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Constrained Fixed Sample Search

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Abstract

We consider a version of the Stigler model of fixed sample price search, where consumer utility depends on whether or not at least one sampled price fulfils a pre-set target. We establish that search intensity is a non-monotonic function of this target. We also point out a hitherto unexamined disadvantage of fixed sample search compared to sequential search.

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1. Introduction

In the classic Stigler (1961) model of fixed sample (or pre-determined) price search, a risk-neutral consumer ends up paying the best (lowest) encountered price. In this paper, we change this setup: here, the consumer obtains additional utility if at least one sampled price (and hence the price paid) is lower than a pre-set target (constraint).

The concept of constrained search was introduced by Bonilla et al. (2019), in the context of sequential search. They derive in detail and fully characterise the optimal reservation value strategy, which turns out to be a function of the constraint (target). Interestingly, despite the discontinuity in the decision problem (stemming from the utility gain obtained by the searcher from fulfilling the target), the reservation value function is shown to be continuous. In addition, the search intensity is non-monotonic: increasing for easy targets, decreasing for tougher ones.

In this paper, we ask: what is the effect of a price target on the optimal fixed sample search strategy? On the one hand, although the analysis is completely different, we establish that the non-monotonicity property of search intensity (here captured by sample size) is robust across the two standard search methods. For relatively high (easy) price targets the searcher will increase the optimal sample size as this target decreases, but will reduce it for relatively low prices as ever harder targets are increasingly difficult to fulfil. On the other hand, our results in this paper also point to a hitherto unexamined disadvantage of the fixed sample search method compared to sequential search.

2. Analysis

A consumer aims to buy one unit of some good, but does not know the price charged by any particular shop. Assuming a continuum of shops, from the perspective of the consumer, prices are distributed according to the cumulative probability distribution function $F(.)$ with support $[\underline{p}, \overline{p}]$. At cost c per shop (same for all shops), the consumer obtains a quote with the exact price charged by a shop.[1] The consumer uses the fixed sample search strategy, choosing the optimal number of shops to contact, given that the objective is to buy at the best possible price. We follow the original Stigler model in that price quotes are received *simultaneously* and no price on the market is considered to be so high as to prevent the purchase.

Crucially, we assume that the consumer receives additional utility (here, a payment reduction of y) if at least one sampled price is no higher than an ex-ante set price target $\widehat{p}$. This is a highly stylised setup, but one that can relatively easily be embedded into a richer framework. For example, one could think of the multiple-good demand problem of a consumer who has a budget constraint and can only afford one good if the price of the other (obtained through search) is low enough.[2]

Let N denote the number of sampled shops. Following the formal analysis in Hey (1979) and the diagrammatic treatment in McKenna (1986), assume that N is a continuous choice variable - it simplifies the analysis of what is of course an integer problem. Let $p(N)$ denote the expected best (lowest) price observed if the sample is of size N. The random variable $p(N)$ is characterised by the cumulative probability distribution function $G(.)$ with support $[\underline{p}, \overline{p}]$.

In our version of the story, if the consumer commits to sampling N shops, the expected overall payment is $p(N) + cN$ if all sampled prices are above $\widehat{p}$, and it is $p(N) + cN - y$ if at least one quote received is no higher than $\widehat{p}$. The consumer's problem is therefore:

$$\min_N L \equiv [1 - G(\hat{p})] \left[\int_{\widehat{p}}^{\overline{p}} p dG(p)/[1 - G(\widehat{p})] + cN \right] + G(\hat{p}) \left[\int_{\underline{p}}^{\widehat{p}} p dG(p)/G(\widehat{p}) + cN - y \right],$$

[1] In contrast, the so-called clearinghouse search model assumes that upon payment of a fixed cost c, a consumer may become fully informed. This is more relevant for online search.

[2] For the pioneering work on this, see Burdett and Malueg (1981) and Manning and Morgan (1982).

where

$$G(\hat{p}) = 1 - [1 - F(\hat{p})]^N.$$

The expected best price if N shops are sampled is:

$$p(N) = \int_{\underline{p}}^{\bar{p}} p dG(p) = \underline{p} + \int_{\underline{p}}^{\bar{p}} [1 - F(p)]^N \, dp,$$

where the latter equality is derived using integration by parts.

