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Mobility, Inequality, and Growth: An Inverted-U Relationship

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Abstract

Empirical studies show that income inequality and economic growth tend to have unclear relationship. To explain the mechanism, this paper presents a simple model in which mobility, inequality, and growth are endogenously determined. In our model, children decide whether or not to acquire education in response to the income transfer from the parent, the each individual's own ability to learn, the wage inequality between educated workers and uneducated workers, and the family background. Their education accumulates knowledge capital that is the driving force of the economic growth. We analytically demonstrate that the inequality has an inverted-U relation with the growth rate through the acceleration of upward mobility.

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1 Introduction

The level of income inequality differs substantially across countries. While the Nordic countries have considerably less inequality, there is extreme inequality in Latin American countries, the Middle East, and South Africa. Furthermore, Anglo-Saxon countries have experienced a rise in inequality since the 1980s.[1] This paper evaluates the economic importance of inequality from the perspective of economic growth.

There are two strands of literature on the relationship between income inequality and economic growth. While many studies have verified the well-known inverted U-shaped Kuznets curve that focuses on causation from growth to inequality, others have focused on the reverse causation from inequality to growth. This paper sheds light on the latter.

Although there are many empirical works in the literature, the inequality-growth nexus is not clear and remains controversial. Some studies have found evidence that the relationship between income inequality and growth might be nonlinear (e.g., Banerjee and Duflo (2003); Chen (2003); Malinen (2013)). Our descriptive data (Fig.1) also show that the relationship between income inequality and economic growth is unclear.

To consider the mechanisms that link inequality to growth, each country's social mobility must be taken into account. According to a famous relationship, called the Great Gatsby Curve in Fig.2, countries with greater income inequality tend to be countries with a greater intergenerational earnings elasticity.[2] Among OECD countries, the Nordic countries have low inequality and high mobility, while the United States and the United Kingdom have high inequality and low mobility.

Theoretically, one clear explanation for why inequality can persist across generations is poor children's disadvantage in educational opportunities. Additionally, many empirical studies demonstrate that education is vital in providing upward mobility for people of lower economic status. Countries with greater income inequality tend to have a higher college premium and a higher intergenerational education correlation. For example, Latin American countries have the highest income inequality in the world. Neidhofer et al. (2018) find that almost 60% of children with high and low education have parents in the same educational class and that only 14% of the individuals with high education come from low education families in Latin America.[3]

This paper provides a simple theory of mobility, inequality, and economic growth. We find that higher social mobility lowers inequality but has an inverted U-shaped relation with the economic growth rate. Consequently, we can indirectly identify an inverted U-shaped relationship between inequality and growth.

This paper relates to the literature that shows complex relationships between inequality and growth.[4] Rehme (2007) builds a model in which public education financed by wealth taxation fosters high-skilled workers. Although the author finds that the level of taxation has an inverted U-shaped relationship with both inequality and growth, there is no clear functional relationship

[1] See Piketty (2014) and Alvaredo et al. (2018).

[2] Hassler et al. (2007) report that a negative correlation between mobility and inequality endogenously arises in their theoretical model.

[3] In addition, Neidhofer et al. (2018) observe a rising trend in educational mobility in Latin America. They conclude that this trend seems to be mainly driven by educational expansions that have particularly benefited children from less educated families.

[4] Aghion et al (1999) show that inequality negatively affects growth when capital markets are imperfect, while growth is independent of inequality in the absence of capital market imperfections.

between growth and inequality. Some studies emphasize that the growth effect of inequality depends on the time span considered. For example, Galor and Moav (2004) argue that inequality enhances growth in the early stages of development, while inequality dampens growth in later stages of development because the engine of growth changes from physical capital accumulation to human capital accumulation. Halter et al. (2014) find that higher inequality positively affects the growth rate in the short run but reduces the growth rate in the future. In contrast, Shin (2012) finds that higher inequality negatively affects growth in the early stage of development but positively affects growth near the steady state. Recently, as another approach, Chu et al. (2019) consider the inequality-growth nexus through pro-patent policy in a Schumpeterian growth model. They find that strengthening patent protection has a positive effect on growth and a positive or an inverted U-shaped effect on inequality in the short run.

This paper contributes to this line of research by identifying an inverted U-shaped relationship between inequality and growth.[5] To the best of our knowledge, no theoretical study explicitly demonstrates such a nonlinearity between them when social mobility operates through education. Maoz and Moab (1999) very closely resembles our paper. They also assume that children can become educated workers by paying a fixed educational cost, which is heterogeneous across children. They show that as the level of inequality decreases in the transitional dynamics, the growth rate initially gradually increases, but the growth rate later decreases as the steady state is approached. However, although there is an inverted U-shaped relationship between inequality and growth, growth in their model is transitional (not long-run growth).[6] In contrast, our model shows an inverted U-shaped relationship between inequality and long-run growth.

