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Mixed duopoly in quantity competition under the optimal privatization rate

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Abstract

This study examines a mixed duopoly in differentiated products in which a partially privatized firm and a private firm simultaneously or sequentially compete in quantity after the government sets the optimal degree of privatization for the partially privatized firm. Comparing the social welfare when the timing of decision making is different, we present the following results. First, social welfare in Cournot equilibrium is equal to that in the Stackelberg equilibrium when a partially privatized firm is the leader. Second, social welfare is the largest in the Stackelberg equilibrium when a partially privatized firm is the follower.

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1. Introduction

We examine a mixed duopoly in differentiated products in which a partially privatized firm and a private firm simultaneously or sequentially compete in quantity after the government sets the optimal degree of privatization for the partially privatized firm. Comparing the social welfare when the timing of decision making is different, we investigate which equilibrium achieves the largest social welfare.

In oligopoly theory, the difference in the timing of firms' decision-making has significantly different consequences, with the Cournot or Bertrand equilibrium classified as a simultaneous-move game and the Stackelberg equilibrium classified as a sequential-move game. Research on the order of moves between firms in oligopolistic competition has a long history. De Fraja and Delbono (1989) demonstrated that social welfare is higher in the Stackelberg equilibrium when a public firm is the leader than in the Cournot equilibrium before and after privatization. Since this seminal paper, a considerable amount of research has examined the Cournot (Bertrand) and Stackelberg equilibrium in quantity (price) competition in mixed oligopolistic settings. Pal (1998) analyzed the endogenous order of moves by adopting the observable delay game of Hamilton and Slutsky (1990) in the context of a mixed oligopoly. Bárcena-Ruiz (2007) extended the argument of the endogenous order of moves to differentiated goods to allow price competition in a simultaneous- or sequential-move game. However, most previous studies focused only on the pure public firm. As a recent exception, Méndez-Naya (2015) considered the situation in which a partially privatized firm and a private firm compete in quantity or price simultaneously or sequentially and presented the endogenous order of moves by firms in a mixed duopoly. A major strand of research on mixed oligopoly addresses the endogenous timing of firms in a mixed oligopoly.[1] However, even in Méndez-Naya (2015), the degree of privatization of a partially privatized firm is exogenously given.

Another strand of research on mixed oligopolies explores the determination of the optimal degree of privatization. Since Matsumura (1998) demonstrated that partial privatization is welfare-maximizing in a mixed duopoly, several scholars have investigated the optimal degree of privatization, including Fujiwara (2007), Ishibashi and Kaneko (2008), and Lin and Matsumura (2012).[2] In reality, the government can determine the optimal degree of privatization of a partially privatized firm through the appropriate adjustment of the firm's shareholding ratio.[3] It is a natural assumption that the government can determine the degree of privatization of a partially privatized firm endogenously to maximize social welfare.

Therefore, combining the two arguments, the different timing of decision making between both firms and the optimal degree of privatization, we investigate what timing, and whether a simultaneous move or sequential move, is desirable for social welfare.[4] Assuming that the government can determine the optimal degree of privatization of the partially privatized firm before such a firm and a private firm compete in a market, we examine which kind of equilibrium leads to higher social welfare.

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[1] For references on this, see Méndez-Naya (2015).

[2] Fujiwara (2007) derived the optimal degree of privatization in a differentiated good market. Ishibashi and Kaneko (2008) analyzed price and quality competition in a mixed duopoly. Lin and Matsumura (2012) extended their model to examine the situation in which there are foreign investors in a privatized firm.

[3] For example, the Japanese government holds about 57% of the stock of Japan Post Holdings, which is a partially privatized company that exclusively deals with postal services.

[4] Due to the limitation of space, we omit the analysis of price competition.

Comparing the social welfare when the timing of decision making is different, we present the following results. First, the social welfare in Cournot equilibrium is equal to that in the Stackelberg equilibrium when a partially privatized firm is the leader. Second, social welfare is the largest in the Stackelberg equilibrium when a partially privatized firm is the follower.

The remainder of this paper is organized as follows. Section 2 presents a model in which a partially privatized firm and a private firm engage in quantity competition simultaneously or sequentially. Section 3 derives the equilibrium results in quantity competition and presents the main results. Section 4 presents concluding remarks.

