Economics Bulletin

Volume 39, Issue 4

A note on price regulation in two-sided markets

Dennis Weisman
Kansas State University

Soheil Nadimi
Kansas State University

Abstract

This paper examines price level regulation in two-sided markets with linear demands. We find that (i) price level regulation increases the price allocation asymmetry when reservation prices differ between the two sides of the market; and (ii) changes in the level of the price cap are divided equally between the two sides of the market whether demands are symmetric or asymmetric. Finally, and potentially important from a policy perspective, the numerical simulations suggest that the efficiency gains from price level regulation are relatively modest for a wide range of cost parameters.

Citation: Dennis Weisman and Soheil Nadimi, (2019) "A note on price regulation in two-sided markets", *Economics Bulletin*, Volume 39, Issue 4, pages 2766-2777
Contact: Dennis Weisman - weisman@ksu.edu, Soheil Nadimi - soheil.nadimi@gmail.com.
Submitted: August 01, 2019. **Published:** December 08, 2019.

1. Introduction

A voluminous literature has emerged on pricing behavior in two-sided markets (Armstrong 2006; Rochet and Tirole 2003, 2006; Weyl 2009a, 2009b, 2010). A prominent example concerns the payment card market wherein the card issuer serves as the intermediary between merchants and consumers (Wright 2004). Complaints over excessive fees charged for these services have led to government regulation or investigations in several countries (Wang 2016; European Commission 2008).

The platform provider in a two-sided markets framework faces two different pricing decisions—the level of the transaction price and the structure of the transaction price. The structure of the transaction price is simply an allocation of the transaction price between the buyer side and the seller side of the market to maximize the volume of usage for any given transaction price.

A principal objective of this paper is to begin to understand the effects of price regulation on the structure of prices in two-sided markets. In certain applications, such as that of the zero-price mandate for broadband (Hemphill 2008),[1] regulation imposes constraints on the structure of prices rather than their level.

This analysis yields several important findings. First, price level regulation increases the price allocation asymmetry when reservation prices differ between the two sides of the market. Second, changes in the level of the price cap are divided equally between the two sides of the market whether demands are symmetric or asymmetric. Third, the numerical simulations suggest that the efficiency gains from price regulation are quite small over a wide range of cost parameters.

The format for the remainder of this paper is as follows. Section 2 explores the general structure of the platform provider's problem in terms of the profit-maximizing transaction price and the allocation of that transaction price to each side of the market. The general structure of the regulator's problem is examined in Section 3. Section 4 investigates the effects of price regulation on the price structure (allocation) and the corresponding welfare effects. Section 5 concludes. The proofs of all formal propositions are contained in the Appendix.

2. The Platform Provider's Problem

Let the transaction price be given by $p = p^B + p^S$, where p^B is the price to the buyer and p^S is the price to the seller. The price structure is determined by the allocation of p between $p^B = \alpha p$ and $p^S = (1-\alpha)p$, where $\alpha \in [0,1]$.[2]

Let $n^B(p^B)$ and $n^S(p^S)$ denote the number of buyers and sellers, respectively, and let c denote the marginal cost of the transaction. Following Rochet and Tirole (2006) and Schmalensee (2002), the profit for the platform provider is given by

[1] The zero-price mandate in broadband markets prohibits paid prioritization that facilitates faster delivery of Internet traffic. The concern is that this practice will create 'fast' and 'slow' lanes on the Internet and harm edge (content) providers (see FCC 2015). In a speech delivered on April 26, 2017, the FCC Chairman announced plans to roll back Title II regulation of the Internet (Pai 2017). In December 2017, the FCC voted in favor of these proposed roll backs (FCC 2018). The Canadian Radio-television and Telecommunications Commission is evaluating similar issues as part of its ongoing proceeding on differential pricing practices related to Internet data plans. See http://www.crtc.gc.ca/eng/archive/2016/2016-192.htm.

[2] In certain cases, one side of the market may receive 'rewards' for participation in the transaction in which case α^* is not bounded on [0,1]. An example of this phenomenon is the cash rewards that users of the Discover card receive based on the amount of their purchases.

$$\pi = n^B(p^B) \times n^S(p^S) \times (p^B + p^S - c). \quad (1)$$

A formal statement of the platform provider's problem is therefore given by

$$\alpha^*, p^* \in \underset{\{\alpha,p\}}{\operatorname{argmax}}\ \pi = n^B(\alpha p)\, n^S\big((1-\alpha)p\big)(p-c). \quad (2)$$

Linear demand functions are employed in the analysis to obtain closed-form solutions and to facilitate the numerical simulations. The linear demand functions for the buyer and seller side of the market are given, respectively, by

$$n^B = a_B - d_B p^B, \quad (3)$$

$$n^S = a_S - d_S p^S, \quad (4)$$

where all parameters are positive with $n^B > 0$ and $n^S > 0\ \forall \alpha \in [0, 1]$.

