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Identifying horizon-based heterogeneity in the cross section of portfolio returns

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Abstract

I introduce an econometric framework to identify and estimate horizon-based heterogeneity in panel data. Using this approach, I identify the horizon-based structure in the cross section of portfolio returns. Accounting for this structure results in a significant improvement in pricing accuracy relative to the standard CAPM and Fama-French three-factor models. The majority of the improvement arises from separately pricing long-horizon and shorter-horizon market exposure.

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1 Introduction

Horizon-based relationships have a long history in economics and finance. Sticky prices, monetary neutrality, interest parity, and the yield curve all imply differential economic relationships over different time horizons. Much of the theoretical literature has explored relationships that operate over the business cycle – the ICAPM model of Merton (1973) features a state variable that describe time-varying investment opportunities while Campbell and Cochrane (1999) introduces a state variable that tracks the business cycle even more explicitly. Changing market exposure over the business cycle was one of the main motivating considerations in Jagannathan and Wang (1996) – which uses a yield spread that strongly forecasts the business cycle as an instrument for the conditional market risk premium. Bansal and Yaron (2004) represents a newer development in the theoretical literature that explores longer-horizon risk relationships.

More recently, the literature has focused on empirically identifying these horizon-based relationships. Adrian and Rosenberg (2008), Engle and Rangel (2009), Engle et al. (2013), and Rangel and Engle (2012) all model volatility risk factors over short and long horizons. A variety of authors have also experimented with sampling frequencies to capture horizon-based heterogeneity in asset pricing models, e.g. Andersen et al. (2005), Harvey (1989), Cochrane (1996), Jagannathan and Wang (1996), Ghysels et al. (2007) among others. A subset of this empirical literature has utilized wavelet-based methods to investigate horizon-based relationships in asset pricing (Gençay et al., 2001a, 2003, 2005; In and Kim, 2012; Ortu et al., 2013)

I propose a method of identifying horizon-based structure in a panel setting. My approach uses a multiresolution analysis associated with a Haar maximal overlap discrete wavelet transform to decompose a regressor variable – an approach most closely related to Ortu et al. (2013). I then propose a sequential test based on the cross-sectional mean of Wald statistics to identify patterns of horizon-based heterogeneity. I apply this framework to excess market returns in an extension to the standard CAPM asset pricing model. The panel test identifies five horizon-based components of excess market returns in the cross section of 100 portfolios sorted on size and book-to-market ratio. I find that separately pricing the horizon-based market exposure leads to an order of magnitude improvement over the standard CAPM market model using a variety of performance measures. The largest improvement in pricing performance arises from separately pricing long-horizon market exposure (horizons 10 years and longer) and market exposure over shorter horizons (under 10 years).

Although I apply this horizon-based framework to financial data, I note that my approach is relevant for any large T panel in which the regression relationship is suspected to vary across time horizon, e.g. when short-run, business-cycle, and long-run effects differ.

2 Methodology and Data

The canonical CAPM model specifies a single source of risk – exposure to the market – captured by β_i in Equation (1) below:

$$r_{it} - r_{ft} = \beta_i \left(r_{mt} - r_{ft} \right) + \epsilon_{it} \tag{1}$$

where r_{mt} denotes the return on the market, r_{ft} denotes the risk-free rate, and i indexes a portfolio. $r_{mt} - r_{ft}$ and $r_{it} - r_{ft}$ indicate excess market returns and excess portfolio returns, respectively. I extend this simple model by allowing the estimated exposure to the market to vary across time horizons. To do so, I propose an additive decomposition of excess market returns

$$r_{mt} - r_{ft} = \mathcal{R}_{1t} + \mathcal{R}_{2t} + \cdots + \mathcal{R}_{kt} \tag{2}$$

where $\mathcal{R}_{jt}$ captures the behavior of excess market returns over a particular time horizon. The decomposition is achieved through a wavelet-based method that corresponds to a partitioning of the frequency domain. The frequency bands associated the partition yield a *horizon-based* interpretation of the components in the time domain. I then allow the coefficient in Equation (1) to vary across components:

$$r_{it} - r_{ft} = \sum_{j=1}^{k} \beta_{ij} \mathcal{R}_{jt} + \epsilon_{it} \tag{3}$$

The estimated coefficients have the same interpretation as in the standard CAPM market model – portfolio exposure to the market – but now that relationship is described by horizon, e.g. portfolio exposure to the market over the business cycle, etc. Of note, this approach to horizon-based inference nests the standard time series regression with the simple restriction that $\beta_{i1} = \beta_{i2} = \cdots = \beta_{ik}$.

I accomplish the decomposition in (2) using a Daubechies-class maximal overlap discrete wavelet transform (MODWT). I provide an overview of the decomposition below with sufficient detail to allow for easy replication. However, for brevity I refer the reader to Percival and Walden (2000) and Gençay et al. (2001b) for additional technical details.

