Economics Bulletin

Volume 39, Issue 2

The systematic risk of gold at different time-scales

Antonis A Michis
Central Bank of Cyprus

Abstract

Gold is frequently cited by investors as a financial asset that can be associated with a negative beta coefficient. I investigate this hypothesis by estimating the beta coefficient of gold at different time-scales and examining the associated implications for investors with different planning horizons. Estimation is performed using maximal overlap discrete wavelet transforms of gold and stock market returns in four major currencies. The results suggest that gold tends to be associated with a negative beta coefficient when considering long-term investment horizons, and this finding is consistent across markets and currencies.

Citation: Antonis A Michis, (2019) "The systematic risk of gold at different time-scales", *Economics Bulletin*, Volume 39, Issue 2, pages 1215-1227
Contact: Antonis A Michis - AntonisMichis@centralbank.cy.
Submitted: July 04, 2018. **Published:** May 15, 2019.

1. Introduction

Gold is frequently considered a hedge or safe haven asset that can improve portfolio performance in times of financial distress, as demonstrated for example in the studies by Baur and McDermott (2010), Baur and Lucey (2010), Joy (2011) and Scott-Ram (2002, p. 137). It is also frequently cited in the finance literature as an asset that can be associated with a negative beta coefficient (Ehrhardt and Brigham, 2011, p. 249).

Since the systematic risk of gold is important for investors, the purpose of this study is to estimate the beta coefficient of gold at different time-scales using a maximal overlap discreet wavelet transform (MODWT) beta estimator. Wavelet transforms are localized in both the time and the frequency domain, which enables the examination of the systematic risk of an asset at different frequency bands (or time-scales). This time-scale decomposition analysis is important when considering investments in gold, since gold is frequently considered as an effective portfolio diversifier over long-term time horizons. It is not possible to obtain similar insights when working with only the observed sampling rates of financial data.

The analysis was performed using gold and stock market returns covering the period September 1995 - August 2018. Particular emphasis was placed on the 2007 - 2008 financial crisis, when there was renewed interest in the use of gold as an investment asset (World Gold Council, 2018). The results based on four major currencies and twenty three years of market data suggest that gold tends to be associated with a negative beta coefficient when considering long-term investment horizons.

2. Multiscale estimation of beta

Gencay et. al. (2005) first proposed a wavelet method to estimate the beta coefficient of stocks at different time-scales, which is consistent with the multi-period version of the Capital Asset Pricing Model (CAPM). In this section a MODWT version of the beta estimator is proposed, which bares some desirable properties and is particularly useful for analysing financial time series like gold returns.

Wavelet analysis is based on new families of orthonormal bases functions that enable the breakdown of time series (or signals) into different time-scales. They are particularly suited for the study of financial time series, since such series usually consist of multiple frequencies, determined by the activities of numerous market participants operating over different time horizons (Masset, 2015).

Using a mother wavelet basis function ψ, a set of wavelets $\psi_{j,k}$ can be obtained by dilating (expansion in the range) and translating (shift in the range) the basis function as follows (see, In and Kim, 2013, p. 17):

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k)\,.$$

The dilation and translation operations simultaneously analyse signals at different points in time and frequencies. This time-frequency analysis is not possible with a Fourier transform, which only operates in the frequency domain. For smaller values of the index j, wavelets become finer and taller scale objects, oscillate more quickly, have less duration and are packed closer.

The wavelets generated from the dilation and translation operations constitute an orthonormal basis in the space of square integrable functions. Any signal in this space can be represented using a linear combination of wavelets:

$$\chi_t = \sum_{j \in Z} \sum_{k \in Z} q_{j,k} \psi_{j,k}(t) \qquad (1)$$

with wavelet coefficients[1]:

$$q_{j,k} = \int_R \chi_t \ \psi_{j,k}(t) \ .$$

The representation in equation (1) provides a deconstruction of the signal into different time-scales proportional to 2^j (a multiresolution analysis). For a signal consisting of $256 = 2^8$ observations, the coefficients at scale 1 (q_1) capture variation over cycles of 2-4 months. Similarly, the coefficients at scales 2 (q_2) and 3 (q_3) are associated with variations over cycles of 4-8 and 8-16 months, respectively, and this is the case up to scale 8. The highest scale 8 is therefore associated with long-term cyclical changes in the data (in excess of 256 months).

Financial time series are most commonly analysed using two wavelet transforms: (i) the discreet wavelet transform (DWT) and (ii) the MODWT. The MODWT is not exactly orthogonal (as is the case with the DWT); however, it is more efficient[2] and, unlike the DWT, has the following important properties: (i) it generates vectors of wavelet coefficients that have lengths equal to that of the actual time series, (ii) it can be applied to time series of any length and (iii) the generated wavelet coefficients are not affected by circular shifts in the actual data (see Kim and In, 2010).