2. The model

We consider a mixed duopoly in which a partially privatized firm and a private firm compete in a differentiated goods market. The partially privatized firm and the private firm are indexed by firm 0 and 1, respectively. Firm 0 maximizes the weighted average of social welfare and profit and firm 1 maximizes its profit. Both firms produce differentiated goods and engage in duopolistic competition. q_i denotes firm i's output, $i = \{0, 1\}$.

We consider an economy that consists of a mixed duopoly market of differentiated goods and a perfectly competitive market of numeraire goods. The utility function of the representative consumer is assumed to be additively separable and linear in the numeraire goods. Following Singh and Vives (1984), the utility function is quadratic, strictly concave, and symmetric with respect to q_0 and q_1 as follows:[5]

$$U(q_0, q_1) = q_0 + q_1 - \frac{1}{2}\left(q_0^2 + q_1^2 + 2bq_0q_1\right). \tag{1}$$

$b \in (0, 1]$ denotes the degree of substitutability between the two goods. When $b = 1$ ($b = 0$), the goods are perfect substitutes (independent). From the utility function (1), we obtain both goods' linear demand functions as follows:

$$p_0 = 1 - q_0 - bq_1, \; p_1 = 1 - q_1 - bq_0. \tag{2}$$

Consumer surplus is $CS \equiv U(q_0, q_1) - p_0q_0 - p_1q_1 = \frac{1}{2}(q_0^2 + q_1^2 + 2bq_0q_1)$.

Both firms have identical technologies with increasing marginal costs. Firm i's cost function is quadratic as follows: $C(q_i) = F + \frac{1}{2}q_i^2$, where F is the fixed cost. For brevity and without loss of generality, we assume $F = 0$. Firm i's profit function is as follows:

$$\pi_i = p_iq_i - \frac{q_i^2}{2}. \tag{3}$$

Producer surplus is $PS \equiv \pi_0 + \pi_1 = p_0q_0 + p_1q_1 - \frac{1}{2}(q_0^2 + q_1^2)$. Social welfare is the sum of consumer and producer surplus, that is, $W \equiv CS + PS = q_0 + q_1 - q_0^2 - q_1^2 - bq_0q_1$.

Following Matsumura (1998), a partially privatized firm aims to maximize the weighted average of social welfare and its own profit. Thus, its objective function is as follows:

$$\Omega = (1 - \alpha)W + \alpha\pi_0, \tag{4}$$

[5] Singh and Vives (1984) considered an economy with a duopolistic market of differentiated goods and a competitive market of a numeraire good, assuming that there is no income effect of the numeraire good market on the duopoly.

where $\alpha \in [0,1]$ denotes the degree of privatization of the partially privatized firm, which determines the weight of the firm's profit in the objective function. When $\alpha = 0$, it is fully nationalized and when $\alpha = 1$, it is fully privatized. A private firm aims to maximize its own profit and a government aims to maximize social welfare. The government can determine the optimal degree of privatization α^* to maximize social welfare.

The timing of the game is the following two-stage game. In the first stage, the government sets the optimal degree of the privatization of the partially privatized firm. In the second stage, each firm sets the quantity level. The solution concept follows the subgame perfect Nash equilibrium.

In the following analysis, we consider the following three scenarios: (i) the simultaneous-move equilibrium, that is, the Cournot equilibrium, (ii) the Stackelberg equilibrium when a partially privatized firm is the leader, and (iii) the Stackelberg equilibrium when a partially privatized firm is the follower.

3. Quantity competition

The partially privatized firm and the private firm choose their output q_0 and q_1 to maximize their objective, namely, (4) and (3), respectively. Solving the first-order conditions for both firms, we obtain their reaction functions as follows:[6]

$$\frac{\partial \Omega}{\partial q_0} = 0 \Rightarrow q_0 = r_0(q_1) \equiv \frac{1 - bq_1}{2 + \alpha}, \tag{5}$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow q_1 = r_1(q_0) \equiv \frac{1 - bq_0}{3}. \tag{6}$$

The first derivatives are $r_0'(q_1) = -b/(2+\alpha) < 0$ and $r_1'(q_0) = -b/3 < 0$.

