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Tax avoidance and asset returns: some theoretical results on the tax clientele effects

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Abstract

Mutual funds are held by investors in both conventional taxable savings accounts and tax-deferred retirement accounts, and some investors fall in higher tax brackets than others. Recent empirical research investigates how the marginal tax rate of a mutual fund affects the fund's tax avoidance behavior and asset returns. This paper studies theoretically the tax clientele effects on the tax avoidance and performance of mutual funds. It finds that when the marginal tax rate of a mutual fund increases, the mutual fund is more prone to deferring the realization of capital gains, the before-tax return decreases, and the after-tax return also decreases.

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1. Introduction

Economists have long paid attention to the important role investment taxes on dividends and capital gains play for mutual fund investors.[1] Mutual funds can reduce the tax burden of their shareholders by various tax management strategies, including deferring the realization of capital gains, accelerating the realization of capital losses, or avoiding securities with high dividend yields. However, such tax management strategies limit investment choices and may impede the before-tax performance of a mutual fund. Therefore, a mutual fund's tax management aims to strike an optimal balance between tax avoidance and investment performance.

One important factor determining the optimal balance between tax avoidance and investment performance must be the marginal tax rate that is relevant for a mutual fund. Mutual funds are held by investors in both conventional taxable savings accounts and tax-deferred retirement accounts, and some investors fall in higher tax brackets than others. Focusing on the tax clientele heterogeneity across mutual funds, a number of recent studies have investigated the tax clientele effects on investment choices, tax avoidance, and both before- and after-tax returns of mutual funds.

For example, Sialm and Starks (2012) evaluate the tax avoidance and returns of mutual funds held by diverse tax clienteles. They find that funds held primarily by taxable investors choose investment strategies that generate lower tax burdens, and hence are more tax efficient, than funds held primarily in tax-deferred accounts. On the other hand, they find no evidence that any investment constraints that may arise from the tax-efficient investment strategies result in a difference in returns between funds held in taxable and tax-deferred accounts. More recently, Sialm and Zhang (2015) present evidence that tax-efficient funds outperform tax-inefficient funds both before and after taxes.

To the best of our knowledge, there exist no theoretical predictions regarding the tax clientele effects on tax avoidance and asset returns with which this new line of empirical studies can be compared. This note presents a theoretical model of a two-period tax management process where at the end of the first period, the mutual fund manager decides – based on the actual return in the first period and the marginal tax rate for the mutual fund – whether to immediately realize the capital gains (losses) in the first period and pay taxes (receive tax credits) or to defer the realization of the capital gains (losses) until the end of the second period.

When the marginal tax rate is below a threshold value, we find that immediately realizing the capital gains (losses) at the end of period 1 is always the preferred choice, regardless of the actual period-1 return. When the marginal tax rate is above the threshold value, on the other hand, we find that there is a critical value of the first-period return, which depends on the marginal tax rate, and deferring the realization of capital gains (losses) is the superior strategy if the actual first-period return is above the critical value, whereas immediately realizing the capital gains (losses) is the superior strategy if the actual first-period return is below the critical value.

Comparative statics analysis with respect to the marginal tax rate yields the following predictions regarding the tax clientele effects on tax avoidance and asset returns. First, when the marginal tax rate of a mutual fund increases, the critical value of the first-period return is lower, implying a higher propensity to defer the realization of capital gains (losses). Second, when the

[1]For example, see Dickson et al. (2000), Gibson et al. (2000), Bergstresser and Poterba (2002), Shoven and Sialm (2004), Christoffersen et al. (2006), and Bergstresser and Pontiff (2013).

marginal tax rate increases, the overall (two-period) before-tax return decreases. Third, when the marginal tax rate increases, the overall after-tax return also decreases.

2. Tax Management in a Two-Period Model of Investment

Many tax management strategies are about transferring taxes to different time periods or different states of the world. Thus, a reduction of the taxes paid today might come at the cost of higher taxes in the future. To capture this tradeoff between immediate and future tax payments, we construct a two-period tax management model.