At their core, wavelet transforms can be understood as a set of related bandpass filters. This set of bandpass filters is constructed such that the filter outputs decompose the variance of a time series across the time horizons associated with the individual filters. A multiresolution analysis (MRA) associated with a wavelet transform is a similar set of bandpass filters but with an additive decomposition in the time series rather than the variance. In general, the impulse response sequences of wavelet transforms and their associated MRA do not have closed-form analytical expressions. However, for expository clarity and reproducibility, I use the MRA from the simplest MODWT wavelet – the Haar wavelet[1] – which does indeed have the following closed form expression for the jth component part $\mathcal{R}_{jt}$ from (2):

$$\mathcal{R}_{jt} =: \sum_{\ell=-2^j+1}^{2^j-1} w_{j,\ell} \left(r_{m,t-\ell} - r_{f,t-\ell}\right) \tag{4}$$

where for $j < k$:

$$w_{j,\ell} = \begin{cases} \frac{2^j - 3|\ell|}{4^j} & |\ell| \leq 2^{j-1} \\ \frac{|\ell| - 2^j}{4^j} & 2^{j-1} < |\ell| \leq 2^j - 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

and for $j = k$:

$$w_{k,\ell} = \begin{cases} \frac{2^j - |\ell|}{4^j} & |\ell| \leq 2^j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Due to the bandpass nature of the filtering operations, the $j < k$ component describes the time domain behavior of the original series over horizons between 2^{j-1} and 2^j sample units (e.g. days, months, etc.). The kth component – the output of a low pass filter – captures long horizon behavior greater than 2^k units. While k is flexible in practice, it is clear from Equations (4) - (6), the length of impulse response sequence for the kth filter is exponential in k, leading to practical, data-determined constraints on k. Furthermore, the choice of k determines the time horizons described by the last component. In this paper I work with $k = 8$, a choice informed by the longest NBER-dated business cycle (for monthly data $2^8 = 128$ months). I present the impulse response sequences for this $k = 8$ component decomposition in Figure 1.

2.1 A Panel Test for Horizon-based Structure

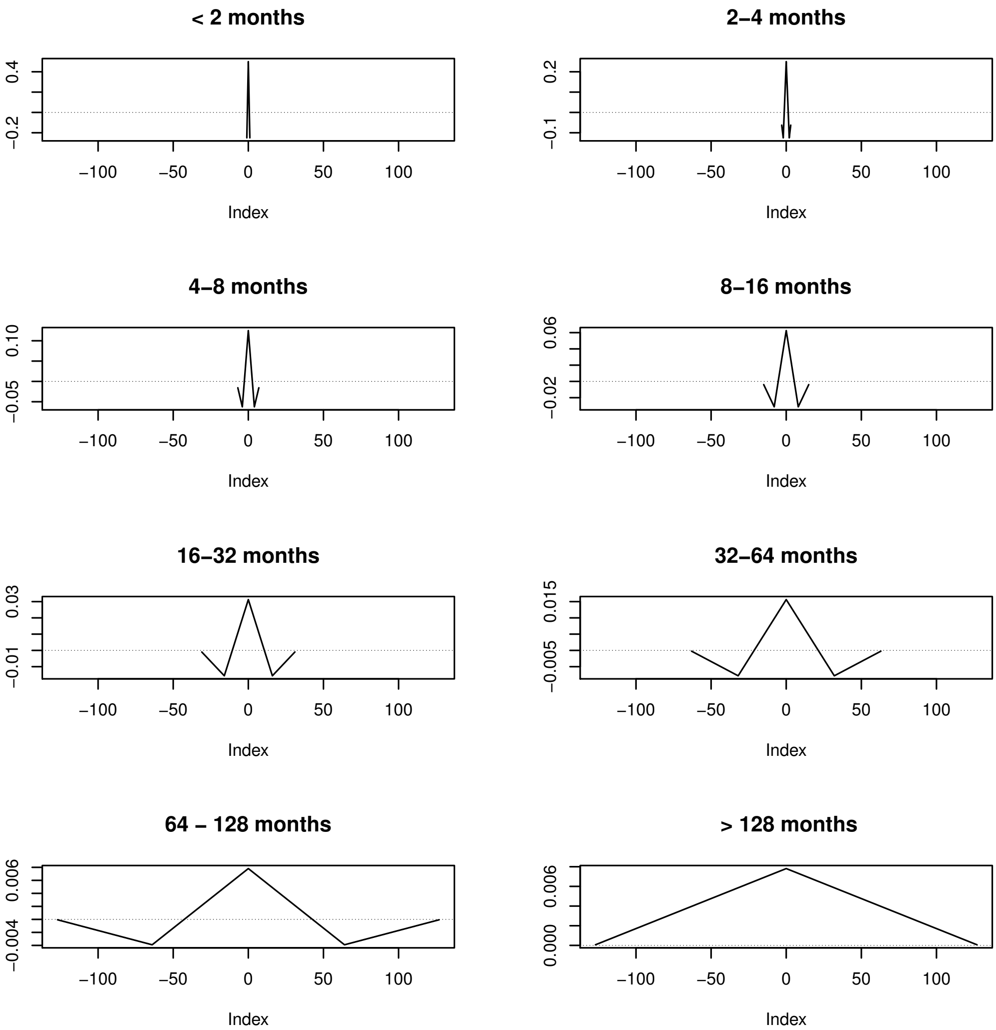
I propose a simple testing procedure to identify horizon-based structure in the cross section of portfolio returns. While this test is presented in the context of cross-sectional asset pricing, it is applicable in more general panel settings in which horizon-based structure is assumed common but unobserved in the panel cross section. This test can be viewed as a model selection procedure with a preference for parsimony – I only decompose the regressor insofar as there is significant evidence that the regression relationship differs across horizons. If there is not sufficient evidence for heterogeneity across horizon, the regressor will not be decomposed and the standard, single factor model will obtain.