Property (iii) is important for financial time series since the generated coefficients can be aligned to match key features and dynamics in the actual time series such as jumps and changes in regime. When working with non-dyadic length time series, Gencay et. al. (2002, p. 144) suggested first padding the data with the last value of the series (mirroring the last observations) to increase their length to the next power of two and then applying the MODWT. In a second stage, the coefficients affected by this boundary value solution are removed from the generated vectors. Even though other methods exist for addressing this boundary value problem, Masset (2015) considers this method the most suitable for stock returns since it can account for the presence of volatility clustering.

In practice, discreet wavelet transforms are computed using an iterative pyramid algorithm that filters the data using a discreet high-pass (wavelet) filter with coefficients ($h_0,...,h_{L-1}$) and a discreet low-pass (scaling) filter with coefficients ($g_0,...,g_{L-1}$). When using the MODWT, the pyramid algorithm starts by filtering the actual time series using the rescaled filters $\tilde{h}_j = h_j / 2^j$ and $\tilde{g}_j = g_j / 2^j$ as follows:

$$\tilde{q}_{1,t} = \sum_{l=0}^{L-1} \tilde{h}_1 \chi_{t-l \bmod N} \qquad \text{and} \qquad \tilde{v}_{1,t} = \sum_{l=0}^{L-1} \tilde{g}_1 \chi_{t-l \bmod N} .^3$$

[1] Z and R represent the sets of integer and real numbers respectively.

[2] The MODWT uses more information and is therefore superior in terms of asymptotic relative efficiency (see, Percival 1995).

[3] The modulus operator, $t - l \bmod N$, is the remainder from the division of $t - l$ with N.

In the second stage of the algorithm, the rescaled filters are applied to the vector of scaling coefficients $\tilde{v}_1$, which generates the second level of wavelet and scaling coefficients vectors $\tilde{q}_2$ and $\tilde{v}_2$. Similarly, in the third stage of the algorithm the rescaled filters are applied on the vector $\tilde{v}_2$ and the procedure is repeated J times to form the matrix of wavelet coefficient vectors $\tilde{q} = [\tilde{q}_1,...,\tilde{q}_J,\tilde{v}_J]'$.

To estimate the systematic risk of an asset at different time-scales, it is necessary to obtain estimates of the associated wavelet variances and covariances. The wavelet variance of time series χ_t, at time-scale λ_j, can be estimated using the following unbiased estimator (see Gencay et. al., 2002, p. 241):

$$\sigma_\chi^2(\lambda_j) = \frac{1}{\tilde{T}_j} \sum_{t=L_j-1}^{T-1} (\tilde{q}_{j,t}^{\chi})^2$$

where $L_j = (2^j - 1)(L+1) + 1$ is the length of the wavelet filter. The number of coefficients unaffected by the boundary is $\tilde{T}_j = T - L_j + 1$; therefore, all the wavelet coefficients affected by the periodic boundary conditions are excluded from this estimator. Accordingly, the following unbiased wavelet covariance estimator can be used to breakdown the covariance between two time series (χ_t and y_t) into different time-scales (see In and Kim, 2013, p. 33):

$$\sigma_{\chi,y}^2(\lambda_j) = \frac{1}{\tilde{T}_j} \sum_{t=L_j-1}^{T-1} (\tilde{q}_{j,k}^{\chi})(\tilde{q}_{j,k}^{y}) .$$

Using these wavelet variance and covariance estimators, the beta coefficient at time-scale λ_j can be calculated as follows:

$$\beta_{\lambda_j} = \frac{\sigma_{x,y}^2(\lambda_j)}{\sigma_x^2(\lambda_j)} . \quad (2)$$

3. Estimation results

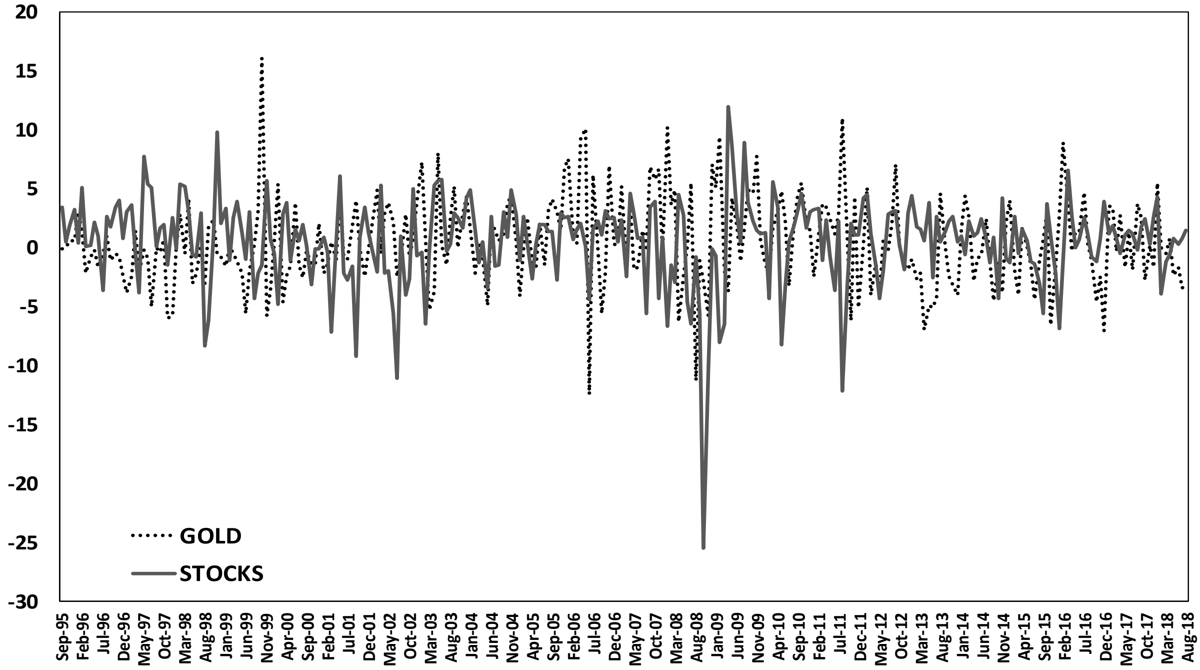
Gold returns were calculated as log differences using a dataset obtained from the World Gold Council. The dataset consisted of monthly gold prices (per troy ounce) for the period September 1995 - August 2018, denominated in four major currencies: US dollar, euro, UK sterling and the Japanese yen. The dataset was also augmented by data from the stock markets of the respective countries, available from the OECD statistical database for the same period.