3.1 Cournot equilibrium

Solving the simultaneous equations (5) and (6) with respect to q_0 and q_1, we obtain the Cournot equilibrium output as follows:

$$\left(q_0^C(\alpha), q_1^C(\alpha)\right) = \left(\frac{3-b}{6-b^2+3\alpha}, \frac{2-b+\alpha}{6-b^2+3\alpha}\right). \tag{7}$$

Table 1 summarizes the equilibrium variables other than output.[7]

Table 1: Cournot equilibrium

firm 0's price	$p_0^C(\alpha)$	$\frac{(3-b)(1+\alpha)}{6-b^2+3\alpha}$
firm 1's price	$p_1^C(\alpha)$	$\frac{2(2-b+\alpha)}{6-b^2+3\alpha}$
firm 0's profit	$\pi_0^C(\alpha)$	$\frac{(3-b)^2(1+2\alpha)}{2(6-b^2+3\alpha)^2}$
firm 1's profit	$\pi_1^C(\alpha)$	$\frac{3(2-b+\alpha)^2}{2(6-b^2+3\alpha)^2}$
social welfare	$W^C(\alpha)$	$\frac{17-8b-2b^2+b^3+(17-7b)\alpha+2\alpha^2}{(6-b^2+3\alpha)^2}$
firm 0's objective	$\Omega^C(\alpha)$	$\frac{34-16b-4b^2+2b^3+(9-4b+5b^2-2b^3)\alpha-2(2-b)(3+b)\alpha^2-4\alpha^3}{2(6-b^2+3\alpha)^2}$

[6] By assumption, the second-order conditions for maximization are necessarily satisfied.

[7] The superscripts C, L, and F denote the equilibrium variables in the Cournot equilibrium, the Stackelberg equilibrium when the partially privatized firm is the leader, and the Stackelberg equilibrium when a partially privatized firm is the follower, respectively.

We determine the optimal degree of privatization α^{C*} to maximize social welfare. Solving the first-order condition of welfare maximization, we obtain α^{C*} as follows:[8]

$$\frac{dW^C}{d\alpha} = 0 \Rightarrow \alpha^{C*} = \frac{b(2-b)}{9-4b} > 0. \tag{8}$$

Substituting α^{C*} into (7) and Table 1, we obtain the variables in the Cournot equilibrium, as shown in Table 2.

Table 2: Cournot equilibrium when α is optimal

firm 0's output	q_0^{C*}	$\frac{9-4b}{2(9-2b^2)}$
firm 1's output	q_1^{C*}	$\frac{3(2-b)}{2(9-2b^2)}$
firm 0's price	p_0^{C*}	$\frac{9-2b-b^2}{2(9-2b^2)}$
firm 1's price	p_1^{C*}	$\frac{3(2-b)}{9-2b^2}$
firm 0's profit	π_0^{C*}	$\frac{9-4b}{8(9-2b^2)}$
firm 1's profit	π_1^{C*}	$\frac{27(2-b)^2}{8(9-2b^2)^2}$
social welfare	W^{C*}	$\frac{17-8b}{4(9-2b^2)}$
firm 0's objective	Ω^{C*}	$\frac{306-330b+113b^2-12b^3}{8(9-4b)(9-2b^2)}$

3.2 Stackelberg equilibrium when the partially privatized firm is the leader

Taking the fact that the private firm reacts following (6) into account, the partially privatized firm sets the output level to maximize its objective. From the first-order condition, we obtain the equilibrium output of the partially privatized firm:

$$\frac{\partial \Omega}{\partial q_0} = (1-\alpha)\Big(\frac{\partial W}{\partial q_0} + \frac{\partial W}{\partial q_1}r_1'\Big) + \alpha\Big(\frac{\partial \pi_0}{\partial q_0} + \frac{\partial \pi_0}{\partial q_1}r_1'\Big) = 0 \Rightarrow \; q_0(\alpha) = \frac{9-4b+b\alpha}{(9-2b^2)(2+\alpha)}. \tag{9}$$

Substituting (9) into $q_1 = r_1(q_0)$, we obtain the private firm's equilibrium output:

$$q_1(\alpha) = \frac{6-3b+(3-b^2)\alpha}{(9-2b^2)(2+\alpha)}. \tag{10}$$

Table 3 summarizes the equilibrium variables other than output.

