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Modelling the Interaction of Liquidity to Price Dynamics

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Abstract

This paper studies the impact of financial liquidity on the macro-economy. We extend a classic macroeconomic model and compute numerical simulations. The model confirms that persistently low inflation can occur despite a high degree of financial liquidity due to a reallocation of cash, normal and risk-free bonds. In that regard, our model uncovers an explanation of a flat Phillips curve. Overall, our approach contributes to a rather disregarded matter in macroeconomic theory.

I would like to thank all conference participants and my research assistant Adrián Elsässer Briones for commenting a preliminary version of this paper. I thank the anonymous reviewers for good comments. All remaining errors are my own responsibility. Address: Reutlingen University and Reutlingen Research Institute (RRI). Email: Bodo.Herzog@Reutlingen-University.de

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1 Introduction

Since the global financial crisis of 2008, there is a lively debate about macroeconomic modelling. There is widespread acknowledgment that the dynamic stochastic general equilibrium models (DSGE) are challenged by reality. Even Blanchard (2018) states that those models performed poorly during the great recession, but there is little agreement on what alternative should be pursued. The models do not integrate the financial sector appropriately and fail to incorporate behaviorial notions (Stiglitz, 2018). As a consequence, the models have difficulties to explain the sluggish recovery and the low inflation. One likely reason is the flawed modelling of financial intermediation (Minsk, 1957; Brunnermeier and Sannikov, 2017).

In this paper, we provide a different approach incorporating liquidity and money inside a classic LM-model. Similar to Calvo (2016), we extend the model by two dimensions. First, we include a Taylor rule and second, we consider the continuous time dynamics of prices in respect to the level of interest rates and financial liquidity.

The main results are: First, a classic macroeconomic model, including financial liquidity, is suitable to examine the aggregate price dynamics contingent to liquidity and growth shocks. Second, our model exhibits evidence to account for low inflation in case of prolonged low interest rates and a high degree of liquidity. This surprising result is driven by the reallocation across cash, normal and risk-free bonds. The macroeconomic conditions unravel this unique anomaly in the aftermath of the global financial crisis.

The article is structured as follows. Section 2, describes the literature. Section 3 discusses the model including the extension. In section 4, we simulate the model. Concluding remarks are in section 5.

2 Literature Review

This paper is related to at least two areas in the literature. First, the literature of macroeconomics, inflation, and monetary policy. This field investigates the zero lower bound (ZLB) on interest rates and argues that inflation targeting are likely to fail (Benhabib et al., 2001; Dupor, 1999). The reason is that the existence of a ZLB implies that rational expectations solutions are not unique and one solution involves a deflationary liquidity trap (McCallum, 2001). There is similar work by Eggertsson and Woodford (2003) and Levin et al. (2010). Yet, our approach is different because we study the anomaly of liquidity and the ZLB on price dynamics.

The second related field of literature is the classic monetary theory. We extend a standard model by Calvo (2016). Our theory includes liquidity, risk-free bonds and a Taylor rule. Similarly to Steinsson (2003) and Hasui et al. (2016), we study financial liquidity, however differently, we focus on the price dynamics. Somewhat related is the work by Cochrane (2017), yet the underlying mechanism is different in our work.

There is likewise a new literature about financial frictions in order to explain some of the modelling flaws. This research builds on earlier work by Bernanke et al. (1999), Kiyotaki and Moore (1997), Bianchi (2011) and others (Samuelson, 1958; Bewley, 1977; Herzog, 2015; Moreira and Savov, 2017). This work is rediscovered by Brunnermeier and Sannikov (2017).

3 Model

The model consists of a liquidity-money (LM) market. The price theory of money is extended by financial liquidity. Hence, the extended model is given by

$$\frac{M}{P} + \xi\left(\frac{M}{P}\right) = L(i - i^m, y). \tag{1}$$

where ξ denotes the liquidity supply function with the first derivative of $\xi'(.) < 0$, M is the money supply, P the price level, y the output and i the nominal interest rate. In order to study the zero-lower bound, the central bank adjusts the interest rate on money i^m. However, the opportunity cost of holding liquidity is defined as $i - i^m$. Thus, the liquidity demand function on the right-hand side is of $L(i - i^m, y)$. Note, a interest rate cut of $i^m \downarrow$ by the central bank causes increasing opportunity cost of holding money $[i - i^m] \uparrow$ and consequently a declining demand of money $L(i - i^m, y) \downarrow$ because of a reallocation of the portfolio towards risk-free bonds. The liquidity demand function has usual first-order derivatives of $\partial L/\partial(i - i^m) < 0$ and $\partial L/\partial y > 0$. Furthermore, if real money supply increases (M/P), the overall impact on the left-hand side of equation (1) is ambiguous.