2 The model

Our model is similar to that of Maoz and Moab (1999). Consider an overlapping generations economy with no population growth. The firms produce a single homogeneous good by employing two types of labor: educated and uneducated workers. Let N_t^E and N_t^U denote the number of educated workers and the number of uneducated workers, respectively. Each generation has a unit mass. Therefore, $N_t^E + N_t^U = 1$. The ratio of educated workers in every period is endogenously determined.

2.1 Households

We consider a discrete-time economy that is consists of overlapping generations of people, each of whom has a single parent and a single child. They live for two periods. In the first period of life (childhood), each individual receives x_t units of goods from his/her parent. Children decide whether to acquire education by paying a fixed cost. Following Maoz and Moab (1999), we assume that individuals cannot borrow or lend. Therefore, they consume the rest of their endowed goods in childhood. In the second period of life (adulthood), each individual works as either an educated or uneducated worker and transfers all wages to his/her child. Without loss of generality, we can

[5] Although Tamai (2015) also shows an inverted U-shaped relationship between inequality and growth, the horizontal axis is growth, and the vertical axis is inequality. Therefore, the result is different from our paper because the horizontal axis is inequality and the vertical axis is growth in our model.

[6] Owen and Weil (1998) also focus on growth in the transitional dynamics.

assume that all adults do not consume goods.[7] Furthermore, we assume that they do not discount the future. Then, the utility function of an individual born at time t is given by $u_t = \ln c_t + \ln x_{t+1}$, where c_t is childhood consumption and x_{t+1} is the adult transfer.

The decision of whether to acquire education differs across individuals for three reasons. First, the amount of income transferred from parents depends on whether the parent is educated. Second, family background affects what goods are required to be educated. Third, even among individuals with the same family background, the innate ability to learn is heterogeneous. Let f_i denote the fixed cost of education for an educated worker's child i. We assume that the fixed cost is uniformly distributed over the interval $[0, 1]$ and that an individual's f is unrelated to that of her parents. In contrast, an uneducated worker's child j requires the fixed cost of education f_j/m, where f_j is also uniformly distributed over the interval $[0, 1]$ and $m > 0$. We assume that the parameter m reflects social mobility, which is determined by cultural factors. If $m < 1$, this means that uneducated workers' children have some kind of disadvantage or face discrimination due to their family background. If $m > 1$, poor families can receive preferential treatment for education, or the economy has some kind of free school for poor children.

The constraints of educated worker's child i are $c_t + e_i f_i = x_t$, where e_i is a binary value defined as follows:

$$e_i = \begin{cases} 1 & \text{(acquires education)} \\ 0 & \text{(otherwise)} \end{cases} \tag{1}$$

and

$$x_{t+1} = w^k_{t+1}, \tag{2}$$

where

$$w^k_{t+1} = \begin{cases} w^E_{t+1} & (e_i = 1) \\ w^U_{t+1} & (e_i = 0). \end{cases} \tag{3}$$

where w^E is the wage of an educated worker and w^U is the wage of an uneducated worker. These wage rates are the expected value of individuals. Later, we consider a rational expectations equilibrium in which expected wage rates are realized in the next period. We normalize the wage of an uneducated worker ($w^U = 1$) and denote the wage of an educated worker by w. Note that w represents the relative wage rate in the economy.

Similarly, the constraint of uneducated worker's child j is

$$c_t + e_i \left(\frac{f_i}{m} \right) = 1. \tag{4}$$

We omit the rest of the constraints facing uneducated worker's child j because they are completely the same as (1)-(3).

A child i born to an educated worker decides whether to acquire education by comparing the utility from being educated with that of being uneducated. The utility when he/she chooses $e_i = 0$

[7] Even if we assume that they consume goods in adulthood, our main results still hold. For example, the inverted U-shaped relationship between growth and inequality still holds under another utility function $u_t = \ln c_t + \gamma \ln x_{t+1} + (1-\gamma) c_{t+1}$, where $\gamma \in [0, 1]$.

is $u_i^u = \ln w_t$, and the utility when he/she chooses $e_i = 1$ is

$$u_i^e = \ln(w_t - f_i) + \ln w_{t+1}.$$

Therefore, the condition for acquiring education is

$$u_i^e \geq u_i^u \Leftrightarrow 0 \leq f_i \leq \min\left[w_t\left(1 - \frac{1}{w_{t+1}}\right), 1\right] \equiv f^E. \tag{5}$$

Therefore, educated workers' children who have $f \in [0, f^E]$ acquire education. Note that $1 - f^E$ is the rate of downward mobility in the model.