Suppose that at the beginning of the first period, a mutual fund – the size of which is normalized to one – has already been invested in a portfolio with a gross return (one plus the net return) $\tilde{R}_1 > 0$ for period 1, where the tilde means that the variable is random. Suppose that the first-period return $\tilde{R}_1$ follows a continuous distribution with a cumulative distribution function (cdf) $F(R_1)$ and a probability density function (pdf) $f(R_1)$. Note that we treat $\tilde{R}_1$ as exogenously given to focus on the tax management decision at the end of period 1. Note also that to simplify expressions, we work with gross returns instead of net returns. Therefore, a "return" in this paper always means a "gross return".

At the end of the first period and based on the actual return in the first period $R_1 > 0$ and the marginal tax rate for the mutual fund $t > 0$, the mutual fund manager decides whether to immediately realize the capital gains (losses) and pay taxes (receive tax credits) or to defer the realization of the capital gains (losses) until the end of the second period. The marginal tax rate of (the shareholders of) a mutual fund should be understood as a "weighted average" of all the relevant marginal tax rates for the constituting investment accounts in the mutual fund.

To simplify the technical aspect of the analysis, we make three assumptions. First, risk neutrality is assumed. In other words, the goal of mutual fund investment is assumed to be maximizing the expected after-tax return for the two-period duration. Second, it is assumed that investment taxes are symmetric between capital gains and capital losses. As a result, we can simply speak of capital gains and taxes with negative capital gains and taxes being understood as capital losses and tax credits, respectively. Third, transaction costs are assumed to be zero.

Suppose that the actual period-1 return is R_1 and the marginal tax rate is *t*. If the manager chooses to immediately realize the capital gains at the end of period 1 by liquidating the mutual fund's stock positions, he must pay taxes on the capital gains, and he also gets to optimally rebalance the portfolio for period 2.[2] In this case, the after-tax return in period 1 is $R_1 - (R_1 - 1)t = R_1(1-t) + t$. Denote the expected period-2 return from an optimally rebalanced portfolio as $\bar{R}_2^{OR}$, where the superscript OR stands for "optimal rebalance". Then the expected overall two-period after-tax return, conditional on the actual period-1 return R_1, is

$$\left[R_1(1-t)+t\right]\left[\bar{R}_2^{OR}(1-t)+t\right]. \tag{1}$$

If, on the other hand, the manager chooses to defer the realization of the capital gains at the end of period 1 by holding on to the mutual fund's stock positions, he postpones tax payment to the end of period 2, and at the same time he cannot rebalance the portfolio for period 2. As a

[2] When we say that the mutual fund manager pays taxes and rebalance portfolios, we mean that he does those things on behalf of the fund investors.

result, he invests the entire period-1 return R_1 to earn an expected period-2 return $\bar{R}_2^{SQ}$ on the status quo portfolio, where the superscript SQ stands for "status quo". Note that it is reasonable to assume $\bar{R}_2^{OR} > \bar{R}_2^{SQ} > 1$, both because the optimally rebalanced portfolio – chosen from a full range of available portfolios including the status quo portfolio – should outperform the status quo portfolio on average (the first inequality), and because the portfolios emerging in the equilibrium must be profitable on average (the second inequality).[3] The expected overall two-period after-tax return in this case, conditional on the actual period-1 return R_1, is

$$R_1\bar{R}_2^{SQ}(1-t)+t\,. \tag{2}$$

Given the actual period-1 return R_1 and the marginal tax rate *t*, the fund manager's optimal choice at the end of period 1 between deterring the realization of capital gains and immediately realizing the capital gains is governed by the proposition below.

<u>Proposition 1.</u> Suppose that the actual period-1 return is R_1 and the marginal tax rate for the mutual fund is t, and let

$$\begin{aligned} t_0 &= \frac{\bar{R}_2^{OR}-\bar{R}_2^{SQ}}{\bar{R}_2^{OR}-1} \\ \hat{R}_1(t) &\equiv \frac{\left(\bar{R}_2^{OR}-1\right)t}{\left(\bar{R}_2^{OR}-1\right)t-\left(\bar{R}_2^{OR}-\bar{R}_2^{SQ}\right)}, \quad \text{for } t>t_0 \end{aligned} \tag{3}$$

Then, (i) immediately realizing the capital gains is preferred if $t \le t_0$ or if $t > t_0$ and $R_1 < \hat{R}_1(t)$; (ii) deferring the realization of capital gains is preferred if $t > t_0$ and $R_1 > \hat{R}_1(t)$; and (iii) the two choices are indifferent if $t > t_0$ and $R_1 = \hat{R}_1(t)$.

