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Rent-seeking contest with two forms of sabotaging efforts

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Abstract

We study standard rent-seeking contests in which there are two forms of sabotaging. This study is conducted for a symmetric model of two contestants with complete information, when the first form of sabotaging negatively affects the probability of winning the contest while the second form increases the marginal cost of the standard rent-seeking effort. We show that only those forms of sabotage that increase the marginal cost of the rent-seeking effort affect the productive rent-seeking effort. Thus, from a designer's point of view, if monitoring is feasible, the only form of sabotaging effort that should be monitored is the second one.

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1 Introduction

Many studies of rent-seeking contests and their forms have been conducted (see, for example, Konrad, 2009 and Congleton and Hillman, 2015). In many of these more than one form of effort is exerted during the rent-seeking contest. One such case is where, in addition to the standard rent-seeking efforts (productive effort), a contestant exerts a further effort of a different nature that might reduce the probability of others winning the contest, or that might increase the opponent contestants' cost in their rent-seeking efforts. Efforts of this nature may be referred to as "sabotaging efforts" (for more details see Chowdhury and Gürtler, 2015). There are many real life examples of the two forms of sabotaging. For instance, negative political campaign advertising that reduces the probability of success of rival contestants (Skarpedas and Grofman,1995; Benhardt and Ghosh, 2020). Examples of the second form could be illegal tricks in labor contests or intentional fouls in sports contests (Amegashie, 2012).The first to study forms of sabotaging that effect the probability of winning in rent-seeking models was Konrad (2000) who showed that the effect on productive effort might be positive or negative in a simultaneous model[1]. This form of sabotaging has also been studied in models of tournaments, where it has been shown that sabotaging might be beneficial from a welfare perspective (for example, see Gürtler and Münster, 2010). The first researcher to study the second form of sabotaging in a sequential rent-seeking model with two contestants was Amegashie (2012), who showed that a sabotaging effort of this form can reduce the productive effort in a contest. In the current model proposed in this paper, we combine both kinds of sabotaging efforts in a sequential model that involves two contestants and two stages. This structure of sabotaging is well known, for example, from elections in post-Communist Russia (Hutcheson, 2007). To the best of our knowledge, no such combinations have been studied before in a rent-seeking model. In the first stage the contestant exerts both forms of sabotaging efforts in the contest, and in the second stage the productive efforts take place. We characterize the equilibrium and show that the only effort that affects the productive effort is the second form of sabotaging, while the effort that directly influences the probability of winning has no effect at all on the productive efforts.

The rest of this paper is organized as follows: In Section 2 we discuss the rent-seeking model. Section 3 presents our main results, and in Section 4 we offer our concluding discussion.

2 The Model

Consider a contest with two risk-neutral players competing to win the contest with one prize V and sabotaging options. In this contest the players have two options for sabotaging. The first option is decrease the rival's probability of winning, denoted by $s_{1,j} \geq 0$ where j stands for the number of the player $j = 1, 2$. The second option is to increase the cost of the productive effort made by $s_{2,j} \geq 0, j = 1, 2$. In the first stage, the players choose the amount of effort that they will exert on each form of sabotaging. Each player pays a marginal cost

[1] In a different form of contest model, the first to study this effect of sabotaging was Lazear (1989).

for each option of sabotaging effort, given by C_1 and C_2, respectively. In the second stage the players exert their productive denoted by $x_j \geq 0, j = 1, 2$.

The utilities of the players are given by

$$\begin{aligned} U_1 &= VP_1 - x_1(1 + s_{2,2}) - C_1 s_{1,1} - C_2 s_{2,1} \\ U_2 &= VP_2 - x_2(1 + s_{2,1}) - C_1 s_{1,2} - C_2 s_{2,2} \end{aligned} \tag{1}$$

where

$$\begin{aligned} P_1 &= \frac{x_1/(1 + s_{1,2})}{x_1/(1 + s_{1,2}) + x_2/(1 + s_{1,1})} \\ P_2 &= \frac{x_2/(1 + s_{1,1})}{x_1/(1 + s_{1,2}) + x_2/(1 + s_{1,1})} \end{aligned}$$

Notice that it is easy to see that $\frac{\partial P_i}{\partial x_i} \geq 0$ and $\frac{\partial P_i}{\partial s_{1,j}} \leq 0$ for $i \neq j$ and $i = 1, 2$. Furthermore, the effect on probability of this form of sabotaging is a special case of Konrad's (2000) simultaneous model. A sabotaging effect of this kind has been modeled before in the team contest case by Doğan et al. (2020) and in the negative campaign case by Benhardt and Ghosh (2020). The increased cost of productive effort is of the form presented by Amegashie (2012). Thus, the effect of both sabotaging efforts is the combined effect of the Konrad (2000) and Amegashie (2012) models.