Table 3: Stackelberg equilibrium when the partially privatized firm is the leader

firm 0's price	$p_0^L(\alpha)$	$\frac{9-2b-b^2+(9-4b-2b^2+b^3)\alpha}{(9-2b^2)(2+\alpha)}$
firm 1's price	$p_1^L(\alpha)$	$\frac{2[3(2-b)+(3-b^2)\alpha]}{(9-2b^2)(2+\alpha)}$
firm 0's profit	$\pi_0^L(\alpha)$	$\frac{(9-4b+b\alpha)[1+(2-b)\alpha]}{2(9-2b^2)(2+\alpha)^2}$
firm 1's profit	$\pi_1^L(\alpha)$	$\frac{3[3(2-b)+(3-b^2)\alpha]^2}{2(9-2b^2)^2(2+\alpha)^2}$
social welfare	$W^L(\alpha)$	$\frac{(17-8b)(1+\alpha)+(1+b)(2-b)\alpha^2}{(9-2b^2)(2+\alpha)^2}$
firm 0's objective	$\Omega^L(\alpha)$	$\frac{17-8b-2(2-b)\alpha-(4-b^2)\alpha^2}{2(9-2b^2)(2+\alpha)}$

The optimal degree of privatization α^{L*} is obtained as follows:

$$\frac{dW^L}{d\alpha} = 0 \Rightarrow \alpha^{L*} = 0. \tag{11}$$

Table 4: Stackelberg equilibrium when α is optimal

firm 0's output	q_0^{L*}	$\frac{9-4b}{2(9-2b^2)}$
firm 1's output	q_1^{L*}	$\frac{3(2-b)}{2(9-2b^2)}$
firm 0's price	p_0^{L*}	$\frac{9-2b-b^2}{2(9-2b^2)}$
firm 1's price	p_1^{L*}	$\frac{3(2-b)}{9-2b^2}$
firm 0's profit	π_0^{L*}	$\frac{9-4b}{8(9-2b^2)}$
firm 1's profit	π_1^{L*}	$\frac{27(2-b)^2}{8(9-2b^2)^2}$
social welfare	W^{L*}	$\frac{17-8b}{4(9-2b^2)}$
firm 0's objective	Ω^{L*}	$\frac{17-8b}{4(9-2b^2)}$

Thus, under the optimal degree of privatization, the partially privatized firm is fully nationalized. Substituting $\alpha^{L*} = 0$ into (9), (11), and Table 3, we obtain the equilibrium variables in this Stackelberg equilibrium, as shown in Table 4.

Comparing Tables 2 and 4, we immediately obtain the following proposition.

Proposition 1. *When the optimal degree of privatization is chosen, the equilibrium variables except for firm 0's objective are equal between Cournot and Stackelberg competition when the partially privatized firm is the leader.*

Proposition 1 implies that Cournot and Stackelberg competition achieve the same equilibrium result when the partially privatized firm is the leader. In particular, social welfare is equal in both cases, that is, $W^{C*} = W^{L*}$. A kind of privatization neutrality theorem holds in the sense that social welfare is equivalent in both cases despite the different degrees of privatization. Since $\alpha^{L*} = 0$ means full nationalization, $W^{L*} = \Omega^{L*}$ holds. Firm 0's objective is larger in this Stackelberg equilibrium than in the Cournot equilibrium, that is, $\Omega^{L*} > \Omega^{C*}$.