The testing procedure is formulated as follows: First, the unrestricted model (3) is estimated for each portfolio i. Individual Wald statistics W_{iT} for the test $\beta_{i1} = \beta_{i2}$ are formed. I then compute the cross-sectional average $\overline{W}_{nT} = n^{-1} \sum_{i=1}^{n} W_{iT}$ of these Wald statistics where

$$\sqrt{\frac{n}{2}} \left(\overline{W}_{nT} - 1\right) \xrightarrow{d} N(0,1) \tag{7}$$

[1]The Haar wavelet is the crudest of the so-called Daubechies-class of discrete wavelets – a popular class of discrete wavelets that yield asymptotically uncorrelated components. The simplicity and closed-form impulse response sequences come at the expense of poor frequency domain resolution. More complicated Daubechies-class wavelets have better frequency domain resolution but are only analytically defined in the frequency domain. See Percival and Walden (2000) for details.

Figure 1: Haar Multiresolution Impulse Response Sequences



This figure presents the impulse response sequences for the multiresolution analysis associated with a $k = 8$ component Haar maximal overlap discrete wavelet transform (MODWT). The time series decomposition is achieved through standard filtering operations – a convolution of the impulse response sequences above and the time series being decomposed.

as length of the time series $T \to \infty$ and then cross-sectional size $n \to \infty$. The asymptotic behavior of this statistic follows from the fact that it is the mean of n $\chi^2(1)$ statistics. Similar panel tests have been utilized to identify noncausality (Dumitrescu and Hurlin, 2012) and stationarity (So et al., 2003). This statistic tests the null that $\beta_{i1} = \beta_{i2}$ under the assumption of cross-sectional homogeneity of scale structure, i.e. that all assets or portfolios have the same structure. While the assumption might easily be violated, it is necessary to meaningfully proceed in a cross-sectional asset pricing study. If the null is not rejected, a new regression is estimated under the enforced null that $\beta_{i1} = \beta_{i2}$, i.e. $\mathcal{R}_{1t}$ and $\mathcal{R}_{2t}$ from Equation (2) are recombined to form a single component of excess market returns. The procedure above is repeated for the test that the coefficient on $\mathcal{R}_{1t} + \mathcal{R}_{2t}$ is equal to the coefficient on $\mathcal{R}_{3t}$. If instead, the test that $\beta_{i1} = \beta_{i2}$ is rejected, then I proceed in testing whether $\beta_{i2} = \beta_{i3}$. This procedure is sequentially iterated over all components in decomposition (2).

2.2 Pricing Horizon-based Structure in the Cross Section

I price the cross section of portfolio returns using the Fama-MacBeth procedure (Fama and MacBeth, 1973; Cochrane, 2005), which estimates the following cross sectional regression for each month in my sample:

$$r_{it} - r_{ft} = \alpha + \sum_{j=1}^{8} \gamma_j \hat{\beta}_{ji} + u_{it} \tag{8}$$

where $\hat{\beta}_{ji}$ is the ith portfolio's estimated exposure to the market over 2^{j-1} to 2^j month horizons for $j < k$ and for $j = k$ the estimated exposure over horizons longer than 2^j months.

I apply this approach to monthly returns on the Fama-French 100 size and book-to-market sorted portfolios[2] from January 1948 to June 2018. Of note, this sample yields appropriate asymptotic behavior of the test statistics in Equation (7) as $T = 654$ and $n = 100$.

3 Results

Table 1 presents estimates of Equation (3). For brevity, only five of the 100 portfolios are presented (unreported portfolios are qualitatively similar). Using the iterated testing procedure outlined above, I identify five components of excess market returns that characterize the horizon-based structure in the cross section of the 100 size/BM portfolios. The first component ($\mathcal{R}_1$) corresponds to very short horizons – under two months. The second component aggregates $\mathcal{R}_2$ and $\mathcal{R}_3$ and corresponds to horizons between two and eight months. The third component aggregates $\mathcal{R}_4$, $\mathcal{R}_5$, and $\mathcal{R}_6$ and corresponds to horizons between eight and 64 months. Notably, this component captures standard business cycle fluctuations – the average NBER-dated business cycle contraction was 11 months over the sample while the average NBER-dated expansion was 58 months. The final two components identified are $\mathcal{R}_7$ (corresponding to horizons between 64 and 128 months) and $\mathcal{R}_8$ (corresponding to "long horizons" greater than 128 months), respectively. Figure 2 provides a graphical representation of these components.