3 The Main Result

In the following section we will characterize the equilibrium in our model.

We will solve for the subgame perfect equilibrium using a backward induction approach beginning with stage 2. The unique solution of this stage is based on the simple solution of a Tullock contest model (see, for example, Konrad, 2009).

$$\begin{aligned} \frac{\partial U_1}{\partial x_1} &= V \frac{x_2/(1 + s_{1,2})(1 + s_{1,1})}{(x_1/(1 + s_{1,2}) + x_2/(1 + s_{1,1}))^2} - (1 + s_{2,2}) = 0 \\ \frac{\partial U_2}{\partial x_2} &= V \frac{x_1/(1 + s_{1,2})(1 + s_{1,1})}{(x_1/(1 + s_{1,2}) + x_2/(1 + s_{1,1}))^2} - (1 + s_{2,1}) = 0 \end{aligned}$$

Rearranging, we get

$$\begin{aligned} x_1 &= \frac{(1 + s_{2,1})(1 + s_{1,2})(1 + s_{1,1})}{((1 + s_{1,2})(1 + s_{2,2}) + (1 + s_{2,1})(1 + s_{1,1}))^2} V \\ x_2 &= \frac{(1 + s_{2,2})(1 + s_{1,2})(1 + s_{1,1})}{((1 + s_{1,2})(1 + s_{2,2}) + (1 + s_{2,1})(1 + s_{1,1}))^2} V \end{aligned} \tag{2}$$

Substituting (2) in (1) we get

$$\begin{aligned} U_1 &= \left(\frac{(1 + s_{2,1})(1 + s_{1,1})}{(1 + s_{1,2})(1 + s_{2,2}) + (1 + s_{2,1})(1 + s_{1,1})} \right)^2 V - C_1 s_{1,1} - C_2 s_{2,1} \\ U_2 &= \left(\frac{(1 + s_{2,2})(1 + s_{1,2})}{(1 + s_{1,2})(1 + s_{2,2}) + (1 + s_{2,1})(1 + s_{1,1})} \right)^2 V - C_1 s_{2,1} - C_2 s_{2,2} \end{aligned}$$

In the first stage, differentiating U_1 with respect to $s_{1,1}$, and U_2 with respect to $s_{1,2}$ and differentiating U_1 with respect to $s_{2,1}$ and U_2 with respect to $s_{2,2}$ we get

$$\begin{aligned} \frac{\partial U_1}{\partial s_{1,1}} &= 2\left(\frac{(1+s_{2,1})^2(1+s_{1,2})(1+s_{2,2})(1+s_{1,1})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^3}\right)V - C_1 \\ \frac{\partial U_2}{\partial s_{1,2}} &= 2\left(\frac{(1+s_{2,2})^2(1+s_{2,1})(1+s_{1,1})(1+s_{1,2})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^3}\right)V - C_1 \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{\partial U_1}{\partial s_{2,1}} &= 2\left(\frac{(1+s_{1,1})^2(1+s_{1,2})(1+s_{2,2})(1+s_{2,1})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^3}\right)V - C_2 \\ \frac{\partial U_2}{\partial s_{2,2}} &= 2\left(\frac{(1+s_{1,2})^2(1+s_{2,1})(1+s_{1,1})(1+s_{2,2})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^3}\right)V - C_2 \end{aligned} \tag{4}$$