The reason for Proposition 1 is as follows: Social welfare in the Cournot and the Stackelberg equilibria when a partially privatized firm is the leader can be expressed as $W^C(\alpha) \equiv W\big(q_0(\alpha), q_1(\alpha)\big)$ and $W^L(\alpha) \equiv W\big(q_0(\alpha), q_1(q_0(\alpha))\big)$, respectively. The first-order conditions in both equilibria are given as follows:

$$\frac{dW^C}{d\alpha} = \frac{\partial W}{\partial q_0}\frac{dq_0^C}{d\alpha} + \frac{\partial W}{\partial q_1}\frac{dq_1^C}{d\alpha} = 0, \tag{12}$$

$$\frac{dW^L}{d\alpha} = \left[\frac{\partial W}{\partial q_0} + \frac{\partial W}{\partial q_1}r_1'\right]\frac{dq_0^L}{d\alpha} = 0. \tag{13}$$

In the Stackelberg equilibrium when a partially privatized firm is the leader, if the government fully nationalizes the public firm by setting $\alpha^L = 0$, the public firm chooses the quantity to maximize social welfare and the term inside the brackets in (12) should be zero, which implies that the first-order condition is satisfied. By contrast, in the Cournot competition, $\alpha^C > 0$ is derived as an interior solution, because $\frac{dq_0^C}{d\alpha} < 0$, $\frac{dq_1^C}{d\alpha} > 0$, and the sign of $\frac{\partial W}{\partial q_i}$ is the same. However, it implies that for both (12) and (13) to be satisfied, α^C must be chosen so that r_1' equals $\frac{dq_1^C}{d\alpha}/\frac{dq_0^C}{d\alpha}$. As a result, the equilibrium outputs must be the same in both cases, that is, $q_0^{C*} = q_0^{L*}$ and $q_1^{C*} = q_1^{L*}$, resulting in the same social welfare.

[8] The second-order conditions of social welfare with respect to output are also necessarily satisfied.

3.3 Stackelberg equilibrium when the private firm is the leader

Taking the fact that the partially privatized firm reacts following (5) into account, the private firm sets the output level to maximize its profit. From the first-order condition, we obtain the private firm's equilibrium output:

$$\frac{\partial \pi_1}{\partial q_1} = 1 - 2q_1 - bq_0 - (1 + br_0')q_1 = 0 \Rightarrow \ q_1(\alpha) = \frac{2 - b + \alpha}{6 - 2b^2 + 3\alpha}. \tag{14}$$

Substituting (14) into $q_0 = r_0(q_1)$, that is, (5), we obtain the equilibrium output of the partially privatized firm:[9]

$$q_0(\alpha) = \frac{6 - 2b - b^2 + (3 - b)\alpha}{(6 - 2b^2 + 3\alpha)(2 + \alpha)}. \tag{15}$$

Table 5 summarizes the equilibrium variables other than output.

Table 5: Stackelberg equilibrium when the private firm is the leader

firm 0's price	$p_0^F(\alpha)$	$\frac{6-2b-b^2+(9-3b-b^2)\alpha+(3-b)\alpha^2}{(6-2b^2+3\alpha)(2+\alpha)}$
firm 1's price	$p_1^F(\alpha)$	$\frac{(2-b+\alpha)(4-b^2+2\alpha)}{(6-2b^2+3\alpha)(2+\alpha)}$
firm 0's profit	$\pi_0^F(\alpha)$	$\frac{[6-2b-b^2+(3-b)\alpha]^2(1+2\alpha)}{2(6-2b^2+3\alpha)^2(2+\alpha)^2}$
firm 1's profit	$\pi_1^F(\alpha)$	$\frac{(2-b+\alpha)^2}{2(6-2b^2+3\alpha)(2+\alpha)}$
social welfare	$W^F(\alpha)$	$\frac{[6-2b-b^2+(3-b)\alpha]^2(1+\alpha)+(2-b+\alpha)(4+b-2b^2+2\alpha)(2+\alpha)^2}{(6-2b^2+3\alpha)^2(2+\alpha)^2}$
firm 0's objective	$\Omega^F(\alpha)$	$\frac{[6-2b-b^2+(3-b)\alpha]^2+2(2-b+\alpha)(4+b-2b^2+2\alpha)(1-\alpha)(2+\alpha)}{2(6-2b^2+3\alpha)^2(2+\alpha)}$