Substituting (3) and (4) into (2) and solving for the profit-maximizing price structure and transaction price level yields Proposition 1.

Proposition 1. Given (3) and (4), the platform provider's profit-maximizing choices for α and p are given by

$$\alpha^* = \frac{2a_B d_S - a_s d_B + c d_B d_S}{a_B d_S + a_s d_B + 2c d_B d_S}. \quad (5)$$

$$p^* = \frac{a_B d_S + a_s d_B + 2c d_B d_S}{3 d_B d_S}. \quad (6)$$

It follows immediately from (5) and (6) that

$$p^{B^*} = \alpha^* p^* = \frac{2a_B d_S - a_s d_B + c d_B d_S}{3 d_B d_S} = \frac{1}{2}p^* - \frac{k}{2 d_B d_S}, \quad (7)$$

$$p^{S^*} = (1-\alpha^*)p^* = \frac{2a_S d_B - a_B d_S + c d_B d_S}{3 d_B d_S} = \frac{1}{2}p^* + \frac{k}{2 d_B d_S}, \quad (8)$$

where $k = [a_S d_B - a_B d_S]$ is independent of p (Weisman 2010). The expressions on the rightmost side of (7) and (8) reflect the *seesaw principle* of two-sided markets as illustrated in Figure 1. Specifically, a higher price on one side of the market is associated with a lower price on the other side of the market with the sign of k determining the side that pays the higher (lower) price. The deviation from a symmetric price allocation is equal, though of opposite sign, on both sides of the market when $k \neq 0$. In addition, it is straightforward to show that $p^{B^*} - p^{S^*} = \frac{a_B}{d_B} - \frac{a_S}{d_S}$. The difference in the equilibrium prices across the two sides of the market is equal to the difference in the reservation prices (Krueger 2009, 276).

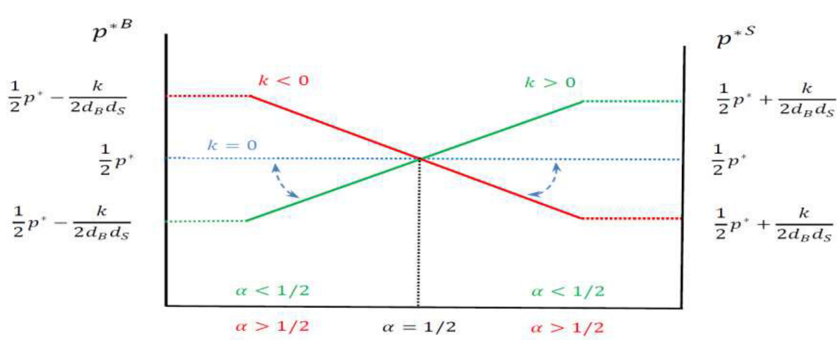

Figure 1. Seesaw Principle for Linear Demands (Platform Provider's Problem)

It is instructive to explore how the price structure rule diverges from a balanced price allocation in which each side of the market bears an equal burden.

Corollary 1. The profit-maximizing price allocation rule can be expressed as

$$\alpha^* = \frac{1}{2} + \frac{3(a_B d_S - a_s d_B)}{2(a_B d_S + a_s d_B + 2c d_B d_S)}. \quad (9)$$

Corollary 1 reveals that the platform provider allocates the price burden equally across the two sides of the market when reservation prices are equal, $\frac{a_B}{d_B} = \frac{a_S}{d_S}$. Otherwise, the greater price burden is borne by the side of the market with the higher reservation price.

The following corollary reveals that demands are equal when both sides of the market exhibit the same degree of price sensitivity.

Corollary 2. For any given transaction price, p, $n^B(p^B) = n^S(p^S)$ in equilibrium when $d_B = d_s$.