Next, we consider the behavior of a representative agent under the assumption of flexible prices. The agent maximizes the discounted expected utility function given by

$$\mathbb{E}[U] = \int_0^\infty [u(c_t) + v(m_t) + h(b_t)]e^{-\rho t} dt, \tag{2}$$

where $u(c_t)$ represents consumption utility, $v(m_t)$ money utility, $h(b_t)$ cash utility from a risk-free bond. All utility functions have standard derivatives such as $u', v', h' > 0$ and $u'', v'', h'' < 0$. Another differentiating feature is the function $h(b_t)$ in this model. The agents' overall (real) wealth is defined by $w_t = m_t + b_t + x_t$, where x_t is a normal bond with no liquidity service. Noteworthy, in equilibrium $x_t = 0$. The interest on x_t is i and the interest on b_t is z. Similarly, the interest on money is of zero.

The real interest rate is determined by the Fisher equation $i_t = r_t + \pi_t$. Thus, the agents' real budget constraint has the following outcome

$$\begin{aligned} \dot{w}_t &= y - c_t + (0 - \pi_t)m_t + (z_t - \pi_t)b_t + (i_t - \pi_t)x_t \\ &= y - c_t + r_t w_t - i_t m_t + (z_t - i_t)b_t. \end{aligned} \tag{3}$$

The final equation is a first-order ordinary differential equation (ODE). It can be rewritten in respect to the wealth dynamics w_t:

$$\dot{w}_t - r_t * w_t = y - c_t - i_t m_t - (i_t - z_t)b_t. \tag{4}$$

Proposition 1 *The first-order inhomogeneous ordinary differential equation (4) has a solution given by*

$$w_t = w_0 + \int_0^\infty [y - c_t - i_t m_t - (i_t - z_t)b_t]e^{-\rho t}dt. \tag{5}$$

The proof of Proposition 1 is in Appendix A. Equation (5) is a familiar expression in standard monetary models. Note b_t is devoted to liquidity. In equilibrium of the standard model, we have $i = z$ and thus the b_t-term drops out. However, in our model, the real liquidity services imply $i > z$ in equilibrium.

The representative agent maximizes the discounted expected utility, subject to the wealth process w_t, such as

$$\begin{aligned} &\max_{c_t, m_t, b_t} \int_0^\infty [u(c_t) + v(m_t) + h(b_t)]e^{-\rho t}dt \\ &\text{s.t.} \qquad w_t = w_0 + \int_0^\infty [y - c_t - i_t m_t - (i_t - z_t)b_t]e^{-\int_0^t r_s ds}dt \end{aligned} \tag{6}$$

The first-order conditions are

$$\frac{\partial L}{\partial c_t} = u'(c_t)e^{-\rho t} - \lambda[1 * e^{-\int_0^t r_s ds}] \Rightarrow u'(c_t)e^{-\rho t} = \lambda R_t$$
$$\frac{\partial L}{\partial m_t} = v'(m_t)e^{-\rho t} - \lambda[i_t * e^{-\int_0^t r_s ds}] \Rightarrow v'(m_t)e^{-\rho t} = \lambda i_t R_t$$
$$\frac{\partial L}{\partial b_t} = h'(b_t)e^{-\rho t} - \lambda[(i_t - z_t)e^{-\int_0^t r_s ds}] \Rightarrow h'(b_t)e^{-\rho t} = \lambda(i_t - z_t)R_t,$$

where $R_t := e^{-\int_0^t r_s ds}$ (Appendix B). In order to close the model, we assume a interest rate rule for the central bank. There are several possibilities. Calvo (2016) considers a simple rule such as $z_t = \theta\pi$ where $\theta > 0$. This can be interpreted in the way that higher inflation leads to higher interest rates and vice versa. We utilize a Taylor rule for modelling the central bank reaction function (Taylor, 1993). The monetary policy rule[1] is