<u>Proof.</u> Directly comparing (1) and (2), we have

$$(1)-(2) = (1-t)\left\{\left(\bar{R}_2^{OR}-1\right)t - R_1\left[\left(\bar{R}_2^{OR}-1\right)t-\left(\bar{R}_2^{OR}-\bar{R}_2^{SQ}\right)\right]\right\}.$$

Therefore, (i) (1) > (2) if $t \le t_0$ or if $t > t_0$ and $R_1 < \hat{R}_1(t)$; (ii) (2) > (1) if $t > t_0$ and $R_1 > \hat{R}_1(t)$; and (iii) (1) = (2) if $t > t_0$ and $R_1 = \hat{R}_1(t)$. *Q.E.D.*

One conventional wisdom on investment tax management is to defer the realization of capital gains and to accelerate the realization of capital losses. Proposition 1 provides both a qualification and a justification for this conventional wisdom. According to Proposition 1, when the marginal tax rate is sufficiently small (i.e. $t \le t_0$), it always pays to immediately realize the capital gains regardless of the size or the sign of the capital gains. The intuition for this finding – which provides a qualification for the conventional wisdom – is straightforward. When the tax rate is small enough, the benefits from optimally rebalancing the portfolio for period 2, which is only possible if the manager chooses to realize the capital gains at the end of period 1, outweighs any tax-saving benefits from deferring the realization of the capital gains. When the tax rate is sufficiently large (i.e. $t > t_0$), on the other hand, Proposition 1 says that the manager should immediately realize "small" capital gains – in the sense of $R_1 < \hat{R}_1(t)$ – but should defer the

[3] In other words, any portfolio emerging in the equilibrium must have an expected return that beats riskless government bonds.

realization of "large" capital gains – in the sense of $R_1 > \hat{R}_1(t)$ – at the end of period 1. This finding offers a justification for the conventional wisdom, with a precise interpretation to "capital gains" and "capital losses" in such a context. Note also that whether the capital gains in period 1 are regarded as "large" or "small" depends on the marginal tax rate a mutual fund faces, because $\hat{R}_1(t)$ depends on *t*. This fact has important implications for the comparative statics analysis in the next section regarding the effects of an increase in the marginal tax rate.

Based on the optimal choice at the end of period 1 described in Proposition 1, we are now ready to present the measures of the mutual fund's overall performance, both before and after tax. We begin with the expected two-period before-tax return. If $t \le t_0$, the two-period before-tax return – conditional on the actual period-1 return R_1 – is $R_1\bar{R}_2^{OR}$; if $t > t_0$, the two-period before-tax return – conditional on the actual period-1 return R_1 – is

$$\begin{matrix} R_1\bar{R}_2^{SQ}, & \text{for } R_1 > \hat{R}_1(t) \\ R_1\bar{R}_2^{OR}, & \text{for } R_1 < \hat{R}_1(t) \end{matrix}, \tag{4}$$

where the two-period before-tax return could be either $R_1\bar{R}_2^{SQ}$ or $R_1\bar{R}_2^{OR}$ when $R_1 = \hat{R}_1(t)$. Since $R_1 = \hat{R}_1(t)$ happens with probability zero, this indeterminacy does not affect the expected two-period before-tax return, which is

$$\bar{\mathbb{R}}^{BT}(t) = \begin{cases} \int_0^\infty R_1\bar{R}_2^{OR} dF(R_1), & t \le t_0 \\ \int_0^{\hat{R}_1(t)} R_1\bar{R}_2^{OR} dF(R_1) + \int_{\hat{R}_1(t)}^\infty R_1\bar{R}_2^{SQ} dF(R_1), & t > t_0 \end{cases}, \tag{5}$$

where the superscript "BT" stands for "before tax".