Having identified the relevant horizon-based structure, I proceed in pricing the cross section using a Fama-Macbeth procedure. For comparison, I also perform the same Fama-Macbeth procedure using two benchmark asset pricing models – the standard CAPM market model and the Fama-French three factor model. Table 2 presents several measures of model fit across the three models. The horizon-based model provides a rather astonishing *order of magnitude* reduction in the sum of squared pricing errors over the Fama-French three factor model. Similarly, the cross-sectional naive R^2 – defined as the ratio of the cross-sectional variance of (time) average fitted returns to the cross-sectional variance of the (time) average realized returns – nearly doubles, with approximately 95% of the cross-sectional variation in excess portfolio returns being captured by the horizon-based approach. Another asset-pricing-specific metric for model performance is the estimated intercept from the cross-sectional regressions α– the expected excess portfolio returns unexplained by the model. Table 2 demonstrates that the (time) average intercept for both the CAPM market model and the Fama-French three factor model are significantly positive. In contrast, the horizon-based model has an insignificant average intercept of -0.01. This stark improvement is visually summarized in Figure 3 – with each point representing the time average of realized to average fitted excess returns for the 100 size/BM portfolios.

One possible source of the dramatic improvement in pricing performance is the increased flexibility of the horizon-based model due to additional parameters. In an attempt to isolate the pricing improvements

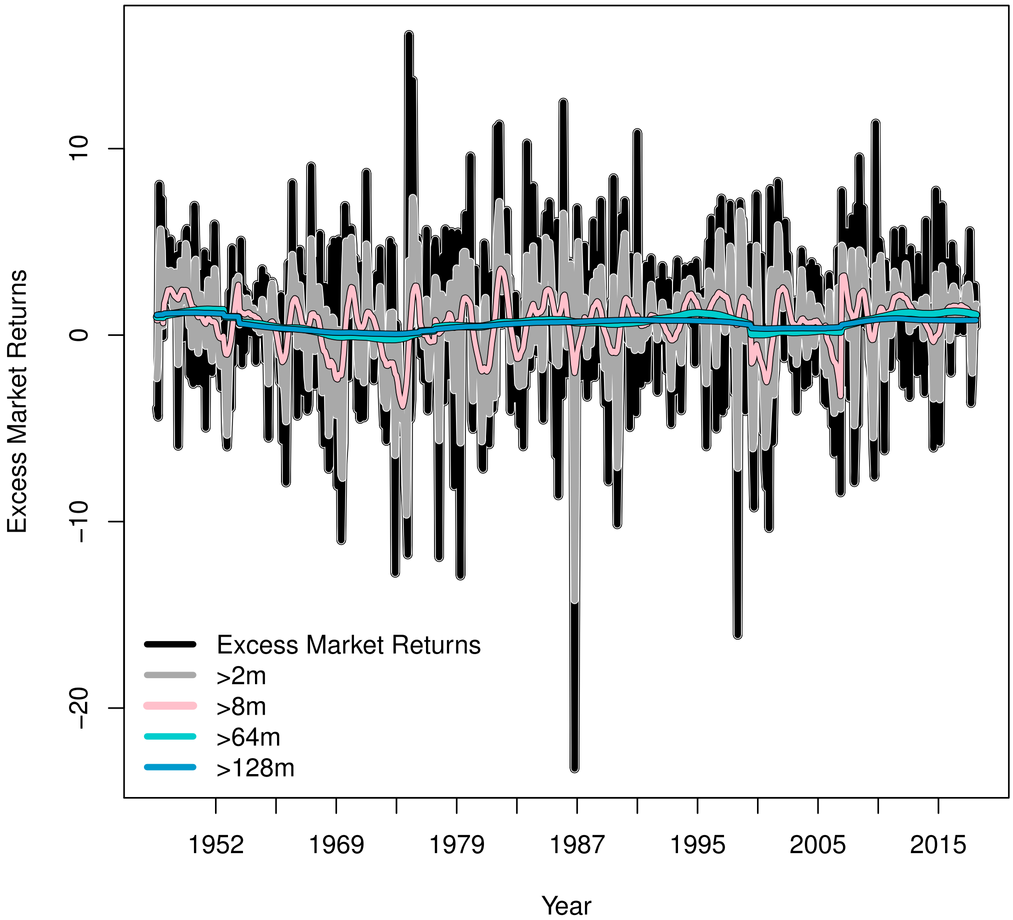
[2] Available at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

Table 1: Time Series Regressions for Selected Portfolios

	Portfolio Excess Returns:				
	ME4.BM5	ME8.BM5	ME6.BM9	ME6.BM5	ME10.BM8
	(1)	(2)	(3)	(4)	(5)
<2m	1.001***	1.027***	1.024***	0.986***	0.997***
	(0.053)	(0.045)	(0.062)	(0.046)	(0.053)
2-4m	1.187***	1.060***	1.279***	1.064***	0.719***
	(0.104)	(0.087)	(0.121)	(0.089)	(0.103)
4-8m	1.493***	1.032***	1.199***	1.377***	0.903***
	(0.182)	(0.151)	(0.211)	(0.156)	(0.179)
8-16m	0.924***	1.243***	0.693**	1.040***	0.960***
	(0.257)	(0.215)	(0.299)	(0.221)	(0.254)
16-32m	1.099***	0.630**	1.212***	0.714**	0.452
	(0.336)	(0.280)	(0.391)	(0.289)	(0.332)
32-64m	1.278**	0.918*	0.996	1.013**	1.492***
	(0.571)	(0.476)	(0.664)	(0.491)	(0.563)
64-128m	−0.223	−1.272*	−0.529	−0.020	1.237
	(0.897)	(0.747)	(1.041)	(0.771)	(0.884)
>128m	1.316***	1.394***	1.340***	1.107***	0.945***
	(0.158)	(0.131)	(0.183)	(0.136)	(0.155)
Observations	684	684	684	684	684
R^2	0.716	0.758	0.643	0.750	0.625