Table 1 Summary statistics of returns

	Stock market index				Gold			
	GER	**JAP**	**UK**	**US**	**GER**	**JAP**	**UK**	**US**
Mean	0.408	0.078	0.282	0.510	0.461	0.471	0.485	0.414
St. Dev	4.936	4.726	3.576	3.749	3.518	3.638	3.740	3.705
Min	-23.383	-24.791	-20.030	-25.472	-11.560	-9.861	-11.024	-12.480
Max	13.042	11.256	9.985	11.927	14.225	15.156	14.046	16.014

Table 1 includes summary statistics for the aforementioned gold and stock returns, separated by country/currency. On average, during the period examined gold returns were better than stock market returns in Germany, Japan and the UK, but slightly worse than stock market returns in the US. Furthermore, gold exhibited a relatively high standard deviation in all currencies, and it was also associated with the highest maximum values per currency, which suggest the existence of considerable price volatility in the short-term. These findings are also illustrated in Figure 1, which exhibits returns in US dollars.

Figure 1 Asset returns in US dollars



To estimate the beta coefficient of gold at different time-scales, the excess returns of the assets were deconstructed using the Daubechies least asymmetric wavelet filter with length 8 in the context of the MODWT. This filter has good frequency localization properties and was also used by Kim and In (2010) and Michis (2015, 2014) for the analysis of similar financial data. Following the wavelet transforms, separate beta coefficients were estimated by time-scale and currency using equation (2) and the wavelet variances (for excess stock market returns) and covariances (between gold and stock market excess returns) presented in Section 2. The estimation results are included in Table 2 together with their robust standard errors that were estimated with the method proposed by Andrews (1991). Furthermore, Table A1 in the Appendix includes 95% confidence intervals for all the estimated coefficients.

The beta coefficients in Table 2 vary by time-scale even when considering each country/currency in isolation. For example, in the case of Germany the beta coefficient is negative when considering short term cycles of 2-4 months (time-scale d1), but it becomes positive in time-scales d3 and d4 that are associated with cycles of 8-16 and 16-32 months, respectively. However, there are two common findings across currencies: (i) over short-term and medium-term cycles (d1-d7) the signs of the beta coefficients vary across markets and cycles and are not always statistically significant, and (ii) the beta coefficients associated with long-term cycles in the data (d8) are negative and statistically significant for all currencies.

The results in Table 2 suggest that the usefulness of gold in reducing portfolio risk varied by time-scale. Gold was not always an effective diversifier over short-term and medium-term cycles (time-scales d1-d7) since it frequently exhibited positive correlation with stock market returns. In contrast, it exhibited good diversification properties over long time horizons (time-scale d8) due its negative correlation with stock market returns. More importantly, these results were consistent across the four major currencies examined in this study. Therefore, gold exhibited characteristics that were more appropriate for investors adopting a long-term "buy and hold" investment strategy, irrespective of whether their investment positions were denominated in US dollars, euros, UK sterlings or Japanese yens.