3. The Regulator's Problem

Following Rochet and Tirole (2003) and Weyl (2009b), the consumer surplus (*CS*) on the *I* side of the market, where *I* = *B, S* is given by

$$CS^I(p^I, p^J) = \left(\frac{n^J}{n^I}\right)\int_{p^I}^{\overline{p^I}} n^I \, d(p^I), \; I = B, S; \;\; J = S, B, I \neq J, \quad (10)$$

where $\overline{p^I}$ is the reservation price for the *I* side of the market. Hence, social welfare (SW) can be expressed as

$$SW = \pi + CS^B + CS^S. \quad (11)$$

A formal statement of the social planner's problem is therefore given by

$$\alpha^{SW}, p^{SW} \in \underset{\{\alpha, p\}}{\operatorname{argmax}} \, SW = \pi + \left(\frac{n^S}{n^B}\right)\int_{p^B}^{\overline{p^B}} n^B \, d(p^B) + \left(\frac{n^B}{n^S}\right)\int_{p^S}^{\overline{p^S}} n^S \, d(p^S), \quad (12)$$

where the reservation prices $\overline{p^B}$ and $\overline{p^S}$ are equal to $\frac{a_B}{d_B}$ and $\frac{a_S}{d_S}$, respectively, when demands are given by (3) and (4).

Proposition 2. Given (3) and (4), the regulator's welfare-maximizing choices for α and p are given by

$$\alpha^{SW} = \frac{4a_B d_S - 2a_s d_B + 2c d_B d_S - d_{B-} d_S}{2(a_B d_S + a_s d_B + 2c d_B d_S - d_{B-} d_S)}. \quad (13)$$

$$p^{SW} = \frac{a_B d_S + a_s d_B + 2c d_B d_S - d_{B-} d_S}{3 d_B d_S}. \quad (14)$$

It follows immediately from (13) and (14) that

$$p^{SW^B} = \alpha^{SW} p^{SW} = \frac{4a_B d_S - 2a_s d_B + 2c d_B d_S - d_{B-} d_S}{6 d_B d_S} = \frac{1}{2} p^{SW} - \frac{k}{2 d_B d_S}. \quad (15)$$

$$p^{SW^S} = (1 - \alpha^{SW}) p^{SW} = \frac{4a_S d_B - 2a_B d_S + 2c d_B d_S - d_{B-} d_S}{6 d_B d_S} = \frac{1}{2} p^{SW} + \frac{k}{2 d_B d_S}. \quad (16)$$

Similar to the platform provider's problem, the expressions on the rightmost side of (15) and (16) illustrate the *seesaw principle* of two-sided markets as shown in Figure 2. Specifically, a higher price on one side of the market is associated with a lower price on the other side of the market with the sign of k determining the side that pays the higher (lower) price. The deviation from a symmetric price allocation is equal, though of opposite sign, on both sides of the market when $k \neq 0$. In addition, it is straightforward to show that $p^{SW^B} - p^{SW^S} = \frac{a_B}{d_B} - \frac{a_S}{d_S}$. The difference in the welfare maximizing prices across the two sides of the market is equal to the difference in the reservation prices.

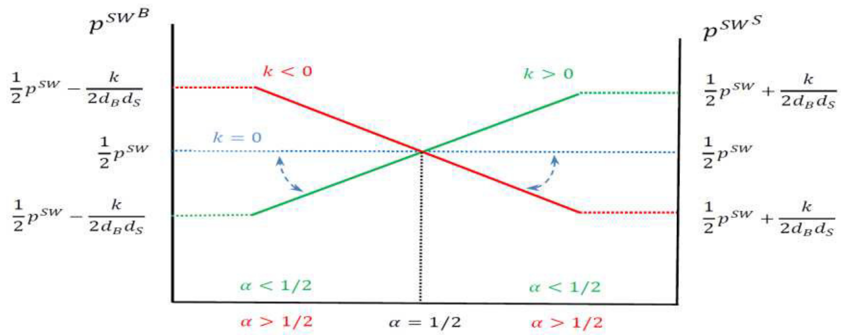

Figure 2. Seesaw Principle for Linear Demands (Regulator's Problem)

Corollary 3. The welfare-maximizing price allocation rule can be expressed as

$$\alpha^{SW} = \frac{1}{2} + \frac{3(a_B d_S - a_S d_B)}{2(a_B d_S + a_S d_B + 2c d_B d_S - d_B - d_S)}. \quad (17)$$

Corollary 3 reveals that the regulator allocates the price burden equally across the two sides of the market when reservation prices are equal, $\frac{a_B}{d_B} = \frac{a_S}{d_S}$. Otherwise, the greater price burden is borne by the side of the market with the higher reservation price.

The following corollary reveals that demands are equal when both sides of the market exhibit the same degree of price sensitivity.