This table presents regressions of excess portfolio returns on the full decomposition of excess market returns. Portfolios are indicated by the quintile of size (ME) and book-market-ratio (BM) – e.g. ME4.BM5 is the portfolio corresponding to the 4th size quintile and 5th BM quintile. Horizon-based components are denoted by their associated horizons. Results are presented for illustration purposes from five of the 100 size/BM sorted portfolios. Alternating gray and white shading is used to indicate components grouped together by the iterated cross sectional testing procedure. Standard errors are presented in parentheses and statistical significance is denoted by: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Figure 2: Components of Excess Market Returns



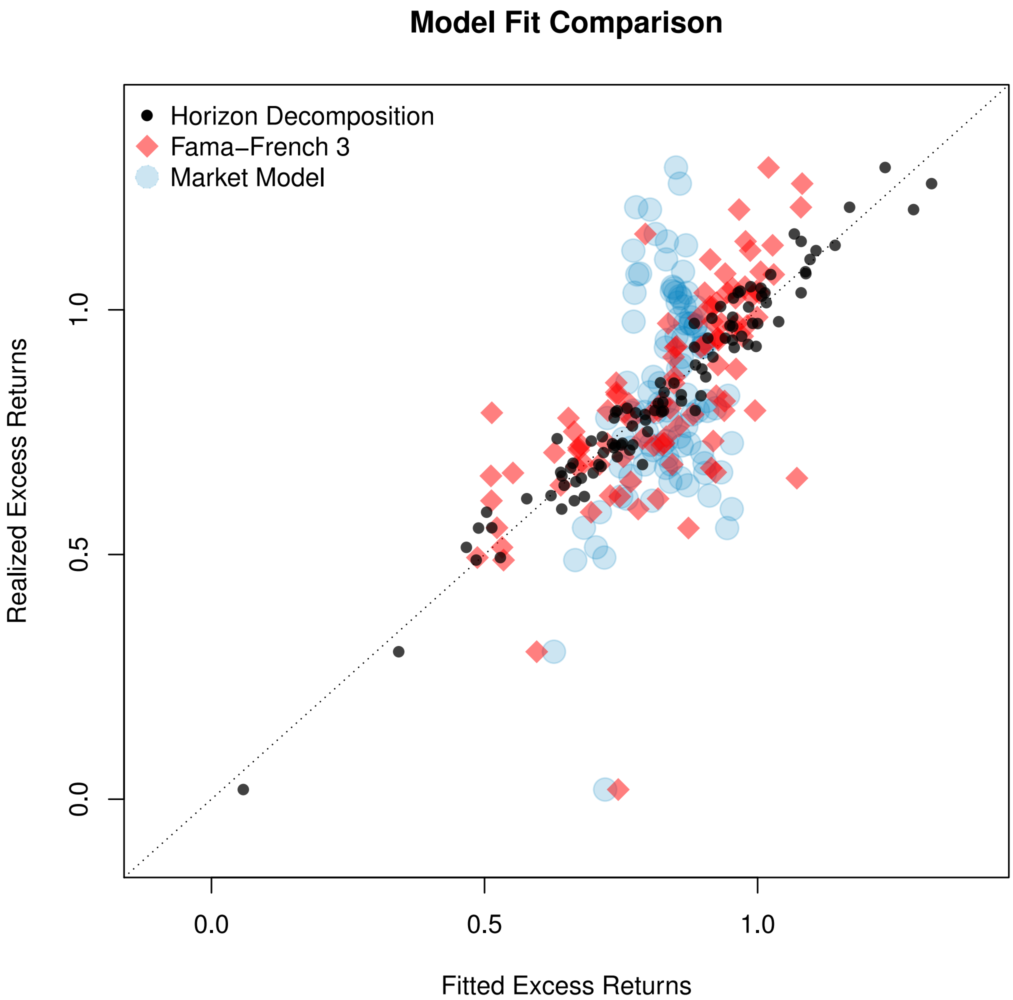
This figure plots components of excess market returns. To aid in visual interpretation, aggregates of the individual components are plotted rather than the components themselves. The black line is the original undecomposed time series. The gray line plots the time series less the first component $\mathcal{R}_1$ (or equivalently, the sum of the second through the last). The pink line plots the time series less the first three components $\mathcal{R}_1$, $\mathcal{R}_2$, and $\mathcal{R}_3$ (or equivalently, the sum of the fourth through the last). The cyan line plots the time series less the first six components $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$, $\mathcal{R}_4$,$\mathcal{R}_5$, and $\mathcal{R}_6$ (or equivalently, the sum of the last two). Finally, the blue line plots the last component.