We then derive the expected two-period after-tax return. Conditional on the actual period-1 return R_1, the two-period after-tax return is $\left[R_1(1-t)+t\right]\left[\bar{R}_2^{OR}(1-t)+t\right]$ if $t \le t_0$. If $t > t_0$, on the other hand, the conditional two-period after-tax return is

$$\begin{matrix} R_1\bar{R}_2^{SQ}(1-t)+t, & \text{for } R_1 > \hat{R}_1(t) \\ \left[R_1(1-t)+t\right]\left[\bar{R}_2^{OR}(1-t)+t\right], & \text{for } R_1 < \hat{R}_1(t) \end{matrix}, \tag{6}$$

Therefore, the expected two-period after-tax return is

$$\bar{\mathbb{R}}^{AT}(t) = \begin{cases} \int_0^\infty \left[R_1(1-t)+t\right]\left[\bar{R}_2^{OR}(1-t)+t\right] dF(R_1), & t \le t_0 \\ \int_0^{\hat{R}_1(t)} \left[R_1(1-t)+t\right]\left[\bar{R}_2^{OR}(1-t)+t\right] dF(R_1) + \int_{\hat{R}_1(t)}^\infty \left[R_1\bar{R}_2^{SQ}(1-t)+t\right] dF(R_1), & t > t_0 \end{cases}, \tag{7}$$

where the superscript "AT" stands for "after tax".

3. The Tax Clientele Effects on Tax Avoidance and Asset Returns

We now study the effects of the marginal tax rate *t* of a mutual fund – the tax clientele effects – on tax avoidance and both before- and after-tax returns of the mutual fund. As is previously pointed out, *t* should be understood as a "weighted average" of all the relevant marginal tax rates for the constituting investment accounts in a mutual fund. *t* differs from one mutual fund to another because of the difference between the two funds in the marginal tax rates of their respective constituting investment accounts. Investment accounts held in mutual funds face different marginal tax rates for two reasons. First, these accounts can be either conventional taxable savings accounts or tax-deferred retirement accounts. Second, some taxable investors fall in higher tax brackets than other taxable investors.

Proposition 2. As the marginal tax rate t increases, a mutual fund is more prone to deferring the realization of capital gains at the end of period-1.

Proof. When t is small such that $t \le t_0$, there is zero propensity to defer the realization of capital gains regardless of the size of the actual period-1 return R_1, according to Proposition 1. When $t > t_0$, from (3),

$$\frac{d\left[\hat{R}_1(t)\right]}{dt} = \frac{-\left(\bar{R}_2^{OR}-1\right)\left(\bar{R}_2^{OR}-\bar{R}_2^{SQ}\right)}{\left[\left(\bar{R}_2^{OR}-1\right)t-\left(\bar{R}_2^{OR}-\bar{R}_2^{SQ}\right)\right]^2} < 0.$$

According to Proposition 1, when $t > t_0$, deferring the realization of capital gains is preferred if and only if $R_1 > \hat{R}_1(t)$. Therefore, a smaller $\hat{R}_1(t)$ implies that the mutual fund is more prone to deferring the realization of capital gains at the end of period 1. *Q.E.D.*

Proposition 3. As the marginal tax rate t increases, a mutual fund's expected two-period before-tax return is unchanged when $t \le t_0$, and decreases when $t > t_0$.

Proof. From (5), $\bar{\mathbb{R}}^{BT}(t)$ is constant when $t \le t_0$; when $t > t_0$,

$$\frac{d\left(\bar{\mathbb{R}}^{BT}(t)\right)}{dt} = \frac{d\left(\hat{R}_1(t)\right)}{dt}\hat{R}_1(t) f\left(\hat{R}_1(t)\right)\left(\bar{R}_2^{OR}-\bar{R}_2^{SQ}\right) < 0,$$

because $d\left(\hat{R}_1(t)\right)\big/dt < 0$, as established in the proof of Proposition 2. *Q.E.D.*

Proposition 4. As the marginal tax rate t increases, a mutual fund's expected two-period after-tax return decreases.

Proof. See the appendix.