Corollary 4. For any given transaction price, p, $n^B(p^B) = n^S(p^S)$ in equilibrium when $d_B = d_s$.

Proposition 3. The welfare-maximizing price is less than the monopolist's profit-maximizing price when demands are given by (3) and (4).

Let $\Delta = p^* - p^{SW}$. It is straightforward to show from (A4) in the Appendix that $\frac{\partial \Delta}{\partial d_B} < 0$ and $\frac{\partial \Delta}{\partial d_S} < 0$. This implies that the difference between the profit-maximizing price and welfare-maximizing price is decreasing with respect to both buyer-side and seller-side price sensitivity. Alternatively stated, the interests of the monopolist and the regulator are more closely aligned with higher degrees of price sensitivity.

Proposition 4. For linear demands, the welfare-maximizing price allocation for the buyer is less (greater) than the monopolist's profit-maximizing price allocation for the buyer when the buyer's reservation price is less (greater) than seller's reservation price.

4. Price Level Regulation

Price regulation is a constraint on the level of the transaction price of the form $p \leq \bar{p}$, where $\bar{p} < p^*$ is the price cap set by the regulator.

Proposition 5. When demands are given by (3) and (4), the monopolist's allocation rule for any given transaction price, p, is given by

$$\alpha^M(p) = \frac{d_B d_S p - a_S d_B + a_B d_S}{2 d_B d_S p} = \frac{1}{2} - \frac{k}{2 d_B d_S p}.\text{[3]} \qquad (18)$$

Once again, the buyer side of the market bears a greater (lesser) price burden when $\frac{a_B}{d_B} > (<) \frac{a_S}{d_S}$. It is straightforward to show that $\frac{\partial \alpha^M}{\partial d_B} < 0$ and $\frac{\partial \alpha^M}{\partial d_S} > 0$. This implies that the allocation of the price burden to the buyer-side of the market is decreasing (increasing) with respect to the buyer-side (seller-side) price sensitivity, *ceteris paribus*. An immediate implication of (18) is that a lower transaction price gives rise to a less balanced price structure when $k \neq 0$, *ceteris paribus*.

To expand upon this last observation, suppose that a price cap is imposed that reduces the transaction price below the profit-maximizing level. In addition, define the degree of price structure asymmetry with respect to the rightmost term in (18) as $\theta = \frac{|k|}{2 d_B d_S p}, 0 \leq \theta \leq \frac{1}{2}$.

$$-\frac{\partial \theta}{\partial p}\bigg|_{k \neq 0} = \frac{|k|}{2 d_B d_S p^2} \geq 0. \qquad (19)$$

Equation (19) suggests Proposition 6.

Proposition 6. Price regulation in a two-sided market increases the price structure asymmetry when $k \neq 0$ and has no effect on the price structure symmetry when $k = 0$.

When $k \neq 0$, the platform provider adopts a more (less) balanced price structure for higher (lower) transaction prices. The choice of a more balanced price structure in response to a higher transaction price prevents large reductions in the number of buyers and sellers given that the platform provider's profit depends on the product of the demands, $n^B(p^B) \times n^S(p^S)$.

Proposition 7. A change in the level of the price cap is divided equally between the two sides of the market whether demands are symmetric or asymmetric.

The following proposition shows that both sides of the market benefit from price level regulation.

Proposition 8. Price level regulation increases consumer surplus on both sides of the market and decreases the monopolist's profit.

Example 1. Table I below illustrates the platform provider's best response price allocation rule, $\alpha^M(p)$, to the regulated transaction price for the specified parameters. The following observations are noteworthy. First, the greater price burden is borne by the buyer side of the market since the reservation price of 15 on the buyer side of the market exceeds the seller-side reservation price of 10 (Corollary 1, Corollary 3 and Proposition 5). Second, the green highlighted row indicates the

[3] The monopolist's choice of price structure (for any given transaction price) is efficient which explains the focus here on price level regulation. See Rochet and Tirole (2003, 1009) and Weyl (2009b) for further discussion. In addition, as Weyl (2009a, 6) observes, 'The price balance chosen by the monopolist may well be optimal and when it is not it may be difficult to determine which direction it would be beneficial for it to move.'

profit-maximizing choices for the platform provider (Proposition 1), and the yellow highlighted row indicates the social-maximizing choices for the regulator (Proposition 2). Third, the welfare-maximizing price is less than the monopolist's profit-maximizing price (Proposition 3). Fourth, the welfare-maximizing price allocation for the buyer is greater than the monopolist's profit-maximizing price allocation for the buyer since the buyer's reservation price is greater than seller's reservation price (Proposition 4). Fifth, $n^B(p^B) = n^S(p^S)$ given that $d_B = d_s$ (Corollary 2 and Corollary 4). Sixth, imposing price regulation on the transaction price simultaneously increases the degree of price structure asymmetry (Proposition 6) and decreases prices on both sides of the market. Seventh, each unit reduction in the transaction price reduces the price on each side of the market by one-half unit (Proposition 7). Finally, price level regulation decreases the monopolist's profit but increases consumer surplus on both sides of the market (Proposition 8).