Table 2 Estimates of systematic risk by time-scale

Scale	GER	JAP	UK	US
d1	-0.082	0.077	-0.035	-0.065
	(0.071)	(0.067)	(0.079)	(0.090)
d2	-0.110*	0.072	-0.005	-0.013
	(0.044)	(0.065)	(0.073)	(0.066)
d3	0.105	0.203*	0.073	0.050
	(0.119)	(0.079)	(0.174)	(0.085)
d4	0.080	0.234*	-0.023	0.233*
	(0.096)	(0.088)	(0.153)	(0.111)
d5	-0.003	0.010	-0.276*	0.046
	(0.065)	(0.118)	(0.043)	(0.093)
d6	-0.063	0.022	-0.379*	-0.186*
	(0.079)	(0.034)	(0.126)	(0.092)
d7	-0.177*	0.088*	-0.271*	-1.085*
	(0.017)	(0.041)	(0.039)	(0.102)
d8	-1.102*	-1.027*	-2.279*	-2.345*
	(0.014)	(0.026)	(0.020)	(0.025)

Significant at: *5% level
Robust st. errors in parentheses

The aforementioned findings were also evaluated using an alternative methodology based on trigonometric filters and least squares coefficient estimates. First, the gold and stock market excess returns were analysed using trigonometric regression filters to extract frequency bands (fb1-fb8) that match the cycles covered by the respective time-scale components (d1-d8) generated by the MODWT. For example, the first frequency band (fb1) was designed to capture variation over cycles of 2-4 months as is the case for time-scale d1 and the second frequency band (fb2) variation over cycles of 4-8 months as is the case for time-scale d2. To see this, consider a time-series $x_t = y_t + \varepsilon_t$ from which we wish to extract the frequency band y_t with a pre-defined period of oscillations in the range $\left[p_u, p_l\right]$. The trigonometric regression filter used in this procedure can be represented as follows (Bloomfield, 2000, p. 11):

$$\hat{y}_t = \sum_{j \in J} \left[a_j \cos\left(\omega_j t\right) + b_j \sin\left(\omega_j t\right) \right]$$

where J (the frequencies we wish to extract) is the set of integers between $j_1 = T / p_u$ and $j_2 = T / p_l$ and the coefficients a_j and b_j can be computed with the ordinary least squares method (OLS)[4]. Christiano and Fitzgerald (2003) derived an alternative representation of this filter using trigonometric identities and a polynomial representation of the form $\hat{y}_t = B_t(L)\,x_t$.

In a second step, separate beta coefficients were estimated with the OLS method in the context of the CAPM, using the extracted frequency bands from the gold and stock market excess returns. The estimated beta coefficients are included in Table 3 together with their robust standard errors that were estimated with the method proposed by Andrews (1991). There are many similarities with the coefficient estimates in Table 2, particularly for frequency band fb8, which is associated with long-term cycles in the data. When compared with the respective coefficients for scale d8 in Table 2 it can be observed that, in this case too, all the beta coefficients are negative, similar in size and statistically significant. Therefore, gold exhibited good diversification properties over long-term time horizons.

Table 3 Estimates of systematic risk by frequency band

Scale	GER	JAP	UK	US
fb1	-0.094	0.105**	-0.033	-0.081
	(0.072)	(0.060)	(0.079)	(0.084)
fb2	-0.156*	0.307*	-0.062	-0.058
	(0.034)	(0.130)	(0.061)	(0.063)
fb3	0.158	0.193*	0.099	0.046
	(0.149)	(0.092)	(0.229)	(0.136)
fb4	0.120	0.390*	0.096	0.372*
	(0.085)	(0.093)	(0.104)	(0.153)
fb5	-0.132	0.257*	-0.488*	-0.024
	(0.099)	(0.115)	(0.042)	(0.087)
fb6	-0.044*	0.014	-0.341*	-0.166*
	(0.054)	(0.042)	(0.052)	(0.045)
fb7	0.449*	-0.013*	0.875*	1.220*
	(0.017)	(0.001)	(0.001)	(0.002)
fb8	-1.165*	-1.074*	-2.359*	-2.440*
	(0.010)	(0.002)	(0.024)	(0.001)

Significant at: *5% level; **10% level
Robust st. errors in parentheses

4. Multiresolution analysis and abnormal returns

In this Section, the dynamic characteristics of gold returns are explored further in order to identify: (i) the most important frequency bands associated with the beta coefficients that help explain variations in actual excess gold returns, (ii) how these frequencies bands evolve over time and (iii) any effects the 2007-2008 financial crisis might have had on gold returns when evaluated over different time-scales.

[4] Assuming T is even, the regression coefficients are:

$$a_j = \begin{cases} \frac{2}{T}\sum_{t=1}^{T}\cos(\omega_j t)\,x_t, & j = 1,\dots,T/2-1 \\ \frac{1}{T}\sum_{t=1}^{T}\cos(\pi t)\,x_t, & j = T/2 \end{cases}, \quad b_j = \begin{cases} \frac{2}{T}\sum_{t=1}^{T}\sin(\omega_j t)\,x_t, & j = 1,\dots,T/2-1 \\ \frac{1}{T}\sum_{t=1}^{T} x_t, & j = T/2 \end{cases}$$

As demonstrated in equation (1), time series signals can be represented using a linear combination of wavelets to form a multiresolution analysis. Based on this principle, a resolution J approximation of a time series x_t can be defined using the fine-scale representation in equation (3) below (Wasserman, 2004, p. 342). It consists of a smooth part constructed using the father wavelets ($\phi_{J,k}$) and a set of detail components constructed using the mother wavelets ($\psi_{j,k}$) that capture the fine-scale oscillations of the signal across J time-scales

$$x_t = \sum_{k \in Z} c_{J,k}\, \phi_{J,k}(x) + \sum_{j=1}^{J} \sum_{k \in Z} q_{j,k}\, \psi_{j,k}(x) = s_J + d_J + \ldots + d_1 . \quad (3)$$