Thus, the optimisation problem simplifies to:

$$\min_N L \equiv p(N) + cN - G(\widehat{p})y. \tag{1}$$

One can think of the optimally chosen sample size N^* as a measure of search intensity. We show that the optimal search intensity is non-monotonic in the target price $\widehat{p}$. As soon as the constraint starts to bite ($\widehat{p} < \overline{p}$), the consumer increases the sample size. However, the incentive to do so decreases after a certain threshold price, when the additional costs exceed the increasingly unlikely benefits of increased sampling.

To see this, first observe that for $\hat{p} \leq \underline{p}$ the target price constraint cannot possibly be fulfilled and, since $F(\hat{p}) = F(\underline{p}) = 0$, the optimal sample size N^* is obtained by solving:

$$\min_N L \equiv \underline{p} + \int_{\underline{p}}^{\bar{p}} [1 - F(p)]^N \, dp + cN.$$

In turn, for $\hat{p} \geq \bar{p}$ the target price constraint is always fulfilled and, since $F(\hat{p}) = F(\bar{p}) = 1$, now N^* is the solution to:

$$\min_N L \equiv \underline{p} + \int_{\underline{p}}^{\bar{p}} [1 - F(p)]^N \, dp + cN - y.$$

As y is just a constant, the optimal sample size is clearly the same in both cases (denote it by N_0), and is obtained from the first-order condition $dL/dN = 0$:

$$\int_{\underline{p}}^{\bar{p}} [1 - F(p)]^N ln(1 - F(p)) dp + c = 0.$$

For $\widehat{p} \in (\underline{p}, \overline{p})$, the first-order condition for an optimum in problem (1) simply equates the expected marginal benefit from an increased sample, and the marginal cost c:

$$-\int_{\underline{p}}^{\bar{p}} [1 - F(p)]^N ln(1 - F(p)) dp - [1 - F(\widehat{p})]^N \ln(1 - F(\widehat{p}))y = c. \tag{2}$$

The expected marginal benefit $MB(N, \widehat{p})$ has two components, as an increase in N has two effects. Ceteris paribus (in particular, for a given $\widehat{p}$), higher search intensity results in a lower expected best price as well as a higher probability of obtaining the additional utility y (the second term being simply $\partial G(\widehat{p})/\partial N$).

Since $\ln(1 - F(p)) < 0$, the expected marginal benefit is positive. It can also be shown to be decreasing in sample size:

$$\frac{\partial MB}{\partial N} = -\int_{\underline{p}}^{\overline{p}} [1 - F(\widehat{p})]^N [\ln(1 - F(\widehat{p}))]^2 dp - [1 - F(\widehat{p})]^N [\ln(1 - F(\widehat{p}))]^2 y < 0.$$

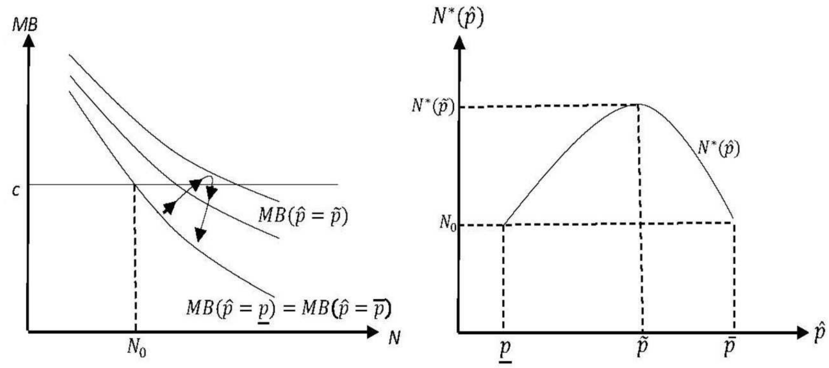
Then, the optimal sample size N^* balances these expected benefits with the additional cost of increasing the sample size (and technically it will of course be one of the two integers nearest to the N that satisfies (2)).