Table I. Platform Provider's Best Response to Price Regulation: $\alpha^M(p)$

a_B	a_S	d_B	d_S	c	p	$\alpha^M(p)$	p^B	p^S	n^B	n^S	π	CS^B	CS^S	SW
30	20	2	2	1	6.00	0.92	5.50	0.50	19.00	19.00	1805.00	90.25	90.25	1985.50
30	20	2	2	1	7.00	0.86	6.00	1.00	18.00	18.00	1944.00	81.00	81.00	2106.00
30	20	2	2	1	8.00	0.81	6.50	1.50	17.00	17.00	2023.00	72.25	72.25	2167.50
30	20	2	2	1	8.67	0.79	6.83	1.83	16.33	16.33	2045.30	66.69	66.69	2178.69
30	20	2	2	1	9.00	0.78	7.00	2.00	16.00	16.00	2048.00	64.00	64.00	2176.00
30	20	2	2	1	10.00	0.75	7.50	2.50	15.00	15.00	2025.00	56.25	56.25	2137.50
30	20	2	2	1	11.00	0.73	8.00	3.00	14.00	14.00	1960.00	49.00	49.00	2058.00
30	20	2	2	1	12.00	0.71	8.50	3.50	13.00	13.00	1859.00	42.25	42.25	1943.50

The final proposition establishes that the gains from price level regulation are increasing with the marginal cost of the transaction. This result derives from the fact that with linear demands the price elasticity increases with price and price is increasing with marginal cost, *ceteris paribus.*[4]

Proposition 9. The percentage increase in efficiency from price level regulation is increasing with the marginal cost of the transaction, c.

Example 2. Relying upon the same set of parameters as in Example 1, but allowing the marginal cost of the transaction, c, to vary, Table II and Figure 3 illustrate the welfare gains from price regulation. Recognize that as c rises, the price allocation becomes more symmetric in that it is closer to ½ on each side of the market.[5] For a relatively wide range of cost values the welfare gains from price regulation are quite small.[6] Specifically, for marginal cost values that are no more than 80% of the sum of the reservation prices, which corresponds to $c \in [0, 20]$, the welfare gains from price regulation are less than 2.5%.[7] This suggests that implementing price regulation may fail the cost-benefit test for a wide range of transaction cost values.

---

[4] See the related discussion following Proposition 3.

[5] The social welfare maximizing price is increasing in c and higher prices lead to greater price structure symmetry (Proposition 6).

[6] The platform provider's profit depends on the product of buyer-side and seller-side demand. An increase in the transaction price reduces quantity demanded on both sides of the market. These 'complementarities' engender a type of self-regulation quality that serves to discipline pricing in two-sided markets. As a result, over a wide range of cost values the incremental social welfare gains from price regulation are *de minimis*.

[7] A similar pattern emerges when we vary the demand intercepts and price-sensitivity parameters.