The optimal degree of privatization α^{F*} satisfies the following equation:

$$\frac{dW^F}{d\alpha} = 0 \Rightarrow X_4\alpha^4 + X_3\alpha^3 + X_2\alpha^2 + X_1\alpha + X_0 = 0, \tag{16}$$

where $X_4 \equiv 27-21b+5b^2 > 0$, $X_3 \equiv 2(81-66b+5b^2+5b^3-b^4) > 0$, $X_2 \equiv 3(108-96b-2b^2+14b^3-b^4) > 0$, $X_1 \equiv 2(108-120b+8b^2+24b^3-6b^4+2b^5-b^6) > 0$, and $X_0 \equiv -4b(2-b)(3-2b)(2-b^2) < 0$. Because (15) is a quartic equation for α, we cannot derive α^{F*} in a concise form.[10] Instead, Fig. 1 presents the result of the numerical calculation of α^{C*}, α^{L*}, and α^{F*}. As shown in the numerical calculation, irrespective of the parameter b, $\alpha^{C*} > \alpha^{F*} > \alpha^{L*} = 0$.

3.4 Welfare comparison

We compare social welfare in the above-mentioned three cases. From Proposition 1 and Fig. 2, we summarize the result as follows:

Proposition 2. *When the government chooses the optimal degree of privatization, social welfare is the largest in the Stackelberg equilibrium when the private firm is the leader. That is,* $W^{F*} > W^{C*} = W^{L*}$.

[9] There are some typographic errors in the derived output levels in Méndez-Naya (2015), although the results remain unchanged. (14) and (15) differ from those derived in the original version.

[10] The exact derivation of α^{F*} is given in the Appendix.

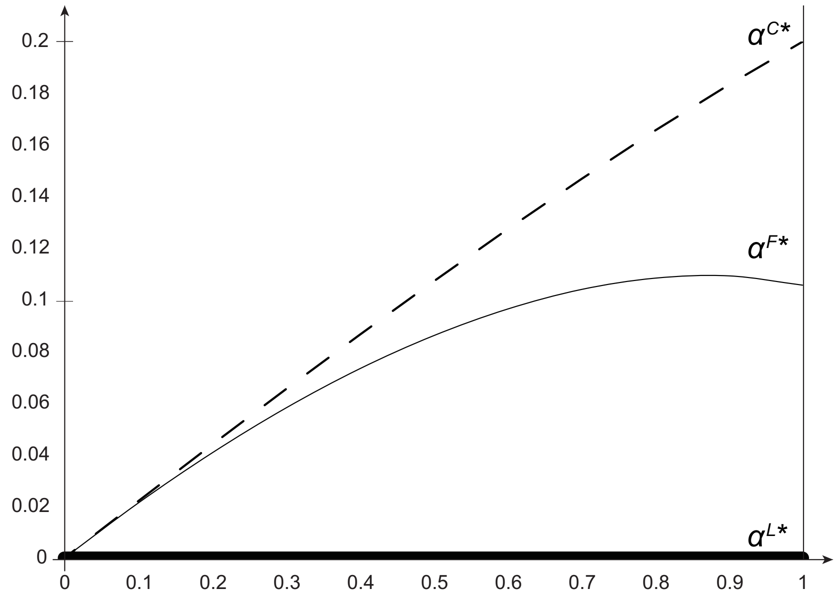

Fig. 1: The optimal degree of privatization ($\alpha^{C*} > \alpha^{F*} > \alpha^{L*} = 0$)