4. Concluding Discussion

The first prediction of our model regarding the tax clientele effects (Proposition 2), namely a mutual fund's tax avoidance propensity increases in the marginal tax rate, seems to be strongly supported by the existing empirical evidence. As mentioned earlier, Sialm and Starks (2012) find that funds held primarily by taxable investors choose investment strategies that are more tax efficient than funds held primarily in tax-deferred accounts. In addition, Kawano (2014) and Lee (2017) find that investors in the upper tax bracket hold portfolios with significantly greater tax-qualified dividend yields, since the 2003 dividend tax rate reductions.[4]

In contrast, the second prediction of our model (Proposition 3), namely the before-tax return decreases in the marginal tax rate of a mutual fund, is not consistent with the existing empirical evidence. Sialm and Starks (2012) find no evidence that any investment constraints that may arise from the tax-efficient investment strategies result in lower before-tax performance

[4]The 2003 Jobs and Growth Tax Relief Reconciliation Act established the preferential tax treatment of qualified dividends relative to long-term capital gains. Dahlquist et al. (2014) also provide evidence supporting the so-called "dividend tax clientele hypothesis".

for funds held by higher tax clienteles. Moreover, Sialm and Zhang (2015) provide empirical evidence that tax-efficient funds, which tend to be held by tax clienteles with higher marginal tax rates, actually outperform tax-inefficient funds on the before-tax basis. According to Sialm and Zhang (2015), even though tax efficiency is achieved through tax avoidance activities that constrain investment opportunities of a mutual fund, and hence put a downward pressure on the before-tax return that can be earned by the fund, tax efficiency is positively correlated with lower trading costs that help provide a upward push to the before-tax return.

Finally, there exists little empirical evidence regarding the third prediction of our model (Proposition 4), namely the after-tax return decreases in the marginal tax rate of a mutual fund. The only exception, to the best of our knowledge, is Sialm and Zhang (2015) who find that tax-efficient funds outperform tax-inefficient funds not only on the before-tax basis but also on the after-tax basis.

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Appendix: Proof of Proposition 4

We demonstrate that for $\overline{\mathbb{R}}^{AT}(t)$ given in (7), $d\left(\overline{\mathbb{R}}^{AT}(t)\right)\Big/dt<0$ for both $t\le t_0$ and $t>t_0$.

(a) $t\le t_0$

$$\begin{aligned}\frac{d\left(\overline{\mathbb{R}}^{AT}(t)\right)}{dt}&=\int_0^\infty\left(1-R_1\right)\left[\overline{R}_2^{OR}(1-t)+t\right]dF(R_1)+\int_0^\infty\left(1-\overline{R}_2^{OR}\right)\left[R_1(1-t)+t\right]dF(R_1)\\&=\left(1-\overline{R}_1\right)\left[\overline{R}_2^{OR}(1-t)+t\right]+\left(1-\overline{R}_2^{OR}\right)\left[\overline{R}_1(1-t)+t\right]\\&<0,\end{aligned}$$

where $\overline{R}_1=E(\tilde{R}_1)$, and the inequality is based on the assumptions that $\overline{R}_1>1$ and $\overline{R}_2^{OR}>1$.

(b) $t>t_0$

$$\begin{aligned}\frac{d\left(\overline{\mathbb{R}}^{AT}(t)\right)}{dt}&=\frac{d\left(\hat{R}_1(t)\right)}{dt}f\left(\hat{R}_1(t)\right)\left\{\left[R_1(1-t)+t\right]\left[\overline{R}_2^{OR}(1-t)+t\right]-\left[R_1\overline{R}_2^{SQ}(1-t)+t\right]\right\}_{R_1=\hat{R}_1(t)}\\&+\int_0^{\hat{R}_1(t)}\frac{d\left\{\left[R_1(1-t)+t\right]\left[\overline{R}_2^{OR}(1-t)+t\right]\right\}}{dt}dF(R_1)+\int_{\hat{R}_1(t)}^\infty\frac{d\left[R_1\overline{R}_2^{SQ}(1-t)+t\right]}{dt}dF(R_1)\end{aligned}$$