Table II. Marginal Cost of Transaction and Efficiency Gains

a_B	a_S	d_B	d_S	c	p^*	p^{SW}	$\alpha^M(p^*)$	$\alpha^M(p^{SW})$	$\pi(p^*)$	$\pi(p^{SW})$	$\%\Delta SW$
30	20	2	2	1	9.00	8.67	0.778	0.788	2048.00	2045.30	0.12
30	20	2	2	2	9.67	9.33	0.759	0.768	1802.52	1799.93	0.13
30	20	2	2	3	10.33	10.00	0.742	0.750	1577.48	1575.00	0.15
30	20	2	2	4	11.00	10.67	0.727	0.734	1372.00	1369.63	0.16
30	20	2	2	5	11.67	11.33	0.714	0.721	1185.19	1182.93	0.18
30	20	2	2	6	12.33	12.00	0.703	0.708	1016.15	1014.00	0.19
30	20	2	2	7	13.00	12.67	0.692	0.697	864.00	861.96	0.22
30	20	2	2	8	13.67	13.33	0.683	0.688	727.85	725.93	0.24
30	20	2	2	9	14.33	14.00	0.674	0.679	606.81	605.00	0.27
30	20	2	2	10	15.00	14.67	0.667	0.670	500.00	498.30	0.31
30	20	2	2	11	15.67	15.33	0.660	0.663	406.52	404.93	0.35
30	20	2	2	12	16.33	16.00	0.653	0.656	325.48	324.00	0.40
30	20	2	2	13	17.00	16.67	0.647	0.650	256.00	254.63	0.47
30	20	2	2	14	17.67	17.33	0.642	0.644	197.19	195.93	0.55
30	20	2	2	15	18.33	18.00	0.636	0.639	148.15	147.00	0.66
30	20	2	2	16	19.00	18.67	0.632	0.634	108.00	106.96	0.81
30	20	2	2	17	19.67	19.33	0.627	0.629	75.85	74.93	1.01
30	20	2	2	18	20.33	20.00	0.623	0.625	50.81	50.00	1.29
30	20	2	2	19	21.00	20.67	0.619	0.621	32.00	31.30	1.71
30	20	2	2	20	21.67	21.33	0.615	0.617	18.52	17.93	2.38
30	20	2	2	21	22.33	22.00	0.612	0.614	9.48	9.00	3.55
30	20	2	2	22	23.00	22.67	0.609	0.610	4.00	3.63	5.86
30	20	2	2	23	23.67	23.33	0.606	0.607	1.19	0.93	11.61
30	20	2	2	24	24.33	24.00	0.603	0.604	0.15	0.00	35.00

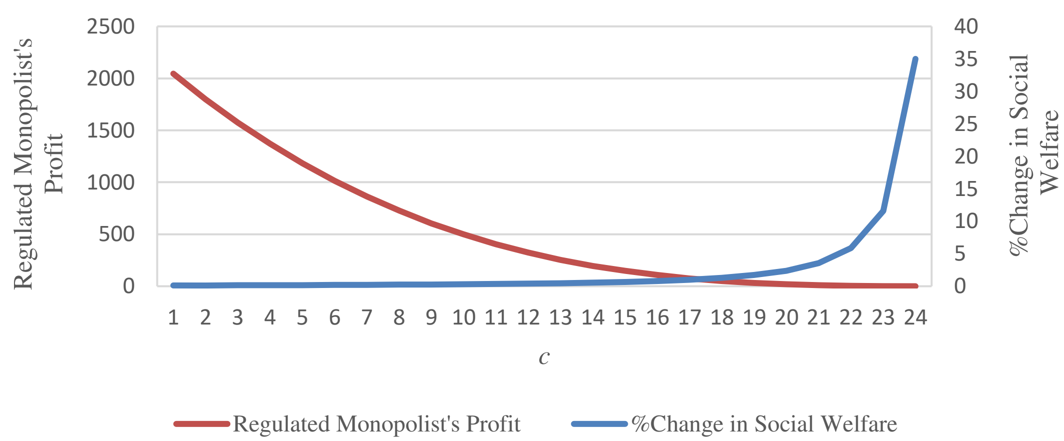

Figure 3. Efficiency Gains from Price Level Regulation

5. Conclusion

This paper analyzes price level regulation in a two-sided markets framework with linear demands. We find that (i) price regulation increases the asymmetry of the price structure when reservation prices differ across the two sides of the market; and (ii) changes in the price cap are apportioned equally between the two sides of the market irrespective of whether demands are symmetric or asymmetric. In addition, the efficiency gains from price regulation are quite small over a wide range of cost parameters. This finding may well cast doubt on the merits of price regulation in two-sided markets if the costs of regulation, broadly defined, are nontrivial.

In terms of future research, it would be interesting to explore the interaction between price level regulation and price structure regulation in a two-sided markets framework with both demand and cost uncertainty. It may also prove worthwhile to explore the welfare effects of price regulation under the Ramsey optimum and when the regulator places a higher weight on consumer surplus than on platform profits. A final, outstanding question concerns whether the findings derived herein for linear market demands hold for more general functional forms.

References

Armstrong, M. (2006) "Competition in Two-Sided Markets" *The Rand Journal of Economics* **37(3)**, 668-691.

Canadian Radio-television and Telecommunications Commission (2016) *Examination of Differential Pricing Practices Related to Internet Data Plans*. Telecom Notices of Consultation CRTC 2016-192.

European Commission (2008) *Antitrust: Commission Initiates Formal Proceedings Against Visa Europe Limited*. MEMO/08/170, Brussels.