$$z_t = c_1(\pi - \pi^*) + c_2 y_t + c_3\bar{z}, \tag{7}$$

where c_1, c_2, and c_3 represent Taylor's constants for the inflation and output gap and $\bar{z}$ denotes the central bank's estimate of the real rate of interest. Using equations (A6) and (A7) and the monetary policy rule in equation (7), we obtain for total liquidity:

$$\frac{Z}{P_t} = m + b = \phi(\rho + \pi_t) + \psi(\rho + (1 - c_1)\pi_t + \pi^* + c_2 y_t + c_3\bar{z} + \epsilon_t), \tag{8}$$

where capital letters denote aggregate variables, such as price level P_t and total liquidity Z and ϵ_t is an i.i.d. random variable. This is an implicit first-order ODE of the price level P_t. Note that $\pi_t = \dot{P}_t/P_t$.

4 Results

In order to simulate the implicit differential equation (8), we need additional assumptions about the utility functions. We assume that the utility functions have logarithmic form. This implies that the inverse functions of $\phi(\rho + \pi_t)$ and $\psi(\rho + (1 - c_1)\pi_t + \pi^* + c_2 y_t + c_3\bar{z}_{t-1} + \epsilon_t)$ are of exponential form. Given that the first-order derivatives of ϕ' and ψ' are both negative, we

[1] Including randomness, we obtain: $z_t = c_1(\pi - \pi^*) + c_2 y_t + c_3\bar{z} + \epsilon_t$.

rewrite the ODE in the following form $\frac{Z}{P_t} = e^{-(\rho+\pi_t)} + e^{-(\rho+(1-c_1)\pi_t+\pi^*+c_2 y_t+c_3\bar{z}+\epsilon_t)}$, and finally, we rewrite the ODE in terms of prices:

$$\dot{P}_t = -\left[\ln\frac{Z}{P_t} + (2\rho + \pi^* + c_2 y_t + c_3\bar{z} + \epsilon_t)\right]\frac{P_t}{(2-c_1)}. \tag{9}$$

We simulate this ODE with the ODE45-algorithm in MATLAB under the following two conditions: (i) the degree of liquidity increases in the range of $Z = [0;5;10;15;20]$, and (ii) the real interest rate ρ varies in the range of $\rho = [0;0.5;1]$. Note, the nominal rate is of $i_t = \rho + \pi_t$. The inflation target is of $\pi^* = 2.0$ and the parameters of the Taylor rule are according to the original policy rule establishes by Taylor (1993, 1999): $c_1 = c_2 = 0.5$ and $c_3 = 0.2$.

Figure (1) summarizes the dynamics of prices to a shock in the policy rate z over time. Note, the left-hand panel denotes the case at the real zero-lower-bound, where $\rho = 0$. Here, the nominal interest rate i_t is determined by $i_t = 0 + \pi_t$ or in steady state $i_t = \pi_t = 0$, where $\pi_t = \dot{P}_t/P_t$. The middle panel assumes a real rate of $\rho = 0.5$ and the right-hand panel captures the case of $\rho = 1$.

The simulation reveals a novel insight of inflation in regard to financial liquidity. There are two robust findings: Firstly, at the zero-lower-bound any degree of liquidity is ineffective as we obtain an immediate pass through to prices (left-hand panel)[2]. Secondly, if rates are positive as in normal times, we observe a weaker pass-through dynamics in case of a high degree of financial liquidity (middle and right-hand panel). In all scenarios, the steady-state level is contingent on both the degree of financial liquidity and the level of interest rates (Figure 1).

What drives this unconventional result is a new liquidity channel of money as well as normal versus risk-free bonds. Indeed, a high degree of liquidity does not necessarily increase spending because households might reallocate the holding of money and normal bonds without liquidity services.[3] This mitigates the price dynamics and steady state price level, particularly at the ZLB. At the ZLB agents start hoarding risk-free bonds instead of investing. This observation might be a new insight to the weak inflation dynamics in the aftermath of the global financial crisis (Minsk, 1957; Admati and Hellwig, 2014).

Furthermore, Figure (1) demonstrates that the steady state price level declines, the lower the degree of liquidity. Surprisingly, the steady state is significantly positive and persistent only at the medium real interest rate

[2]The same pattern occurs with negative real interest rates.

