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Comment on "Price and quantity competition in network goods duopoly: A reversal result"

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Abstract

Based on the utility function of Hoernig (2012), who introduces network externalities, Pal (2014) considers some of the classic issues related to Cournot and Bertrand equilibria, e.g., Singh and Vives (1984), and demonstrates that profit under Bertrand equilibrium is higher than that under Cournot equilibrium if the degree of network externalities is sufficiently large. In this note, focusing on the role of consumer expectations, i.e., active and passive expectations, of network sizes, we demonstrate that the main result of Pal (2014) does not hold under active expectations. Furthermore, we compare profits, outputs, and consumer surplus in equilibrium in the cases of active and passive expectations.

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1. Introduction

Using the utility function of Hoernig (2012), who introduces network externalities, Pal (2014) considers some of the classic issues related to Cournot and Bertrand equilibria, e.g., Singh and Vives (1984), and in particular demonstrates that profit under Bertrand equilibrium is higher than that under Cournot equilibrium if the degree of network externalities is sufficiently large.

We focus on the role of consumer expectations of network sizes because expectations are important in any market with network externalities. Related to this point, Hurkens and López (2014) reexamine the literature on mobile termination in the presence of network externalities, e.g., Laffont et al. (1998) and Gans and King (2001), and resolve the puzzle, that is, profit decreases and consumer surplus increases when termination charges increase, if consumer expectations are assumed *passive* and required to be fulfilled in equilibrium (*italics* added). In other words, the result of Gans and King (2001) depends on the role of consumer expectations, i.e., consumers having *responsive* expectations, not *passive* expectations.

Following the definitions of Hurkens and López (2014, p. 1007), responsive expectations means that firms first compete in prices (or in quantities), then consumers form expectations about network sizes and finally consumers make optimal purchasing decisions, given the prices and their expectations.[1] Passive expectations means that consumers first form expectations about network sizes and firms then compete in prices (or in quantities); finally, consumers make optimal purchasing decisions, given their expectations. These decisions then lead to actual market shares and network sizes. Thus, in equilibrium, realized and expected network sizes are the same (see Katz and Shapiro, 1985).

Furthermore, related to the definitions, in considering price competition on two-sided platforms, Gabszewicz and Wauthy (2004) assume passive and active (i.e., responsive) beliefs for network sizes on the other side platform. To examine the effect of different levels of information on two-sided platform profits, Hagiu and Halaburda (2014) also assume responsive expectations.[2]

Adapting the passive and responsive (active) expectations terminology presented

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[1] In the case of price competition, consumers realize and expect that when one firm lowers its price it will increase its market share and become the larger network. That is, consumers must adjust their expectations in response to a price change. It is presumed that given these changed expectations, optimal purchasing decisions will lead realized and expected network sizes to coincide. Thus, for all prices, expectations are required to be self-fulfilling.

[2] In addition to the literature cited in Hurkens and López (2014), there are the following studies. Suleymanova and Way (2012) consider the effect of consumer expectations on price competition in a Hotelling model with network externalities, assuming that expectations are strong (stubborn) and weak (price-sensitive), instead of *passive* and *responsive*. Toshimitsu (2017) analyzes the relationship between consumer surplus and the timing of consumer expectations in the case of a monopoly with network externalities. He assumes that consumers *ex ante* (*ex post*) form expectations of network sizes before (after) the monopolist's decision. Thus, the *ex ante* (*ex post*) expectations correspond to the passive (responsive) expectations in this paper.

by Hurkens and López (2014),[3] we demonstrate that Pal (2014), who assumes passive expectations, derives the main result, i.e., Proposition 1, however, this does not hold under active expectations.[4] Furthermore, we compare the equilibrium profits, outputs, and consumer surplus, in the cases of passive and active expectations. In particular, we demonstrate that profit under active expectations is larger than that under passive expectations if the degree of network externalities is sufficiently large and that regardless of the mode of competition, consumer surplus under active expectations is larger than that under passive expectations.

The remainder of this paper is organized as follows. In Section 2, using the formulas of Pal (2014), we present the demand and inverse demand functions under active expectations. Then, we derive the equilibrium outcomes in the cases of Cournot and Bertrand competition and compare them. Furthermore, we consider the equilibrium outcomes in the cases of passive and active expectations. In Section 3, we summarize our results.

2. The Model

2.1 Fulfilled equilibria under active expectations

Pal (2014) assumes passive expectations, which implies that consumers do not believe the announced output levels of firms; in other words, firms are not able to commit to their output levels. Conversely, we consider the case of active expectations, which implies that firms can commit to the announced output levels before consumers make their purchase decisions. Thus, substituting $y_i = x_i, \quad y_j = x_j, \quad i \neq j, i, j = 1,2,$ into equations (1a) and (1b) of Pal (2014), we derive the following demand and inverse demand functions:

$$x_i = \frac{\alpha - p_i + \beta p_j}{1-n}, \tag{1}$$

$$p_i = \frac{\alpha(1+\beta) - (1-n)(x_i + \beta x_j)}{1-\beta^2}, \quad i \neq j, i, j = 1,2. \tag{2}$$

To simplify the analysis, we assume that the marginal cost of production is zero, i.e., $c = 0,$ because we observe low and even negligible marginal operating costs in a network industry, such as telecommunications and Internet businesses.

Based on equations (1) and (2), we demonstrate the equilibrium Cournot and Bertrand outcomes in Table 1. We also summarize the corresponding results from Pal (2014) in Table 2.

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[3] Hereafter, following the referee's suggestion, corresponding to "passive" expectations, we use "active" expectations instead of "responsive" expectations.