In order to address point (i) and identify the most important frequency bands associated with the beta coefficients, the excess stock market returns ($r_{m,t}$) were decomposed into eight separate time-scale components in line with equation (3) above: $r_{m,t} = s_8 + d_8 + \ldots + d_1$. All time-scale components were then inserted as independent variables into the following model which resembles the CAPM

$$r_{g,t} = a + \beta_1 d_1 + \beta_2 d_2 + \beta_3 d_3 + \beta_4 d_4 + \beta_5 d_5 + \beta_6 d_6 + \beta_7 d_7 + \beta_8 d_8 + \gamma s_8 + \varepsilon_t .$$

Table 4 Coefficient estimates for time-scale components

Scale	GER	JAP	UK	US
Intercept	17270*	170.4	17880*	11490
	(7003)	(851.53)	(6738)	(15128)
d1	-0.056	0.087	-0.040	-0.089
	(0.085)	(0.080)	(0.096)	(0.118)
d2	-0.204*	0.026	-0.012	0.022
	(0.101)	(0.136)	(0.168)	(0.154)
d3	0.222	0.238	0.139	-0.045
	(0.141)	(0.173)	(0.251)	(0.226)
d4	0.044	0.283	-0.042	0.410
	(0.167)	(0.202)	(0.376)	(0.280)
d5	0.206	-0.032	0.031	0.131
	(0.300)	(0.304)	(0.506)	(0.408)
d6	1.077*	0.027	1.241*	0.390
	(0.512)	(0.417)	(0.713)	(0.924)
d7	0.104	0.457	0.719	1.685
	(0.932)	(0.589)	(1.288)	(2.112)
d8	-1.919**	-2.015*	-4.199*	-4.956*
	(1.149)	(0.939)	(1.866)	(1.883)
s8	-45990*	-2593*	-73960*	-24450*
	(18648)	(12993)	(27877)	(32190)
R-squared:	0.064	0.044	0.079	0.071

Significant at: *5% level; **10% level

Robust st. errors in parentheses

It is important to emphasize that the dependent variable in this model refers to actual excess gold returns ($r_{g,t}$) and is not decomposed into separate time-scale components, as is the case for the independent variable. The OLS coefficient estimates are included in Table 4 together with their robust standard errors. In line with the results in Section 3, the coefficient estimates for time-scale components d8 are all negative and statistically significant. This is also the case for the smooth components s8 that represent the long-run average levels in the data as in kernel regression. In contrast, the majority of the coefficients in the other time-scales are not statistically significant and in most cases vary by currency. These results suggests that the systematic risk of gold is primarily related with long-term movements in excess market returns that tend to be negatively related with actual excess gold returns.

Table 5 Varying coefficients models: estimation results

	GER	JAP	UK	US
Parametric coefficients:				
Intercept	-0.005	0.001	0.008	0.037*
	(0.022)	(0.024)	(0.025)	(0.010)
Approximate significance of smooth terms (F-test):				
$f(t).r_{m,t}$	134.7*	73.3*	218.6**	3361*
R-sq.(adj)	0.607	0.467	0.696	0.971

Significant at: *5% level; **10% level

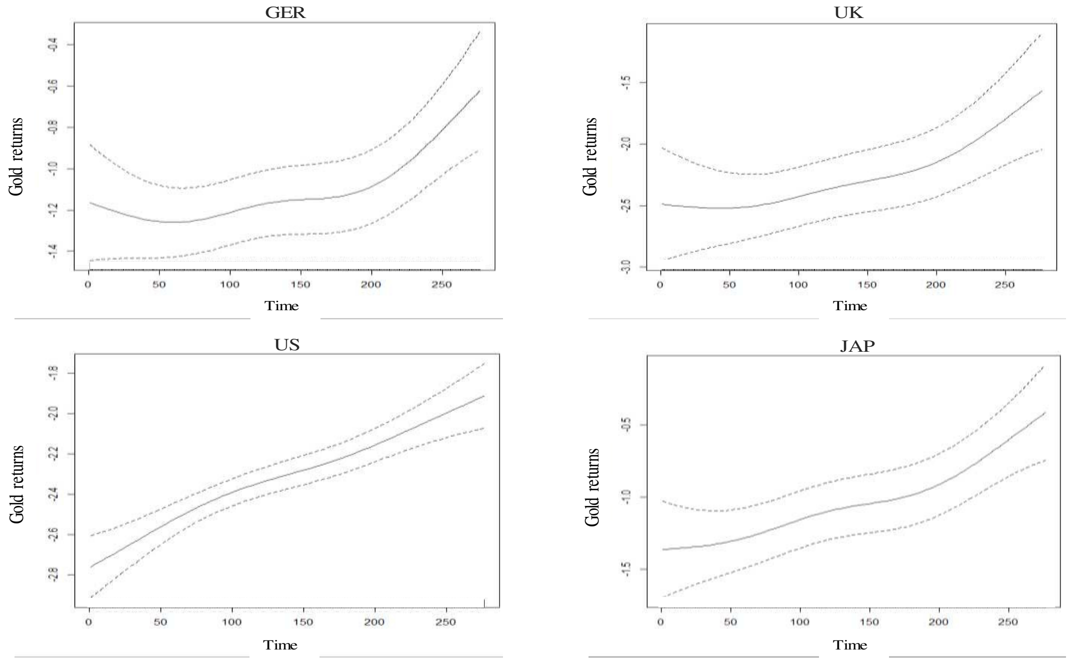
Point (ii) above regarding the evolution of the important frequency bands in time can be addressed by estimating a varying coefficient model at time-scale d8 where the beta coefficient is allowed to vary smoothly with a time trend (t). This analysis is possible because the vectors of wavelet coefficients generated from the MODWT have lengths equal to that of the actual time series and can be aligned to match key features and dynamics in the actual data. The varying coefficient CAPM model at time-scale d_8 can be represented as follows:

$$r_{g,d_8} = a + f(t).r_{m,d_8} + \varepsilon_t \, .$$

The variables r_{g,d_8} and r_{m,d_8} denote the scale d8 wavelet coefficients generated from the gold and stock market excess returns data respectively and the smooth time function $f(t)$ is allowed to interact with the independent variable.

The model was estimated with the penalized iteratively re-weighted least squares method and the smoothing parameter was selected based on the generalized cross validation (GCV) criterion (see Wood, 2006, pp. 138 and 175). The estimation results are summarised in Table 5 and the time-varying beta coefficients are depicted in Figure 2 together with their 95% confidence bands. All the estimated smooth terms are statistically significant and in most cases the models provided a good fit to the data. The time varying beta coefficients have negative values across the time periods considered in the analysis. However, they also exhibit distinct positive trends that tend to be more pronounced after time period 209 which refers to January 2013 in the sample. This time point coincides with the beginning of a declining trend in the price of gold (it peaked in 2012) that was mainly related with outflows by Exchange Traded Funds (ETFs) and the unwinding of their positions in gold (O' Connor et. al., 2015). Similarly, Baur (2013) suggested that the creation of gold ETFs (that began in 2003) has led to structural changes in the market for gold. For this reason, the examination of the relationship between the price of gold and ETF demand can provide valuable insights in understanding the price volatility and the systematic risk of gold in recent years and constitutes a fruitful area for future research.

Figure 2 Varying coefficient models: smooth terms



Since the dataset used in this study includes the 2007-2008 financial crisis, a period of extreme volatility in financial markets, it would be of interest to investigate on a scale-by-sale basis what effects the crisis might have had on gold returns. For this purpose, a multiscale version of the standardized cumulative abnormal returns (SCAR) test was developed that is based on the standard market model and the Student's t-distribution (Cuthbertson and Nitzsche, 2004 pp. 206-207). To see this, for each time-scale j consider the following equations for gold returns

$$r_{g,j,t} = a_j + \beta_j \, r_{m,j,t} + \varepsilon_{j,t} \qquad t = T_0,...,T_1$$
$$\boldsymbol{r}_{g,j} = \boldsymbol{X}_{m,j} \, \boldsymbol{\theta}_j + \boldsymbol{\varepsilon}_j$$

where $\boldsymbol{r}_{g,j} = \left[r_{g,j,t_0+1},...,r_{g,j,t_1} \right]$ is a vector of length $(t_1 - t_0)$ and $\boldsymbol{X}_{m,j} = \left[\mathbf{1}, \boldsymbol{r}_{m,j} \right]$ is a $(t_1 - t_0) \times 2$ matrix consisting a vector of ones and the $(t_1 - t_0) \times 1$ vector of market returns $\boldsymbol{r}_{m,j}$ over time-scale j. It is also assumed that $\varepsilon_{j,t} \sim NIID(0, \sigma_j^2)$. Based on this representation, the time interval $(t_0,...t_1,...,t_2,...,t_3)$ covered by the sample size of the study is divided into three periods: the estimation window $(t_0,...,t_1)$, the 2007-2008 financial crisis window $(t_1 + 1,...,t_2)$ and the post crisis window $(t_2 + 1,...,t_3)$. The financial crisis window was set as the period between

February 2007 and April 2011, which is the financial crisis timeline defined by the Federal Reserve Bank of St. Louis[5].