[3]This is particularly visible in high saving countries with negative yields (Germany).

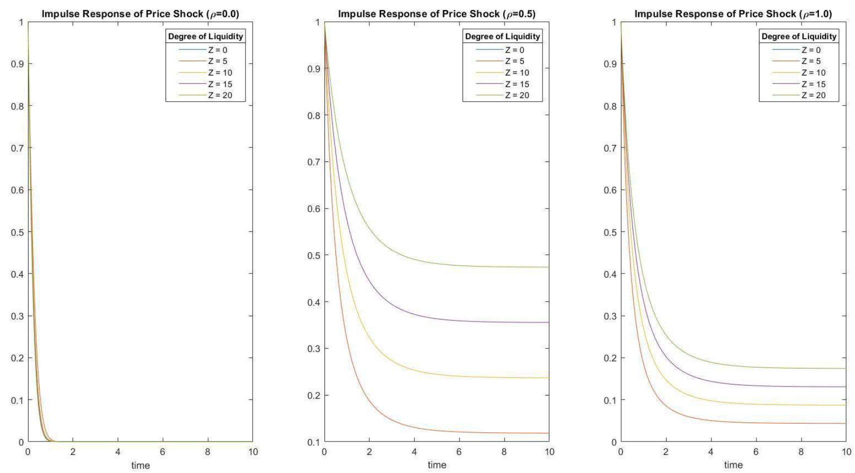

Figure 1: Simulation Results

levels. Under those circumstances, the price level even grows along with the degree of liquidity (middle panel). Consequently, the simulation exercise provides an explanation of persistently low inflation at the zero lower bound.

Note, the simulation result is likely contingent on the assumption of log-preferences, which affect the cross-derivatives of i and z, and the utilization of separable household preferences on m, b and c. Nonetheless, the model discovers a new interaction-channel between liquidity and prices. A prolonged monetary expansion at the ZLB might not lift prices; at least in our framework.

5 Conclusion

We extend a classic macroeconomic model with liquidity. We find that the inflation dynamics is reliant on the interest rate level and the degree of financial liquidity. In our model, a policy of low interest rates is conducive to a sharp drop of the steady state price level. We corroborate the possibility of a rapid pass through of liquidity to persistently low prices by simulation. Under those circumstances, expansionary monetary policy is virtually insufficient to lift inflation. Consequently, the model offers one explanation for a flat Phillips curve.

Nonetheless, there is the need for further research. It would be interesting to investigate this mechanism in an equilibrium framework. We believe this is a promising attempt to learn more about this relationship.

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Appendix A

Proof of Proposition 1. The general solution of the first-order inhomogeneous ODE can be decomposed into the sum of a homogenous (hom) and a particular (par) solution: $w_t = w_t^{hom} + w_t^{par}$.

First, we compute the homogenous solution of equation (4), where $w_t - r_t w_t = 0$. Here the solution is simply $w_t^{hom} = c * e^{\int_0^t r_s ds} = w_0 * e^{\int_0^t r_s ds}$ together with the initial condition: $w_0 = c * e^{\int_0^0 r_s ds} = ce^0 = c$.

Second, we compute a particular solution. Assume $w^{par} = c_t * e^{\int_0^t r_s ds}$. Next, compute the derivative in respect to time:

$$\dot{w}_t = c_t' * e^{\int_0^t r_s ds} + c_t * e^{\int_0^t r_s ds} * r_t.$$

Next, substitute the budget constraint $\dot{w}_t = r_t w_t + y - c_t - i_t m_t + (z_t - i_t) b_t$ from equation (4) on the left-hand side. We obtain

$$r_t w_t + y - c_t - i_t m_t - (i_t - z_t) b_t = c_t' * e^{\int_0^t r_s ds} + \underbrace{c_t * e^{\int_0^t r_s ds}}_{=w_t} * r_t$$

$$y - c_t - i_t m_t - (i_t - z_t) b_t = c_t' * e^{\int_0^t r_s ds},$$

or $c_t' = [y - c_t - i_t m_t - (i_t - z_t) b_t] e^{-\int_0^t r_s ds}$. Finally, integration of the last equation yields

$$c_t = \int_0^\infty [y - c_t - i_t m_t - (i_t - z_t) b_t] e^{-\int_0^t r_s ds} dt,$$

and the particular solution is of

$$w_t^{par} = c_t * e^{\int_0^t r_s ds} = \left[\int_0^\infty [y - c_t - i_t m_t - (i_t - z_t) b_t] e^{-\int_0^t r_s ds} dt \right] * e^{\int_0^t r_s ds}.$$