Federal Communications Commission (2015) In the Matter of Protecting and Promoting the Open Internet. *Report and Order on Remand, Declaratory Ruling, and Order*. GN Docket No. 14-28.

Federal Communications Commission (2018) In the Matter of Restoring Internet Freedom. *Declaratory Ruling, Report and Order*. WC Docket No. 17-108.

Hemphill, S.C. (2008) "Network Neutrality and the False Promise of Zero-Price Regulation" *Yale Journal on Regulation* **25(2)**, 135-179.

Krueger, M. (2009) "The Elasticity Pricing Rule for Two-Sided Markets: A Note" *Review of Network Economics* **8(3)**, 271-278.

Pai, A. (2017) *The Future of Internet Freedom*. Remarks of FCC Chairman Ajit Pai at the Newseum, Washington, D.C. Available at https://www.fcc.gov/document/chairman-pai-speech-future-internet-regulation.

Rochet, J and J. Tirole (2003) "Platform Competition in Two-Sided Markets" *Journal of the European Economic Association* **1(4)**, 990-1029.

Rochet, J and J. Tirole (2006) "Two-Sided Markets: A Progress Report" *Rand Journal of Economics* **37(3)**, 645-667.

Schmalensee, R. (2002) "Payment Systems and Interchange Fees" *The Journal of Industrial Economics* **50(2)**, 103-122.

Wang, Z. (2016) "Price Cap Regulation in Two-Sided Markets: Intended and Unintended Consequences" *International Journal of Industrial Organization* **45**, 28-37.

Weisman, D.L. (2010) "Optimal Price Allocations in Two-Sided Markets" *Review of Network Economics* **9(3)**, 1-8.

Weyl, G.E. (2009a) "The Price Theory of Two-Sided Markets" Working Paper. Available at SSRN: https://ssrn.com/abstract=1324317 or http://dx.doi.org/10.2139/ssrn.1324317.

Weyl, G.E. (2009b) "Monopoly, Ramsey and Lindahl in Rochet and Tirole" *Economics Letters* **103(2)**, 99-100.

Weyl, G.E. (2010) "A Price Theory of Multi-Sided Platforms" *American Economic Review* **100(4)**, 1642-1672.

Wright, J. (2004) "Determinants of Optimal Interchange Fees in Credit Payment Systems" *The Journal of Industrial Economics* **52(1)**, 1-26.

Appendix

Proof of Proposition 1.

Given (3) and (4), differentiating the profit of the platform provider in (1) with respect to α and p and setting the expressions equal to zero yields

$$\frac{\partial\pi}{\partial\alpha} = [-d_B p n^S + d_S p n^B](p - c) = 0. \tag{A1}$$

$$\frac{\partial\pi}{\partial p} = -[d_B \alpha n^S + d_S(1-\alpha)n^B](p - c) + n^B n^S = 0. \tag{A2}$$

Solving (A1) and (A2) simultaneously yields (5) and (6). □

Proof of Corollary 1.

Simplifying (5) yields (9). □

Proof of Corollary 2.

(A1) yields

$$\alpha(p) = \frac{d_B d_S p - a_s d_B + a_B d_S}{2 d_B d_S p}. \tag{A3}$$

Given $p^B = \alpha p$ and $p^S = (1-\alpha)p$, the proof is immediate upon substituting (A3) into (3) and (4) and setting $d_B = d_s$. □

Proof of Proposition 2.

Given (12), differentiating social welfare with respect to α and p, and solving the resulting equations simultaneously yields (13) and (14). □

Proof of Corollary 3.

Simplifying (13) yields (17). □

Proof of Corollary 4.

Given (12), differentiating social welfare with respect to α and setting the resulting expression equal to zero yields (A3). The proof is immediate upon substituting (A3) into (3) and (4) and setting $d_B = d_s$. □

Proof of Proposition 3.

(6) and (14) yields

$$\Delta = p^* - p^{SW} = \frac{(d_B + d_S)}{3 d_B d_S} > 0. \; \square \tag{A4}$$

Proof of Proposition 4.

(5) and (13) yield

$$\Delta = \alpha^{SW} - \alpha^* = \frac{3(d_B + d_S)\,(a_B d_S - a_S d_B)}{2(a_B d_S + a_s d_B + 2c d_B d_S - d_B - d_S)(a_B d_S + a_s d_B + 2c d_B d_S)}. \tag{A5}$$

The denominator in (A5) is positive since $a_B d_S + a_s d_B + 2c d_B d_S > 0$ and $p^{SW} > 0$ implies that $a_B d_S + a_s d_B + 2c d_B d_S - d_B - d_S > 0$. Hence, $\Delta > (<)\ 0$ when $\frac{a_B}{d_B} > (<) \frac{a_s}{d_s}$. □

Proof of Proposition 5.