Table 2: Model Comparison Metrics

	Model:		
	Horizon	FF3	Market
SSPE	0.193	2.102	3.787
R^2	0.954	0.498	0.096
Adj R^2	0.951	0.477	0.077
Mean $\widehat{\alpha}$	-0.011	1.141***	1.359***
	(0.254)	(0.236)	(0.289)
	(0.302)†	(0.546)†	(0.640)†

This table present various measures of model fit for the proposed horizon-based model, the Fama-French 3 factor model, and the standard CAPM market model. SSPE is the sum of squared pricing errors, defined as the cross-sectional sum of squared differences in the times series average of fitted and realized returns. R^2 is defined as the ratio of the cross-sectional variance of (time) average fitted returns to the cross-sectional variance of the (time) average realized returns. The adjusted R^2 corrects for the number of parameters estimated in the cross section. The mean $\widehat{\alpha}_i$ is the (time) average estimated intercept from the cross sectional regressions. Fama-Macbeth standard errors are reported below in parentheses. † Indicates Shanken-corrected standard errors (Shanken, 1992) that control for errors in the estimated regressors. Statistical significance based on the uncorrected errors is denoted by: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Inference based on the Shanken-corrected errors is qualitatively similar – the mean $\hat{\alpha}$ from the Horizon model is not significant. The mean $\hat{\alpha}$ from the Fama-French and Market models are statistically different than zero at the 5% level (rather than the 1% level).

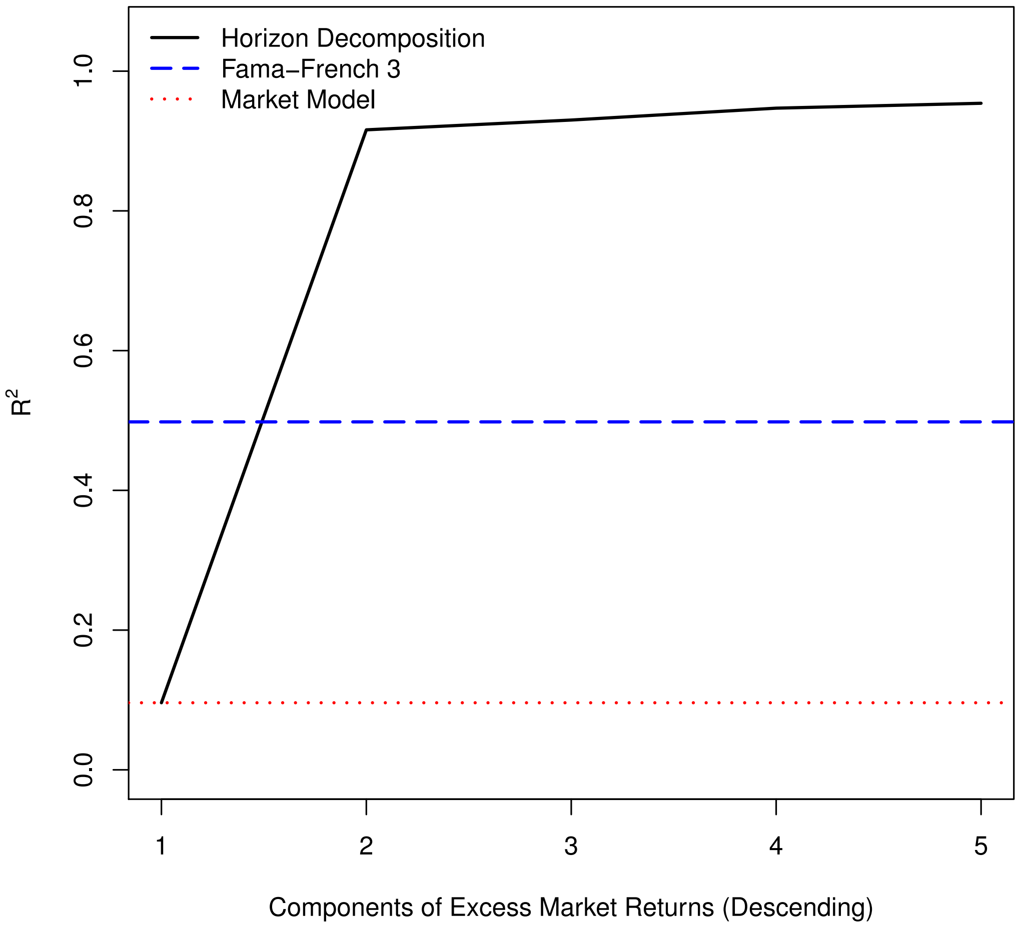
Figure 3: Model Performance Comparison



This figure plots the time series average of fitted excess portfolio returns from three models against the time series average of realized excess portfolio returns.