The abnormal returns (AR) of gold over the financial crisis window were estimated as follows:

$$AR_{g,j,t} = r_{g,j,t} - \hat{r}_{m,j,t} = r_{g,j,t} - (\hat{\alpha}_j + \hat{\beta}_j r_{m,j,t})$$
$$\boldsymbol{AR}_{g,j} = \boldsymbol{r}^*_{g,j} - \boldsymbol{X}^*_{m,j}\,\hat{\boldsymbol{\theta}}_j$$

where $\boldsymbol{r}^*_{g,j} = [r_{g,j,T_1+1},...,r_{g,j,T_2}]'$ and $\boldsymbol{X}^*_{m,j} = [\mathbf{1}^*, \boldsymbol{r}^*_{m,j}]$ with $\mathbf{1}^* = [1_{T_1+1},...,1_{T_2}]$ and $\boldsymbol{r}^*_{m,j} = [r_{m,j,T_1+1},...,r_{m,j,T_2}]$. To measure "normal gold returns", the estimation window sample and the gold market model were used to generate the following vector of estimated coefficients $\hat{\boldsymbol{\theta}}_j = [\hat{a}_j, \hat{\beta}_j]$. Using the same notation, the cumulative abnormal returns (CAR) for each time-scale j over the financial crisis window were calculated as follows:

$$CAR_{g,j} = \sum_{t=T_1+1}^{T_2} AR_{g,j} = \mathbf{1}^{*'}\,\boldsymbol{AR}_{g,j}\,.$$

Under the null hypothesis of no abnormal returns

$$CAR_{g,j} \sim N(0, \sigma^2_{CAR_{g,j}})$$

where $\sigma^2_{CAR_{g,j}} = \mathbf{1}^{*'}\boldsymbol{V}_{m,j}\,\mathbf{1}^*$ and $\boldsymbol{V}_{m,j} = [\boldsymbol{I} - \boldsymbol{X}^*_{m,j}(\boldsymbol{X}'_{m,j}\boldsymbol{X}_{m,j})^{-1}\boldsymbol{X}^{*'}_{m,j}]\sigma^2_j$. The SCAR test is based on the following ratio

$$SCAR_{g,j} = \frac{CAR_{g,j}}{\hat{\sigma}^2_{CAR_{g,j}}}\,.$$

Under the null hypothesis of no CAR this test follows the Student's t-distribution with $(T_1 - T_0 - 2)$ degrees of freedom.

The SCAR test results by time-scale and currency are included in Table 6. Statistically significant values of the test were obtained only for time-scales d6, d7 and d8. When evaluated over time-scale d8 the SCAR test generated statistically significant and positive values in all currencies. In contrast, it was mostly associated with negative CAR when evaluated over time-scales d6 and d7. Therefore, cumulative gold returns exhibited positive abnormal behaviour only when evaluated over long-term cycles. This finding is consistent with the results of Section 3, where gold was found to exhibit good diversification properties over time-scale d8 due its negative correlation with stock market returns. It also provides some support to the safe heaven hypothesis frequently cited in the literature according to which gold is considered by many investors as an asset that can protect wealth from financial market crashes. Therefore its long-run average price should be expected to appreciate as the macroeconomic conditions in a country deteriorate (Baur and Lucey 2010 and Baur and McDermott 2010).

[5] Financial Crisis Timeline: https://www.stlouisfed.org/financial-crisis/full-timeline

Table 6 Standardised CAR test resuts

Scale	GER	JAP	UK	US
d1	0.015	0.021	0.019	0.018
d2	0.033	0.047	0.041	0.039
d3	0.022	0.109	0.010	0.111
d4	-0.095	-0.151	-0.122	-0.144
d5	0.011	0.266	0.025	0.323
d6	-3.077*	-2.776*	-2.051*	-2.485*
d7	-3.623*	-5.752*	-21.545*	8.382*
d8	10.060*	7.950*	16.054*	26.974*

Significant at: *5% level (null hypothesis: no abnormal returns)

These properties of gold are strongly related with its unique supply and demand characteristics. Gold has a relatively inelastic supply in the short-run since mining production processes tend to be slow and uncertain. Additional supply sources involve scrap and central bank sales, however annual new gold supplies in the market remain small relative to its existing stock (O' Connor et. al. 2015). Gold demand is mainly associated with three sources: industrial fabrication (e.g. electronics, dentistry), jewellery and investments. Of these sources investment demand is the most rapidly growing component, primarily due to the hedging and safe haven properties of gold.

In addition to being a safe haven asset, gold is also considered as a hedge for the US dollar (Joy 2011 and Reboredo 2013). This is because many investors prefer to exchange their dollars for gold when the dollar is losing value due to negative developments in the macroeconomic conditions of the US economy. Unlike common stocks, gold does not represent a claim on the future cash flows or assets of a corporation that are influenced by the general economic conditions in a country. It is also a real asset without a counterparty liability claim and as a chemical component it cannot be cancelled out of existence as is the case for stocks (O' Connor et. al. 2015).

Finally, as a hard currency with an inelastic supply, gold has the ability to hold its value during inflationary times in the US economy. Feldstein (1980) and Fortune (1987) suggested two channels through which inflation influences gold prices (the opportunity cost effect and the substitution effect channels respectively) that involve gradual price and interest rate adjustments. All these countercyclical demand characteristics of gold are closely related with changes in the macroeconomic conditions of the US economy that extend well beyond the short-term investment horizons frequently considered for many financial assets.