and

$$\begin{aligned} \frac{\partial^2 U_1}{\partial s_{1,1}^2} &= 2\left(\frac{(1+s_{2,1})(1+s_{1,2})(1+s_{2,2})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^2}\right)^2 V \\ &\quad -4\left(\frac{(1+s_{2,1})^3(1+s_{1,2})(1+s_{2,2})(1+s_{1,1})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^4}\right)V \\ \frac{\partial^2 U_2}{\partial s_{1,2}^2} &= 2\left(\frac{(1+s_{2,2})(1+s_{2,1})(1+s_{1,1})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1}1+s_{1,1}))^2}\right)^2 V \\ &\quad -4\left(\frac{(1+s_{2,2})^3(1+s_{2,1})(1+s_{1,1})(1+s_{1,2})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^4}\right)V \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{\partial^2 U_1}{\partial s_{2,1}^2} &= 2\left(\frac{(1+s_{1,1})(1+s_{1,2})(1+s_{2,2})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^2}\right)^2 V \\ &\quad -4\left(\frac{(1+s_{1,1})^3(1+s_{1,2})(1+s_{2,2})(1+s_{2,1})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^4}\right)V \\ \frac{\partial^2 U_2}{\partial s_{2,2}^2} &= 2\left(\frac{(1+s_{1,2})(1+s_{2,1})(1+s_{1,1})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^2}\right)^2 V \\ &\quad -4\left(\frac{(1+s_{1,2})^3(1+s_{2,1})(1+s_{1,1})(1+s_{2,2})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^4}\right)V \end{aligned} \tag{6}$$

$$
\begin{aligned}
\frac{\partial^2 U_1}{\partial s_{1,1}\partial s_{2,1}} &= 2\left(\frac{(1+s_{1,1})(1+s_{2,1})(1+s_{1,2})^2(1+s_{2,2})^2}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^4}\right)V \\
&+2\left(\frac{(1+s_{2,2})^2(1+s_{1,2})^2(1+s_{2,1})(1+s_{1,1})-2(1+s_{1,1})^2(1+s_{2,1})^2(1+s_{1,2})(1+s_{2,2})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^4}\right)V \\
\frac{\partial^2 U_2}{\partial s_{1,2}\partial s_{2,2}} &= 2\left(\frac{(1+s_{2,2})(1+s_{2,1})^2(1+s_{1,1})^2(1+s_{1,2})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^4}\right)V \\
&+2\left(\frac{(1+s_{2,1})^2(1+s_{1,1})^2(1+s_{2,2})(1+s_{1,2})-2(1+s_{1,2})^2(1+s_{2,2})^2(1+s_{2,1})(1+s_{1,1})}{((1+s_{1,2})(1+s_{2,2})+(1+s_{2,1})(1+s_{1,1}))^4}\right)V
\end{aligned} \tag{7}
$$

Similarly to Fu and Lu (2009) and Amegashie (2012), our focus is on the symmetric equilibrium. Accordingly, $s_{1,1} = s_{1,2} = s_1$ and $s_{2,1} = s_{2,2} = s_2$.Thus, by rewriting (3) and (4) we get

$$\frac{\partial U_1}{\partial s_1} = \frac{\partial U_2}{\partial s_1} = \frac{V}{4(1+s_1)} - C_1 \tag{8}$$

and

$$\frac{\partial U_1}{\partial s_2} = \frac{\partial U_2}{\partial s_2} = \frac{V}{4(1+s_2)} - C_2. \tag{9}$$

Thus the second derivative is a symmetric case given by

$$\frac{\partial^2 U_1}{\partial s_1^2} = \frac{\partial^2 U_2}{\partial s_1^2} = -\frac{V}{4(1+s_1)^2} \tag{10}$$

and

$$\frac{\partial^2 U_1}{\partial s_2^2} = \frac{\partial^2 U_2}{\partial s_2^2} = -\frac{V}{4(1+s_2)^2}. \tag{11}$$

Finally, the symmetric mixed derivatives are given by

$$\frac{\partial^2 U_1}{\partial s_{1,1}\partial s_{2,1}} = \frac{\partial^2 U_2}{\partial s_{1,2}\partial s_{2,2}} = 0. \tag{12}$$

Notice that according to (10) ,(11) and (12) the sabotaging efforts are separable (mixed derivatives equals to zero) and $\frac{\partial^2 U_1}{\partial s_1^2} = \frac{\partial^2 U_2}{\partial s_1^2} < 0$, $\frac{\partial^2 U_1}{\partial s_2^2} = \frac{\partial^2 U_2}{\partial s_2^2} < 0$. In other words, second order conditions for each player are satisfied. In addition, since we are considering a symmetric equilibrium, we can rewrite (2) as follows

$$x_1 = x_2 = x = \frac{V}{4(1+s_2)} \tag{13}$$

In order to analyze the equilibrium we need the following propositions .