Note that $\left\{\left[R_1(1-t)+t\right]\left[\overline{R}_2^{OR}(1-t)+t\right]-\left[R_1\overline{R}_2^{SQ}(1-t)+t\right]\right\}_{R_1=\hat{R}_1(t)}$ is simply (1) – (2) at $R_1=\hat{R}_1(t)$, and therefore equals 0. So

$$\begin{aligned}\frac{d\left(\overline{\mathbb{R}}^{AT}(t)\right)}{dt}&=\int_0^{\hat{R}_1(t)}\frac{d\left\{\left[R_1(1-t)+t\right]\left[\overline{R}_2^{OR}(1-t)+t\right]\right\}}{dt}dF(R_1)+\int_{\hat{R}_1(t)}^\infty\frac{d\left[R_1\overline{R}_2^{SQ}(1-t)+t\right]}{dt}dF(R_1)\\&=\int_0^{\hat{R}_1(t)}\left(1-R_1\right)\left[\overline{R}_2^{OR}(1-t)+t\right]dF(R_1)+\int_0^{\hat{R}_1(t)}\left(1-\overline{R}_2^{OR}\right)\left[R_1(1-t)+t\right]dF(R_1)\\&+\int_{\hat{R}_1(t)}^\infty\left(1-R_1\overline{R}_2^{SQ}\right)dF(R_1)\end{aligned}$$

It is obvious that $\int_0^{\hat{R}_1(t)}\left(1-\overline{R}_2^{OR}\right)\left[R_1(1-t)+t\right]dF(R_1)<0$. So to prove $\frac{d\left(\overline{\mathbb{R}}^{AT}(t)\right)}{dt}<0$, it is sufficient to show $\int_0^{\hat{R}_1(t)}\left(1-R_1\right)\left[\overline{R}_2^{OR}(1-t)+t\right]dF(R_1)+\int_{\hat{R}_1(t)}^\infty\left(1-R_1\overline{R}_2^{SQ}\right)dF(R_1)<0$.

Indeed,

$$
\begin{aligned}
&\int_0^{\hat{R}_1(t)}\left(1-R_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]dF(R_1)+\int_{\hat{R}_1(t)}^{\infty}\left(1-R_1\bar{R}_2^{SQ}\right)dF(R_1) \\
&=\int_0^{\hat{R}_1(t)}\left(1-R_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]dF(R_1)+\int_{\hat{R}_1(t)}^{\infty}\left(1-R_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]dF(R_1) \\
&+\int_{\hat{R}_1(t)}^{\infty}\left\{\left(1-R_1\bar{R}_2^{SQ}\right)-\left(1-R_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]\right\}dF(R_1) \\
&=\int_0^{\infty}\left(1-R_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]dF(R_1)+\int_{\hat{R}_1(t)}^{\infty}\left\{\left(1-R_1\bar{R}_2^{SQ}\right)-\left(1-R_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]\right\}dF(R_1) \\
&=\left(1-\bar{R}_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]+\int_{\hat{R}_1(t)}^{\infty}\left\{\left(1-R_1\bar{R}_2^{SQ}\right)-\left(1-R_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]\right\}dF(R_1) \\
&\le\int_{\hat{R}_1(t)}^{\infty}\left\{\left(1-R_1\bar{R}_2^{SQ}\right)-\left(1-R_1\right)\left[\bar{R}_2^{OR}(1-t)+t\right]\right\}dF(R_1) \\
&=\int_{\hat{R}_1(t)}^{\infty}\left\{\left(1-\bar{R}_2^{OR}\right)+\left(\bar{R}_2^{OR}-1\right)t-R_1\left[\left(\bar{R}_2^{OR}-1\right)t-\left(\bar{R}_2^{OR}-\bar{R}_2^{SQ}\right)\right]\right\}dF(R_1) \\
&\le\int_{\hat{R}_1(t)}^{\infty}\left\{\left(\bar{R}_2^{OR}-1\right)t-R_1\left[\left(\bar{R}_2^{OR}-1\right)t-\left(\bar{R}_2^{OR}-\bar{R}_2^{SQ}\right)\right]\right\}dF(R_1) \\
&\le 0,
\end{aligned}
$$

where the last inequality is based on the fact that $\left(\bar{R}_2^{OR}-1\right)t-R_1\left[\left(\bar{R}_2^{OR}-1\right)t-\left(\bar{R}_2^{OR}-\bar{R}_2^{SQ}\right)\right]<0$ when $t>t_0$ and $R_1 > \hat{R}_1(t)$ (see the definition of $\hat{R}_1(t)$ in equation (3)). *Q.E.D.*