5. Conclusions

The price of gold exhibits different characteristics in the short-term than in the long-term. Accordingly, the systematic risk of gold should be expected to vary by time-scale. The empirical results in this study, using data covering the period September 1995 – August 2018, confirm this assertion. The sign and magnitude of the beta coefficient of gold varied by time-scale and currency. However, it was always negative when evaluated over time-scale d8, which is associated with long term cycles in the data. Consequently, in the long-term, excess gold returns moved in the opposite direction from excess stock market returns, and gold was therefore an effective portfolio diversifier.

Appendix

Table A1 95% confidence intervals for beta coefficients

Scale	GER	JAP	UK	US
d1	(-0.221, 0.057)	(-0.054, 0.208)	(-0.190, 0.121)	(-0.241, 0.111)
d2	(-0.197, -0.024)	(-0.055, 0.199)	(-0.147, 0.137)	(-0.143, 0.116)
d3	(-0.129, 0.339)	(0.049, 0.357)	(-0.070, 0.215)	(-0.117, 0.217)
d4	(-0.108, 0.268)	(0.062, 0.406)	(-0.323, 0.276)	(0.016, 0.450)
d5	(-0.130, 0.124)	(-0.221, 0.241)	(-0.360, -0.192)	(-0.136, 0.228)
d6	(-0.217, 0.091)	(-0.044, 0.088)	(-0.627, -0.131)	(-0.366, -0.006)
d7	(-0.211, -0.143)	(0.007, 0.169)	(-0.347, -0.195)	(-1.286, -0.884)
d8	(-1.131, -1.074)	(-1.078, -0.975)	(-2.319, -2.239)	(-2.395, -2.296)

References

Andrews D.W.K. (1991), "Heteroskedasticity and autocorrelation consistent covariance matrix estimation", *Econometrica*, 59, 817–858.

Baur, D.G. and Lucey, B.M. (2010), "Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold", *Financial Review*, 45, 217–229.

Baur, D.G. and McDermott, T.K. (2010), "Is gold a safe haven? International evidence", *Journal of Banking and Finance*, 34, 1886–1898.

Baur, D. G. (2013), "Exchange-traded funds on gold – A free lunch?", *SSRN Electronic Journal*, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2297756##

Bloomfield, P. (2000), Fourier analysis of time series: An introduction, 2nd edition, John Wiley and Sons: New York.

Christiano, L. and Fitzgerald, T.J. (2003), "The band pass filter", *International Economic Review*, 44(2):435-65.

Cuthbertson, K. and Nitzsche, D. (2004), Quantitative Financial Economics: stocks, bonds and foreign exchange, John Wiley and Sons: Chichester.

Ehrhardt, M. C. and Brigham, E.F. (2011), Corporate finance: a focused approach, 4th ed. OH: South-Western, Cengage Learning.

Feldstein, M. (1980), "Inflation, tax rules, and the prices of land and gold", *Journal of Public Economics*, 14, 309-317.

Fortune, J. N. (1987), "The inflation rate of the price of gold, expected prices and interest rates", *Journal of Macroeconomics*, 9, 71-82.

Gencay R Selcuk F and Whitcher B (2002). An Introduction to Wavelets and Other Filtering Methods in Finance and Economics. Academic Press: New York.

Gencay, R. Selcuk, F. and Whitcher, B. (2005), "Multiscale systematic risk", *Journal of International Money and Finance*, 24, 55-70.

In, F. and Kim, S. (2013), An introduction to wavelet theory in finance: a wavelet multiscale approach, Singapore: World Scientific.

Joy, M. (2011), "Gold and the US dollar: hedge or heaven?", *Finance Research Letters*, 8, 120-131.

Kim, S. and In, F. (2010), "Portfolio allocation and the investment horizon: a multiscaling approach", *Quantitative Finance*, 10, 443-453.

Masset, P. (2015), "Analysis of financial time series using wavelet methods", in: Lee, C-F and Lee, J.C. (Ed.), *The Handbook of Financial Econometrics and Statistics*, New York: Springer.

Michis, A. A. (2014), "Investing in gold: individual asset risk in the long-run", *Finance Research Letters,* 11, 369-374.

Michis, A. A. (2015), "Multiscale analysis of the liquidity effect in the UK economy", *Computational Economics*, 45, 615-633.

O' Connor, F. A., Lucey, B. M., Batten J. A. and Baur, D. G. (2015), "The financial economics of gold – a survey", *International Review of Financial Analysis*, 41, 186-205.

Percival, D. B. (1995), "On estimation of the wavelet variance", *Biometrika*, 82, 619-631.

Reboredo, J. C. (2013), "IS gold a safe haven or a hedge for the US dollar? Implications for risk management", *Journal of Banking & Finance*, 37, 2665 – 2676.

Scott-Ram, R. (2002), "Investing in gold and precious metals", in: Jobman, D.R., (Ed.), *The Handbook of Alternative Assets*, New York: John Wiley and Sons.

Wasserman, L. (2004), All of statistics: A concise course in statistical inference, New York: Springer.

Wood, S. (2006), Generalized Additive Models: An Introduction with R, CRC Press: Florida.

World Gold Council (2018), "The relevance of gold as a strategic asset", London, UK.