Given (2), differentiating the profit of the platform provider with respect to p and setting the expression equal to zero yields (18). □

Proof of Proposition 6.

It follows immediately from (19) that

$$-\frac{\partial\theta}{\partial p} = \begin{cases} > 0, & k \neq 0 \\ 0, & k = 0. \end{cases} \quad \square \tag{A6}$$

Proof of Proposition 7.

Using (18) to obtain expressions for p^B and p^S yields

$$p^B = \alpha^M(p)p = \frac{1}{2}p - \frac{k}{2d_B d_S}. \tag{A7}$$

$$p^S = \left(1 - \alpha^M(p)\right)p = \frac{1}{2}p + \frac{k}{2d_B d_S}. \tag{A8}$$

Differentiating (A7) and (A8) with respect to p, obtain

$$\frac{\partial p^B}{\partial p} = \frac{\partial p^S}{\partial p} = \frac{1}{2}. \quad \square \tag{A9}$$

Proof of Proposition 8.

Substituting (A7) and (A8) into (10) and differentiating the resulting equations with respect to p yields

$$\frac{\partial cs^B}{\partial p} = \frac{d_S p}{4} - \frac{a_S}{4} - \frac{a_B d_S}{4d_B}. \tag{A10}$$

$$\frac{\partial cs^S}{\partial p} = \frac{d_B p}{4} - \frac{a_B}{4} - \frac{a_S d_B}{4d_S}. \tag{A11}$$

Since $p < \frac{a_B}{d_B} + \frac{a_S}{d_S}$ (the sum of reservation prices), (A10) and (A11) are negative.
Substituting (A7) and (A8) into (1) and then differentiating the resulting expression with respect to p yields

$$\frac{\partial\pi}{\partial p} = \frac{d_B d_S}{4} \underbrace{\left[\frac{a_B}{d_B} + \frac{a_S}{d_S} - p\right]}_{+} \left[\frac{a_B}{d_B} + \frac{a_S}{d_S} + 2c - 3p\right] > 0, \tag{A12}$$

recognizing that the term in the second bracket is positive $\forall p < p^* = \frac{a_B d_S + a_S d_B + 2c d_B d_S}{3d_B d_S}$. □

Proof of Proposition 9.

Substituting (5) and (6) into (11) yields

$$SW^{Monopolist} = \frac{(a_S d_B + a_B d_S - c d_B d_S)^2[3(d_B + d_S) + 2(a_S d_B + a_B d_S - c d_B d_S)]}{54(d_B d_S)^2} > 0, \tag{A13}$$

recognizing that $c \leq p < \frac{a_B}{d_B} + \frac{a_S}{d_S}$.
Substituting (13) and (14) into (11) yields

$$SW^{Regulator} = \frac{[d_B + d_S + 2(a_S d_B + a_B d_S - c d_B d_S)]^3}{216(d_B d_S)^2} > 0, \tag{A14}$$

recognizing that $c \leq p < \frac{a_B}{d_B} + \frac{a_S}{d_S}$.

The percentage change in social welfare under price level regulation is given by

$$\%\Delta SW = \left(\frac{SW^{Regulation} - SW^{Monopolist}}{SW^{Monopolist}}\right) \times 100 =$$

$$\frac{25(d_B+d_S)^2[d_B+d_S+6(a_S d_B+a_B d_S-c d_B d_S)]}{(a_S d_B+a_B d_S-c d_B d_S)^2[3(d_B+d_S)+2(a_S d_B+a_B d_S-c d_B d_S)]} > 0. \qquad \text{(A15)}$$

Differentiating (A15) with respect to c yields

$$\frac{\partial \%\Delta SW}{\partial c} = \frac{150 d_B d_S([d_B+d_S][d_B+d_S+2(a_S d_B+a_B d_S-c d_B d_S)])^2}{(a_S d_B+a_B d_S-c d_B d_S)^3[3(d_B+d_S)+2(a_S d_B+a_B d_S-c d_B d_S)]^2} > 0, \qquad \text{(A16)}$$

recognizing that $c \leq p < \frac{a_B}{d_B} + \frac{a_S}{d_S}$. □