Furthermore, when graphed as a function of N, the downward sloping $MB(N, \widehat{p})$ curve is identical for both $\widehat{p} = \underline{p}$ and $\widehat{p} = \overline{p}$, and it moves as $\hat{p}$ changes. To see this, please note that:

$$\frac{\partial MB}{\partial \widehat{p}} = [1 - F(\widehat{p})]^{N-1} f(\widehat{p}) y \left[N \ln(1 - F(\widehat{p})) + 1\right].$$

As $\widehat{p}$ approaches $\underline{p}$ from the right, the above derivative is positive; in turn, as $\widehat{p}$ approaches $\overline{p}$ from the left, the derivative approaches $-\infty$. Since $\ln(1 - F(\widehat{p}))$ is strictly monotonic and decreasing, $\partial MB/\partial \widehat{p}$ is zero for a unique target price $\widehat{p} \in (\underline{p}, \overline{p})$, which we denote by $\widetilde{p}$. Figure 1(a) captures the above:

Figure 1



(a) Determination of optimal sample size (N^*)

(b) Optimal sample size

For $\widehat{p} \in (\widetilde{p}, \overline{p})$ the marginal benefit curve tilts and shifts to the right as $\widehat{p}$ decreases. In turn, for $\widehat{p} \in (\underline{p}, \widetilde{p})$ it tilts and it shifts to the left as $\widehat{p}$ decreases. A decrease of $\widehat{p}$ from $\overline{p}$ creates incentives to increase the sample size, so as to decrease expected best price and improve the chances of fulfilling a relatively easy target price. However, the effect of this on both $p(N)$ and $G(\widehat{p})$ is decreasing, so at some point it is not worth it. It is this second negative effect that is novel here, and it stems from the fact that it is of course increasingly difficult to hit relatively low pre-set price targets.

We can further characterise the optimal strategy (also see Figure 1 (b)). Equation (2) determines the optimal sample size N^* as a function of $\widehat{p}$. Using implicit differentiation one then obtains:

$$\frac{\partial N^*}{\partial \hat{p}} = \frac{-[1 - F(\hat{p})]^{N^*-1} f(\hat{p})[N^* ln(1 - F(\hat{p})) + 1]y}{[1 - F(\hat{p})]^{N^*} \left[\ln(1 - F(\hat{p}))\right]^2 y + \int\limits_{\underline{p}}^{\overline{p}} [1 - F(p)]^{N^*} \left[ln(1 - F(p))\right]^2 dp}.$$

The $N^*(\widehat{p})$ function has two kinks, one at each extreme of the price range on the market. For $\widehat{p} = \overline{p}$, the left-hand limit of the above derivative is zero, while for $\hat{p} = \underline{p}$ the right-hand limit is positive, given by:

$$\lim_{\widehat{p} \to \underline{p}} \frac{\partial N^*}{\partial \hat{p}} = \frac{f(\hat{p})y}{\int\limits_{\underline{p}}^{\overline{p}} [1 - F(p)]^{N^*} [ln(1 - F(p))]^2 dp}.$$

One can already conclude that (i) the optimal sample size N^* is continuous in the price target $\widehat{p}$, (ii) $N^* = N_0$ in both cases when the constraint is ignored - either because it is impossible or because it is spurious, and (iii) search intensity picks up $(N^* > N_0)$ as soon as a meaningful price target is in place. In other words, the optimal sample size function $N^*(\widehat{p})$ is indeed non-monotonic for $\widehat{p} \in (\underline{p}, \overline{p})$ and has one stationary point.[3]

To further investigate the uniqueness of the peak search intensity, please note from (3) that $\partial N^*/\partial \widehat{p} = 0$ for $N^* ln(1 - F(\hat{p})) + 1 = 0$. There is only one interior maximum N^* if this equation has a unique solution, which is indeed the case provided $N^* ln(1 - F(\hat{p}))$ is monotonic in $\widehat{p}$. To verify this, first observe that:

$$\frac{\partial [N^* ln(1 - F(\hat{p}))]}{\partial \hat{p}} = \frac{\partial N^*}{\partial \hat{p}} ln(1 - F(\widehat{p})) - \frac{N^* f(\hat{p})}{1 - F(\hat{p})}.$$