Proposition 1 *Let* $\frac{V}{4} < \min\{C_1, C_2\}$; *then there exists a symmetric equilibrium in which the sabotaging effort is equal to zero and the productive effort equals the standard Tullock effort,* $x_1 = x_2 = x = \frac{V}{4}$.

Proof. From (8) and (9) we can see that in the case where $\frac{V}{4} < \min\{C_1, C_2\}$ we get $\frac{\partial U_1}{\partial s_1}, \frac{\partial U_2}{\partial s_1}, \frac{\partial U_1}{\partial s_2} \frac{\partial U_2}{\partial s_2} < 0$. In other words $s_1 = s_2 = 0$, and thus $x_1 = x_2 = x = \frac{V}{4}$, yielding the result. ■

Notice that if there is no sabotaging according to (13) the productive effort will be equal to standard rent-seeking efforts. This result is consistent with the Amegashie (2012) model, which includes only the second form of sabotaging. The following result will address the case where there might be a sabotaging effort, and it's effect on the productive effort.

Proposition 2 *The only sabotaging effort that might decrease the productive effort is the effort that increases the marginal cost of the productive effort.*

Proof. Follows strictly from (13). ■

The result of Proposition 2 is in line with Konrad (2000), who stated that the effect of the first form of sabotaging on the productive effort is ambiguous. The intuition behind the result is as follows: the fact that the first form of sabotaging directly affects success is a function of the Tullock contest form (ratio-form); where there is symmetry, the effect of this form of sabotaging on productive effort is nullified. Meanwhile, the second form is directly affected by the marginal cost of the productive effort, and, as mentioned in Amegashie (2012), this effort has a very high marginal effect on the cost of productive effort, and thus on the effort itself.

Notice that it is easy to see that in the current symmetric setting we get $\frac{\partial^2 P_1}{\partial s_{1,1} \partial x_1} = \frac{\partial^2 P_2}{\partial s_{1,2} \partial x_2} = 0$. According to Konrad's (2000) simultaneous model, this condition will ensure that if the first form of sabotaging effort is feasible, it will have no effect on the marginal rent-seeking effort (for more discussion on the effect of sabotaging on marginal-rent-seeking efforts see Konrad (2000)). Thus, in the general case of a standard sequential rent seeking contest success function with sabotaging, as long as $\frac{\partial^2 P_j}{\partial s_{1,j} \partial x_j} \geq 0$ (as in Konrad (2000)), the first form of sabotaging might have no negative effect on marginal rent-seeking efforts. In other words, if the first form of sabotaging is feasible, and $\frac{\partial^2 P_j}{\partial s_{1,j} \partial x_j} \geq 0$, the rent-seeking effort might not decrease as the result of such sabotaging.

4 Concluding Discussion

It is well known that sabotaging efforts in contests can have negative effects on welfare in general and, in particular, on designers of contests in which there are benefits only from productive efforts. One of the options to reduce the effects of sabotaging is to increase the cost of sabotaging (Chowdhury and Gürtler, 2015). This can be done by applying monitoring to increase the cost of sabotaging, as considered in Minchuk et al. (2018). In the case outlined in this paper, the only monitoring that affects the designers' revenue is the form that increases the marginal cost of sabotaging, and thus reduces the designers' revenue. In other words, this form of monitoring effort should be applied if it is profitable for the designer (for more details on this form of monitoring see Minchuk et al., 2018). On the other hand, no efforts are needed by the contest designer to monitor the first form of sabotaging, since it has no effect on the productive effort, in contrast to the second form of sabotaging effort which does. Thus, if the question is: should we apply regulation (if it is feasible) to control negative campaigns or/and illegal tricks in the elections, the answer is straight-forward-we should do so only for the illegal tricks. For example, in the USA, some states apply regulations limiting negative campaigning by candidates. In those campaigns, the states use their laws to regulate the time, place, and manner of speeches (Ferguson, 1997). Our model shows that this form of regulation might be inefficient (as noted in some cases by Ferguson, 1997) since it has no effect on productive efforts (standard campaigns without negativity).

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