[4] Pal (2014) also analyzes the endogenous selection of strategic variables, i.e., quantity contracts or price contracts, and shows that choosing a quantity contract is optimal for firms. Although we do not examine this issue, we can prove that the Pal's (2014, Proposition 2) result holds under active expectations.

Table 1: Equilibrium outcomes under active expectations

	Cournot competition	Bertrand competition
Quantity	$x^C_{active} = \frac{\alpha(1+\beta)}{(1-n)(2+\beta)}$	$x^B_{active} = \frac{\alpha}{(1-n)(2-\beta)}$
Price	$p^C_{active} = \frac{\alpha}{(1-\beta)(2+\beta)}$	$p^B_{active} = \frac{\alpha}{2-\beta}$
Profit	$\pi^C_{active} = \frac{\alpha^2(1+\beta)}{(1-n)(1-\beta)(2+\beta)^2}$	$\pi^B_{active} = \frac{\alpha^2}{(1-n)(2-\beta)^2}$
Consumer surplus	$CS^C_{active} = \frac{1-n}{1-\beta}\left(x^C_{active}\right)^2$	$CS^B_{active} = \frac{1-n}{1-\beta}\left(x^B_{active}\right)^2$

Table 2: Equilibrium outcomes under passive expectations (Pal, 2014)

	Cournot competition	Bertrand competition
Quantity	$x^C_{passive} = \frac{\alpha(1+\beta)}{2-n+\beta(1-n)}$	$x^B_{passive} = \frac{\alpha}{2-n-\beta}$
Price	$p^C_{passive} = \frac{\alpha}{(1-\beta)\{2-n+\beta(1-n)\}}$	$p^B_{passive} = \frac{\alpha}{2-n-\beta}$
Profit	$\pi^C_{passive} = \frac{\alpha^2(1+\beta)}{(1-\beta)\{2-n+\beta(1-n)\}^2}$	$\pi^B_{passive} = \frac{\alpha^2}{(2-n-\beta)^2}$
Consumer surplus	$CS^C_{passive} = \frac{1-n}{1-\beta}\left(x^C_{passive}\right)^2$	$CS^B_{passive} = \frac{1-n}{1-\beta}\left(x^B_{passive}\right)^2$

2.2 Comparison: The role of consumers' expectations

In Table 1, we show the same result as Lemma 1 in Pal (2014), i.e., $p^C_{active} > p^B_{active}$ and $x^B_{active} > x^C_{active}$. However, with respect to profit-ranking in the cases of Cournot and Bertrand competition, we derive the same result as Singh and Vives (1984), regardless of the presence of network externalities, i.e., $\pi^C_{active} > \pi^B_{active}$. That is, Proposition 1 in Pal (2014) does not hold under active expectations. In particular, equations (1) and (2) are rewritten as follows:

$$x_i = \frac{1}{1-n}\left(\alpha - p_i + \beta p_j\right), \tag{1′}$$

$$p_i = \frac{1-n}{1-\beta^2}\left(A - x_i - \beta x_j\right), \quad i \neq j, i, j = 1,2, \tag{2′}$$

where $A \equiv \frac{1+\beta}{1-n}$. Under active expectations, in which the firms can commit to their outputs and consumers believe them, the firms can internalize the externalities from

consumer expectations of network sizes. This differs from the passive expectations case. As shown by equations (1′) and (2′), the demand and inverse demand functions are formally the same linear functions as those in Singh and Vives (1984).

From the perspective of the role of consumer expectations, we should compare the equilibrium outcomes in the cases of Cournot and Bertrand competition. Using Tables 1 and 2, with respect to profits, we derive the results as the following lemma.

Lemma 1

(i) $\pi^C_{active} > (<)\pi^C_{passive} \Leftrightarrow n > (<) \dfrac{\beta(2+\beta)}{(1+\beta)^2} < 1,$

(ii) $\pi^B_{active} > (<)\pi^B_{passive} \Leftrightarrow n > (<)\beta(2-\beta) < 1.$

In the cases of Cournot and Bertrand competition, if the degree of network externalities is sufficiently large, the profit in the case of active expectations is larger than that in the case of passive expectations. As mentioned above, because firms can internalize network externalities, the larger the degree of network externalities, the larger the profit, compared with the case of passive expectations.

Furthermore, to discuss the relationship between consumer welfare and the role of consumer expectations, with respect to outputs and consumer surplus, we obtain the following results in Lemma 2.[5]

Lemma 2

(i) $x^C_{active} > x^C_{passive}$ and $x^B_{active} > x^B_{passive}$,

(ii) $CS^C_{active} > CS^C_{passive}$ and $CS^B_{active} > CS^B_{passive}$.

Regardless of the mode of competition, outputs and consumer surplus in the case of active expectations are larger than those in the case of passive expectations.[6] In particular, competition is more intense under active expectations compared with under passive expectations. Thus, under the active expectations case, where firms can commit to the output levels in advance, firms have incentives to increase their output levels compared with the passive expectations case. Accordingly, consumer surplus in the case of active expectations is larger than that in the case of passive expectations.

3. Concluding Remarks

Focusing on the role of consumer expectations of network sizes, we have reconsidered the main result of Pal (2014, Proposition 1); in the case of passive expectations, profit under Bertrand equilibrium is higher than that under Cournot equilibrium if the degree

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[5] Taking equation (2) in Pal (2014), consumer surplus is given by $CS^m_k \equiv \dfrac{1-n}{1-\beta} x^m_k$, $m = C, B,$ and $k = active, passive.$

[6] With respect to prices, we obtain as follows: $p^C_{active} < p^C_{passive}$ and $p^B_{active} < p^B_{passive}$.

of network externalities is sufficiently large. However, we have demonstrated that in the case of active expectations, regardless of the degree of network externalities, profit under Bertrand equilibrium is lower than that under Cournot equilibrium.

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