Substitute $\partial N^*/\partial \widehat{p}$ from (3), re-arrange and factorise, to obtain:

$$\frac{\partial [N^* ln(1 - F(\hat{p}))]}{\partial \hat{p}} = \frac{A - B}{C} < 0,$$

where

$$A \equiv N^* f(\hat{p}) \int_{\underline{p}}^{\bar{p}} [1 - F(p)]^{N^*} \left[ln(1 - F(p))\right]^2 dp \quad (> 0)$$

$$B \equiv [1 - F(\widehat{p})]^{N^*} f(\widehat{p}) ln(1 - F(\widehat{p})) y \quad (< 0)$$

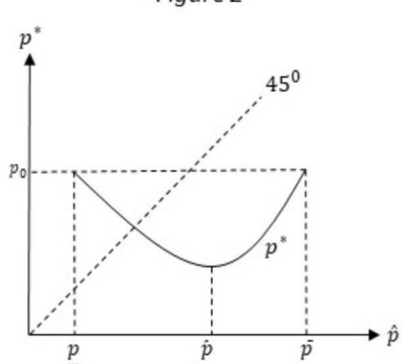
$$C \equiv [F(\hat{p}) - 1] \left[[(1 - F(\hat{p})]^{N^*} \left[ln(1 - F(\hat{p}))\right]^2 y + \int_{\underline{p}}^{\bar{p}} [1 - F(p)]^{N^*} \left[ln(1 - F(p))\right]^2 dp \right] (< 0)$$

This proves that the search intensity function $N^*(\widehat{p})$ has a unique interior maximum, given by:

$$N^*(\widetilde{p}) = -\frac{1}{\ln(1 - F(\widetilde{p}))}.$$

What is the best price the consumer can expect to obtain for any optimally chosen sample size, given a price target $\widehat{p}$? Denote this expected best price by $p^* \equiv p(N^*)$. Since - ceteris paribus - we have $\partial p(N)/\partial N < 0$, it follows that p^* has a minimum for $\widehat{p} = \widetilde{p}$. As $N^* = N_0$ for both $\widehat{p} = \underline{p}$ and $\widehat{p} = \overline{p}$, the expected best price p^* is also the same (denote it by p_0) in both extreme cases. Generically therefore, p^* is higher than $\widehat{p}$ for low values of $\widehat{p}$. Figure 2 illustrates this, and depicts the case where $p^* = \widehat{p}$ once.

Figure 2



Expected lowest price

[3]The *rate* at which the optimal search intensity changes with $\widehat{p}$ depends on the properties of f and F. In particular, the effect of $\widehat{p}$ on the change in hazard rate $f(\widehat{p})/[1 - F(\widehat{p})]$ depends on log-concavity. This is true also for constrained sequential search and the shape of the reservation strategy function there.

Crucially, this implies that for any parameter values there exists a range of price targets $\widehat{p}$ such that $p^*(\widehat{p}) > \widehat{p}$. Or, putting it differently: there are no parameter values for which $p^*(\widehat{p}) \leq \widehat{p}$ for *all* $\widehat{p} \in (\underline{p}, \overline{p})$. To see this, note that a decrease in the search cost shifts down the p^* curve; as expected, if $c = 0$ we obtain $p^* = \underline{p}$ for all $\widehat{p} \in [\underline{p}, \overline{p}]$. However, even with almost negligible search frictions (smallest positive c), the p^* function is U-shaped (albeit very flat) and the expected best price is still higher than at least the lowest possible target ($\underline{p}$).