In summary, we obtain the general solution

$$
\begin{aligned}
w_t &= w_t^{hom} + w_t^{par} \\
&= w_0 * e^{\int_0^t r_s ds} + [\int_0^\infty [y - c_t - i_t m_t - (i_t - z_t) b_t] e^{-\int_0^t r_s ds} dt] * e^{\int_0^t r_s ds} \\
&= \left(w_0 + \int_0^\infty [y - c_t - i_t m_t - (i_t - z_t) b_t] e^{-\int_0^t r_s ds} dt \right) * e^{\int_0^t r_s ds}.
\end{aligned}
$$

Ruling out a Ponzi game $\lim_{t\to\infty} w_t e^{-\int_0^t r_s ds} \geq 0$, implies a non-negative wealth such as:

$$
\lim_{t\to\infty} w_t e^{-\int_0^t r_s ds} = \lim_{t\to\infty} \left(w_0 + \int_0^\infty [y - c_t - i_t m_t - (i_t - z_t) b_t] e^{-\int_0^t r_s ds} dt \right) \geq 0.
$$

Hence, we obtain $w_t = w_0 + \int_0^\infty [y - c_t - i_t m_t - (i_t - z_t) b_t] e^{-\int_0^t r_s ds} dt$. After assuming that $r = \rho = \int_0^t r_s ds = constant$, we obtain equation (5) □

Appendix B

Solution of Optimization. Set up the Lagrange function

$$
L(c_t, m_t, b_t) = \int_0^\infty [u(c_t) + v(m_t) + h(b_t)] e^{-\rho t} dt - \lambda \left[\bar{w} - w_0 - \int_0^\infty [y - c_t - i_t m_t - (i_t - z_t) b_t] e^{-\int_0^t r_s ds} dt \right]
$$

and compute the first-order conditions:

$$
\frac{\partial L}{\partial c_t} = u'(c_t) e^{-\rho t} - \lambda [1 * e^{-\int_0^t r_s ds}] \Rightarrow u'(c_t) e^{-\rho t} = \lambda R_t \tag{A1}
$$

$$
\frac{\partial L}{\partial m_t} = v'(m_t) e^{-\rho t} - \lambda [i_t * e^{-\int_0^t r_s ds}] \Rightarrow v'(m_t) e^{-\rho t} = \lambda i_t R_t \tag{A2}
$$

$$
\frac{\partial L}{\partial b_t} = h'(b_t) e^{-\rho t} - \lambda [(i_t - z_t) e^{-\int_0^t r_s ds}] \Rightarrow h'(b_t) e^{-\rho t} = \lambda (i_t - z_t) R_t, \tag{A3}
$$

where $R_t = e^{-\int_0^t r_s ds}$. Next, we define total liquidity by $Z_t = M_t + B_t$. The central bank sets the interest rate z_t on the liquid bond b_t. The real variables are indicated by lower case letters such as $m_t = \frac{M_t}{P_t}$ and $b_t = \frac{B_t}{P_t}$. Therefore, total liquidity is

$$Z_t = m_t * P_t + b * P_t = (m_t + b_t)P_t. \tag{A4}$$

Solving for m_t and b_t in equation (A2) and (A3), while assuming that $r = \rho = \int_0^t r_s ds = constant$, yields the following conditions

$$u'(c_t) = \lambda R_t e^{\rho t} = \lambda e^{-\rho t} e^{\rho t} = \lambda \tag{A5}$$

$$v'(m_t) = \lambda i_t \Rightarrow m := \phi(i_t) = \phi(\rho + \pi_t) \tag{A6}$$

$$h'(b_t) = \lambda(i_t - z_t) \Rightarrow b := \psi(i_t - z_t) = \psi(\rho + \pi_t - z_t), \tag{A7}$$

where $\phi' < 0$ and $\psi' < 0$ $\square$