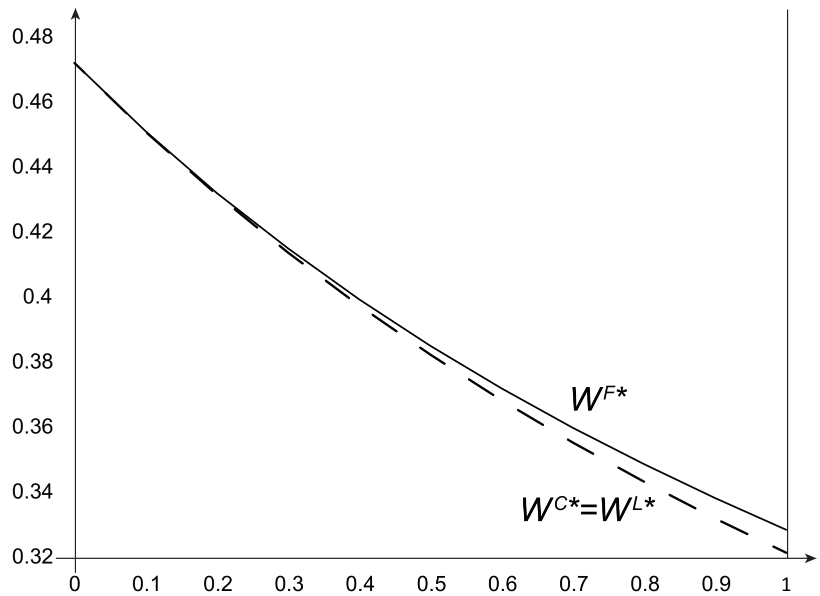

Fig. 2: Social welfare ($W^{F*} > W^{C*} = W^{L*}$)

Proposition 2 implies that the government prefers the case in which a partially privatized firm is the Stackelberg follower to other cases if it can adjust the optimal privatization rate. In other words, the government prefers the partially privatized firm to become the second-mover.

It is somewhat difficult to provide an intuitive explanation for Proposition 2 because the equilibrium cannot be derived explicitly. However, the basic logic of why Proposition 2 holds is the same as the previous result shown in the exogenous degree of privatization. Hamada (2016) demonstrated that in the competition between a fully nationalized public firm and a private firm, the Stackelberg equilibrium when the public firm is the follower achieves the highest social welfare. Thus, the second-mover advantage exists for the public firm. The second-mover advantage for the public firm applies even when a partially privatized firm and a private firm engage in market competition, as suggested in Pal (1998). Even in our model in which the government sets the optimal degree of privatization, as the Stackelberg follower, the partially privatized firm has the option to produce more to maximize social welfare after the private firm chooses its output level.

4. Concluding remarks

This study examines a mixed duopoly in differentiated products in which a partially privatized firm and a private firm simultaneously or sequentially compete in quantity after the government sets the optimal degree of privatization of the partially privatized firm. Comparing the social welfare when the timing of decision making is different, we present the following results. First, social welfare in Cournot equilibrium is equal to that in the Stackelberg equilibrium when a partially privatized firm is the leader. Second, however, the social welfare is the largest in the Stackelberg equilibrium when a partially privatized firm is the follower.

Finally, we conclude by discussing the possible extension of our results. First, we presented the results by simplifying the model in which a linear demand function and a quadratic cost function are assumed and there are only two firms, a partially privatized firm and a private firm. Although many existing studies of mixed oligopoly have adopted the same model specification, understanding whether we can generalize our results in a more general setting of demand and cost functions is left as a future research task. Second, we do not endogenize the timing of decision making. If we consider the observable delay game developed by Hamilton and Slutsky (1990), we can revisit the endogenous timing by firms in a mixed oligopoly model when the government sets the optimal degree of privatization. Because the endogenization of the timing of firms' strategic choice is an important topic, investigating the endogenous timing of decision making with the optimal degree of privatization would be another challenging issue.

References

[1] Bárcena-Ruiz, J.C. (2007) "Endogenous timing in a mixed duopoly: Price competition" *Journal of Economics* **91**, 263–272.

[2] De Fraja, G. and F. Delbono. (1989) "Alternative strategies of a public enterprise in oligopoly" *Oxford Economic Papers* **41**, 302–311.

[3] Fujiwara, K. (2007) "Partial privatization in a differentiated mixed oligopoly" *Journal of Economics* **92**, 51–65.

[4] Hamada, K. (2016) "Alternative strategies of a public enterprise in oligopoly revisited: An extension to stackelberg competition" *Bulletin of Economic Research* **68**, 85–96.

[5] Hamilton, J.H. and S.M. Slutsky. (1990) "Endogenous timing in duopoly games: Stackelberg or Cournot equilibria" *Games and Economic Behavior* **2**, 29–46.

[6] Ishibashi, K. and T. Kaneko. (2008) "Partial privatization in mixed duopoly with price and quality competition" *Journal of Economics* **95**, 213–231.

[7] Lin, M.H. and T. Matsumura. (2012) "Presence of foreign investors in privatized firms and privatization policy" *Journal of Economics* **107**, 71–80.