Finally, we ask what happens if the utility gain is not fixed but depends on the gap between the target price and the price paid.[4] Consider the simplest version, with $y = \widehat{p} - p$, naturally defined only for $\widehat{p} \in [\underline{p}, \overline{p}]$. Then, we have:

$$\min_N L \equiv [1 - G(\widehat{p})] \left[\int_{\widehat{p}}^{\overline{p}} p dG(p) / [1 - G(\widehat{p})] + cN \right] +$$

$$+ G(\widehat{p}) \left[\int_{\underline{p}}^{\widehat{p}} p dG(p)/G(\widehat{p}) + cN - \int_{\underline{p}}^{\widehat{p}} [\widehat{p} - p] dG(p)/G(\widehat{p}) \right].$$

The second term above incorporates the expected prize $\widehat{p} - p$, and the problem becomes:

$$\min_N L \equiv p(N) + cN - \int_{\underline{p}}^{\widehat{p}} [\widehat{p} - p] dG(p) = p(N) + cN - \int_{\underline{p}}^{\widehat{p}} G(p) dp,$$

where the latter obtains through integration by parts. Finally, substituting for $G(p)$ and $p(N)$ as previously derived, we obtain:

$$\min_N L \equiv \underline{p} + \int_{\underline{p}}^{\overline{p}} [1 - F(p)]^N dp + cN - \int_{\underline{p}}^{\widehat{p}} \{1 - [1 - F(p)]^N\} dp.$$

It turns out that N^* is now a strictly monotonic, increasing function of $\widehat{p}$. Intuitively, as soon as the target price kicks in ($\widehat{p} > \underline{p}$), there is an incentive to increase search intensity. But now, although the marginal effects on the expected best price and on the probability of hitting the target decrease, the effect of the prize $\widehat{p} - p$ dominates. For higher (easier) targets the gap between the target and any given price becomes wider, thus leading to an increase in optimal sample size.

3. Discussion

How do the results for constrained fixed sample search compare with those for constrained sequential search?

First, our analysis here, together with the findings in Bonilla et al. (2019) reveal a robust result: in both cases, search intensity is a non-monotonic function of a constraint (target) whose attainment guarantees extra utility. Second, it is well-known that in many settings, sequential search performs better than fixed sample search, the main disadvantage of the latter method being its rigidity, as it implies a commitment to buy, and (only) after having gathered information from a pre-set sample.[5] In this paper, we point out a further apparent drawback of fixed sample search. The underlying reason for this drawback is that with constrained fixed sample search, there is no search intensity that would guarantee the fulfilling of *all* relevant targets *ex-post*. With pre-determined search and an active (i.e. not impossible or spurious) constraint (target),

[4]We thank an anonymous referee for suggesting this question.

[5]This has been noted in several standard (both micro and macro-economics) textbooks. See Phlips (1988) and Sargent (1987) for examples.

there is always a positive probability that the *realised* outcome does not fulfil this target, regardless of search intensity. In sharp contrast, with constrained sequential search there is *always* a range of (relatively easy) targets that the searcher is able to and does in fact *commit to* fulfil.

To make the above comparison meaningful, we have taken the basic Stigler model and augmented it with the key element that characterises constrained search: an exogenous price target with a one-off fixed utility jump, should this target be achieved. One could also relax another key assumption of the Stigler model, and allow the consumer to potentially refuse to buy. For this to constitute a well-defined choice, one would require either the presence of an outside option, or the embedding of our problem into a sequential framework. However, such a setup would lead to the idea of a reservation price.[6] Crucially, this is conceptually different from a target price as we define it, be it exogenous (like here) or endogenous. Such a modified framework would obscure the comparison between (constrained) fixed sample and (constrained) sequential search, as in the latter the commitment to refuse an offer that falls short of the target is implicit and endogenous.

We believe that constrained (fixed sample and sequential) search methods could fruitfully be integrated into richer economic models. In the context of price search, we already mentioned the potential application to consumer demand theory. Bonilla et al.(2019) and Bonilla and Kiraly (2013) obtain interesting results in models where constrained job search provides the link between two frictional markets. More generally, constrained search is relevant whenever access to another market (frictional or not) or further options are conditional on securing an appropriate "ticket" first, through search.

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[6]For example, Gastwirth (1976) uses a model where price observations (drawn from a pre-determined sample) arrive sequentially, and not simultaneously as in our setup. He shows that the optimal strategy is to choose *both* a reservation price *and* a sample size (which gets "truncated", should the reservation price be attained). However, there is no target price as we model it.