating the indeterminacy of an equilibrium growth path is sensitive to slight changes in the non-linearity of time discount rate and tax rate.

# 4. Conclusion

This study examines the relationship between dynamic stability and fiscal policy using an endogenous growth model with productive government expenditures and a socially determined time preference rate. Following empirical studies from the literature, we assume that the time preference rate depends on consumption and income. Then, fiscal policy influences the dynamic stability, not only through marginal and average productivity of private capital but also via the time preference rate.

This study derives the sufficient conditions for the existence of multiple equilibrium paths and shows that deep parameters of the time preference function seriously affects the conditions for indeterminacy. In particular, the (i) non-linearity of the time preference function and the (ii) upper and lower bounds of the time preference rate were essential to produce instability in the dynamic equilibrium.

Furthermore, we investigate the compatibility between the Barro tax rule and economic stabilization. Under certain conditions, the Barro tax rule actualizes welfare maximization and stabilization of the dynamic equilibrium. Indeed, numerical analysis shows such a case. However, it also demonstrates the indeterminacy under the Barro tax rule. Hence, the government will fail to attain both welfare maximization and economic stabilization by adopting the Barro tax rule. Consequently, the compatibility of the Barro tax rule and economic stabilization is sensitive to shocks in parameters.

Finally, we would like to describe future directions of this research. In this paper, we did not explicitly treat labor supply or implicitly assume an exogenous supply of labor that is normalized to unity. However, labor supply affects household incomes and, therefore, the aggregate income. Furthermore, income tax influences the labor supply. Therefore, fiscal policy will involve the dynamic stability of a macroeconomic equilibrium through the channels delineated by this study as well as additional channels caused by incorporating an endogenous labor supply into the model. This paper provides an analytical basis for future research.

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