[8] Matsumura, T. (1998) "Partial privatization in mixed duopoly" *Journal of Public Economics* **70**, 473–483.

[9] Méndez-Naya, J. (2015) "Endogenous timing in a mixed duopoly model" *Journal of Economics* **116**, 165–174.

[10] Pal, D. (1998) "Endogenous timing in a mixed oligopoly" *Economics Letters* **61**, 181–185.

[11] Singh, N. and X. Vives. (1984) "Price and quantity competition in a differentiated duopoly" *RAND Journal of Economics* **15**, 546–554.

Appendix

A.1 The derivation of α^{F*}

We can derive the solutions of the quartic equation using the Ferrari method. The quartic equation (16) is transformed as follows:

$$\alpha^4 + Y_3\alpha^3 + Y_2\alpha^2 + Y_1\alpha + Y_0 = 0, \tag{A.1}$$

where $Y_3 \equiv \frac{X_3}{X_4}$, $Y_2 \equiv \frac{X_2}{X_4}$, $Y_1 \equiv \frac{X_1}{X_4}$, and $Y_0 \equiv \frac{X_0}{X_4}$. Substituting $\widehat{\alpha} \equiv \alpha + \frac{Y_3}{4}$ into (A.1), we obtain the following equation:

$$\widehat{\alpha}^4 + Z_2\widehat{\alpha}^2 + Z_1\widehat{\alpha} + Z_0 = 0, \tag{A.2}$$

where $Z_2 \equiv -\frac{3}{8}Y_3^2 + Y_2$, $Z_1 \equiv \frac{1}{8}Y_3^3 - \frac{1}{2}Y_3Y_2 + Y_1$, and $Z_0 \equiv -\frac{3}{256}Y_3^4 + \frac{1}{16}Y_3^2Y_2 - \frac{1}{4}Y_3Y_1 + Y_0$. By adjusting the coefficients appropriately, (A.2) can be transformed as follows:

$$(\widehat{\alpha}^2 + H)^2 = (I\widehat{\alpha} + J)^2, \tag{A.3}$$

where H satisfies $4(2H - Z_2)(H^2 - Z_0) = Z_1^2$, $I^2 \equiv 2H - Z_2$, and $J^2 \equiv H^2 - Z_0$. (A.3) is arranged as follows:

$$(\widehat{\alpha}^2 + I\widehat{\alpha} + H + J)(\widehat{\alpha}^2 - I\widehat{\alpha} + H - J) = 0 \tag{A.4}$$

$$\Leftrightarrow \widehat{\alpha} = \frac{-I \pm \sqrt{I^2 - 4(H+J)}}{2}, \frac{I \pm \sqrt{I^2 - 4(H-J)}}{2}. \tag{A.5}$$

The unique solution that satisfies $\widehat{\alpha} \in [\frac{Y_3}{4}, 1 + \frac{Y_3}{4}]$ from $\alpha \in [0, 1]$ is as follows:

$$\widehat{\alpha} = \frac{I + \sqrt{I^2 - 4(H-J)}}{2} \Leftrightarrow \alpha^{F*} = \frac{I + \sqrt{I^2 - 4(H-J)}}{2} - \frac{Y_3}{4}. \tag{A.6}$$

Through a long calculation process, it is shown that $\alpha^{F*} < \alpha^{C*}$ for all $b \in (0, 1]$. Table A.1 shows the numerical calculation of the optimal privatization rates and social welfare.

Table A.1: The optimal privatization rates and social welfare

b	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
α^{C*}	0	0.0221	0.0439	0.0654	0.0865	0.1071	0.1273	0.1468	0.1655	0.1833	0.2
α^{L*}	0	0	0	0	0	0	0	0	0	0	0
α^{F*}	0	0.0213	0.0407	0.0581	0.0733	0.0861	0.0964	0.1040	0.1084	0.1093	0.1057
W^{C*}	0.4722	0.451	0.4316	0.4138	0.3975	0.3824	0.3684	0.3554	0.3433	0.3320	0.3214
W^{F*}	0.4722	0.4512	0.4322	0.4151	0.3994	0.3851	0.3720	0.3599	0.3487	0.3383	0.3285

($W^{C*} = W^{L*}$ always holds.)