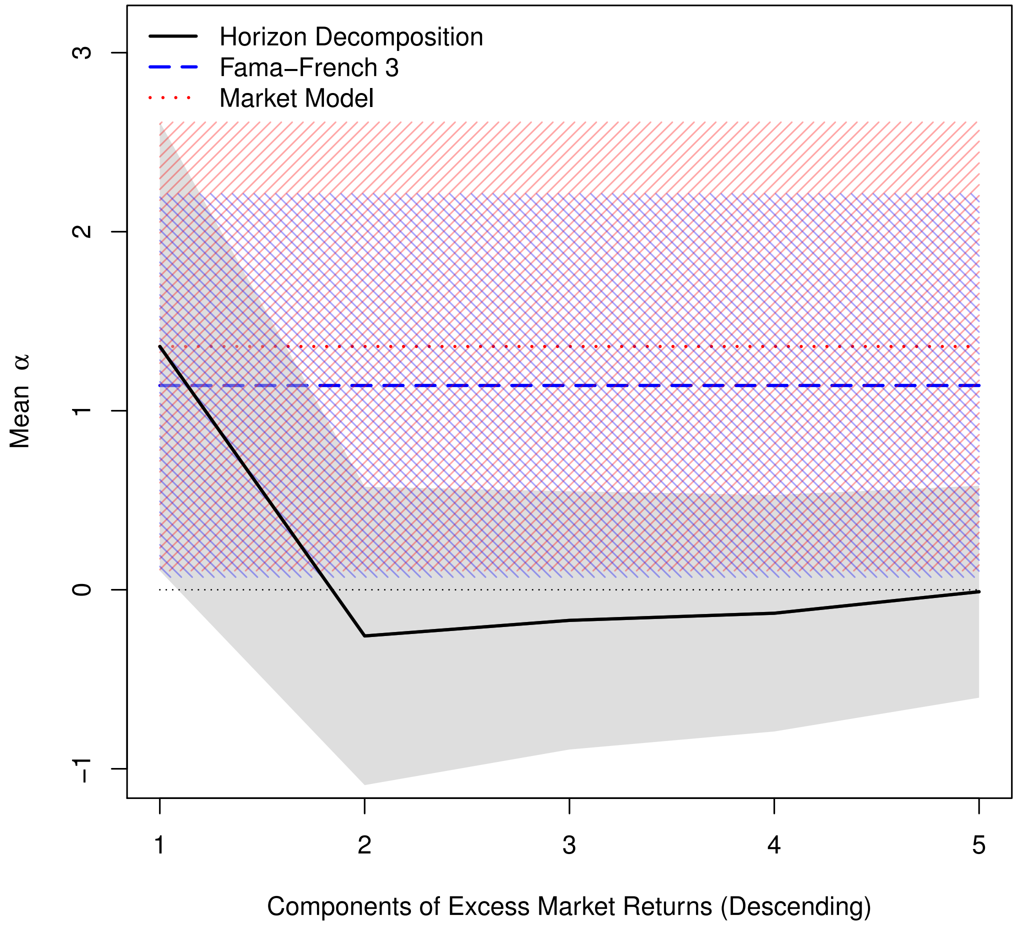
(and rule out additional parameters as the drivers of pricing performance), I sequentially re-estimate the Fama-Macbeth procedure, each time increasing the extent to which excess market returns are decomposed. As a baseline, I consider a single component of excess market returns which corresponds to the CAPM market model. I then decompose excess market returns into two components: a long-horizon component $\mathcal{R}_8$ corresponding to horizons greater than 128 months, and a shorter-horizon component $\sum_{i=1}^{7} \mathcal{R}_i$ which corresponds to horizons less than 128 months. I then consider three components of excess market returns – $\mathcal{R}_8$, $\mathcal{R}_7$, and $\sum_{i=1}^{6} \mathcal{R}_i$ (>128 months, 64-128 months, and <64 months, respectively) – four components of excess market returns – $\mathcal{R}_8$, $\mathcal{R}_7$, $\sum_{i=4}^{6} \mathcal{R}_i$, and $\sum_{i=1}^{3} \mathcal{R}_i$ (>128 months, 64-128 months, 8-64 months, and <8 months, respectively) – and five components of excess market returns which corresponds to the model identified by my iterated testing procedure – $\mathcal{R}_8$, $\mathcal{R}_7$, $\sum_{i=4}^{6} \mathcal{R}_i$, $\sum_{i=2}^{3} \mathcal{R}_i$, and $\mathcal{R}_1$ (>128 months, 64-128 months, 8-64 months, 2-8 months, and <2 months, respectively). Figure 4 demonstrates that the vast majority of improved fit comes from pricing long-run and short-run market exposure separately, with the cross-sectional R^2 jumping from 9.6% for a single component to 91.6% for two components. Clearly the improved performance is not solely arising from additional parameters as this two-factor model significantly outperforms the Fama-French three factor model with a cross-sectional R^2 of 49.8%. I document similar behavior for the mean intercept in Figure 5. The mean intercept is no longer differentiable from zero with the separate pricing of long-horizon market exposure. The graph further demonstrates that the point estimate of the average intercept converges to zero with the inclusion of the remaining horizon-based market exposure factors. In sum, I interpret these findings as evidence that the improved performance of the horizon-based model is arising from accounting for an underlying horizon-based structure in the cross section of returns, rather than simply due to improved fit from additional model parameters.

Figure 4: Diagnosing Model Performance – R^2



This figure plots R^2 as components of excess market returns are sequenctially decomposed from longest horizon to shortest horizon. One component corresponds to undecomposed excess market returns which is equivalent to the standard market model. Two components corresponds to $\mathcal{R}_8$ (associated with horizons $>$ 128 months) and the sum of the remaining components (associated with horizons $<$ 128 months). Three components corresponds to $\mathcal{R}_8$ (associated with horizons $>$ 128 months), $\mathcal{R}_7$ (associated with 64-128 month horizons), and the sum of the remaining components (associated with horizons $<$ 64 months). Four components corresponds to $\mathcal{R}_8$, $\mathcal{R}_7$, the sum of $\mathcal{R}_4$-$\mathcal{R}_6$ (associated with 8-64 month horizons), and the sum of the remaining components (associated with horizons $<$ 8 months). Finally, five components corresponds the preferred model selected by the iterated testing procedure: a component associated with horizons $>$ 128 months, a component associated with 64-128 month horizons, a component associated with 8-64 month horizons, a component associated 2-8 month horizons, and a component associated with horizons $<$ 2 months. For comparison, R^2 for the Fama-French 3 factor model and the standard CAPM market model are included.

Figure 5: Diagnosing Model Performance – Mean α



This figure plots the cross sectional mean α as components of excess market returns are sequentially decomposed from longest horizon to shortest horizon. 95% asymptotic confidence intervals based on Shanken-corrected (EIV-corrected) errors are included as well. One component corresponds to undecomposed excess market returns which is equivalent to the standard market model. Two components corresponds to $\mathcal{R}_8$ (associated with horizons $>$ 128 months) and the sum of the remaining components (associated with horizons $<$ 128 months). Three components corresponds to $\mathcal{R}_8$ (associated with horizons $>$ 128 months), $\mathcal{R}_7$ (associated with horizons 64-128 months), and the sum of the remaining components (associated with horizons $<$ 64 months). Four components corresponds to $\mathcal{R}_8$, $\mathcal{R}_7$, the sum of $\mathcal{R}_4$-$\mathcal{R}_6$ (associated with horizons 8-64 months), and the sum of the remaining components (associated with horizons $<$ 8 months). Finally, five components corresponds the preferred model selected by the iterated testing procedure: a component associated with horizons $>$ 128 months, a component associated with horizons 64-128 months, a component associated with 8-64 months, a component associated 2-8 months, and a component associated with $<$ 2 months. For reference, the mean α for the Fama-French 3 factor model and the standard CAPM market model are included.

References

Adrian, T. and Rosenberg, J. (2008). Stock returns and volatility: Pricing the short-run and long-run components of market risk. *Journal of Finance*, 63(6):2997–3030.

Andersen, T. G., Bollerslev, T., Diebold, F. X., and Wu, J. (2005). A framework for exploring the macroeconomic determinants of systematic risk. *American Economic Review*, 95(2):398–404.

Bansal, R. and Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4):1481–1509.

Campbell, J. Y. and Cochrane, J. (1999). Force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2):205–251.

Cochrane, J. H. (1996). A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy*, 104(3):572–621.

Cochrane, J. H. (2005). *Asset Pricing: Revised Edition*. Princeton University Press.

Dumitrescu, E.-I. and Hurlin, C. (2012). Testing for granger non-causality in heterogeneous panels. *Economic Modelling*, 29(1):1450–1460.

Engle, R. F., Ghysels, E., and Sohn, B. (2013). Stock market volatility and macroeconomic fundamentals. *Review of Economics and Statistics*, 95(3):776—-797.

Engle, R. F. and Rangel, J. G. (2009). High and low frequency correlations in global equity markets. Working Papers 2009-17, Banco de México.

Fama, E. F. and MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy*, 81(3):607–36.

Gençay, R., Selçuk, F., and Whitcher, B. (2001a). Scaling properties of foreign exchange volatility. *Physica A: Statistical Mechanics and its Applications*, 289(1):249–266.

Gençay, R., Selçuk, F., and Whitcher, B. (2003). Systematic risk and timescales. *Quantitative Finance*, 3(2):108–116.

Gençay, R., Selçuk, F., and Whitcher, B. (2005). Multiscale systematic risk. *Journal of International Money and Finance*, 24(1):55–70.

Gençay, R., Selçuk, F., and Whitcher, B. J. (2001b). *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*. Academic press.

Ghysels, E., Sinko, A., and Valkanov, R. (2007). Midas regressions: Further results and new directions. *Econometric Reviews*, 26:53–90.

Harvey, C. R. (1989). Time-varying conditional covariances in tests of asset pricing models. *Journal of Financial Economics*, 24(2):289–317.

In, F. and Kim, S. (2012). *An Introduction to Wavelet Theory in Finance: A Wavelet Multiscale Approach*. World Scientific Publishing Company.

Jagannathan, R. and Wang, Z. (1996). The conditional capm and the cross-section of expected returns. *Journal of Finance*, 51(1):3–53.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica*, 41(5):867–87.

Ortu, F., Tamoni, A., and Tebaldi, C. (2013). Long-run risk and the persistence of consumption shocks. *The Review of Financial Studies*, 26(11):2876–2915.

Percival, D. B. and Walden, A. T. (2000). *Wavelet Methods for Time Series Analysis*, volume 4. Cambridge University Press.

Rangel, J. G. and Engle, R. F. (2012). The factor–spline–garch model for high and low frequency correlations. *Journal of Business & Economic Statistics*, 30(1):109–124.

Shanken, J. (1992). On the estimation of beta-pricing models. *The Review of Financial Studies*, 5(1):1–33.

So, K., Pesaran, M. H., and Shin, Y. (2003). Testing for unit roots in heterogeneous panels. *The Journal of Econometrics*, 115(1):53–74.