Similarly, we can obtain the threshold value for uneducated workers' children as follows:

$$u_j^e \geq u_j^u \Leftrightarrow 0 \leq f_j \leq \min\left[m\left(1 - \frac{1}{w_{t+1}}\right), 1\right] \equiv f^U. \tag{6}$$

Therefore, an uneducated worker's child who has $f \in [0, f^U]$ acquires education. Here, f^U is the rate of upward mobility in the model.

Note that (5) and (6) imply that if the relative wage is one (i.e., $w = 1$), no individual has an incentive to acquire education. Thus, conversely, a sufficient wage gap provides children incentives to acquire education.

2.2 Production

The final goods are produced at constant returns to scale and are perfectly competitive. The production function is $Y_t = H_t E_t^{\alpha} U_t^{1-\alpha}$, where E_t is the number of educated workers, U_t is the number of uneducated workers, and H_t is the knowledge capital in the economy. We assume that $1/2 < \alpha < 1$ holds. This means that educated workers can contribute more to production than uneducated workers.

Profits are given by $\pi = H_t E_t^{\alpha} U_t^{1-\alpha} - w_t E_t - U_t$. The first-order conditions for profit maximization are

$$w_t = \alpha\left(\frac{Y_t}{E_t}\right), \tag{7}$$

$$1 = (1-\alpha)\left(\frac{Y_t}{U_t}\right). \tag{8}$$

2.3 Knowledge capital

We assume that knowledge capital evolves as follows:

$$H_{t+1} = \left[1 + A\left(f^U N_t^U\right)^{\beta}\left(f^E N_t^E\right)^{1-\beta}\right] H_t, \tag{9}$$

where $A > 0$ is a scale parameter. This implies that the knowledge capital in the next period depends on the number of educated children, $f^U N_t^U$ and $f^E N_t^E$, in the current period. $\beta \in (0, 1)$

is the degree to which the education of uneducated workers' children contributes to total knowledge capital.

2.4 Labor Market Equilibrium

The conditions for labor market equilibrium are

$$E_t = N_t^E, \tag{10}$$

$$U_t = N_t^U. \tag{11}$$

3 Mobility, Inequality, and Growth

In the rest of the paper, we assume that m is sufficiently small such that $f^U < 1$ holds.

3.1 Dynamics

From (7), (8), (10) and (11), we obtain

$$w_t = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-N_t^E}{N_t^E}\right). \tag{12}$$

The number of educated workers in the next period is the sum of children who acquire education in the current period. Therefore, it evolves as follows:

$$N_{t+1}^E - N_t^E = f^U N_t^U - \left(1-f^E\right) N_t^E. \tag{13}$$

By substituting (5), (6) and (12) into (13), we obtain

$$N_{t+1}^E - N_t^E = 0 \Leftrightarrow w_{t+1} = \frac{\alpha/(1-\alpha)+m}{\alpha/(1-\alpha)+m-N_t^E/(1-N_t^E)}. \tag{14}$$

The RHS in (14) is an increasing function of N_t^E and is illustrated as an upward curve in Fig. 3. If the economy is in the region above this line, $N_{t+1}^E > N_t^E$ holds. In contrast, if the economy is in the region below this line, $N_{t+1}^E < N_t^E$ holds.

By moving forward one period in (12) and using (5), (6), and (13), we obtain

$$\begin{aligned} w_{t+1} &= \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{1}{(m+(w_t-m)N_t^E)(1-1/w_{t+1})}-1\right] \\ \Leftrightarrow \quad &\left[\left(\frac{1-\alpha}{\alpha}\right)w_{t+1}+1\right]\left(1-\frac{1}{w_{t+1}}\right)\left(\frac{\alpha}{1-\alpha}+m\right) = \frac{1}{1-N_t^E} \\ \Leftrightarrow \quad & w_{t+1} = \phi(N_t^E). \end{aligned} \tag{15}$$

This implicit function ϕ is increasing in N_t^E and satisfies $\phi(0) > 1$. The economy must lie on the curve of (15) in Fig.3. The steady state always exists because when $N_t^E = 1/[1+(\alpha/(1-\alpha)+m)^{-1}] \equiv \overline{N^E}$, the RHS of (14) goes to infinity, while w_{t+1} in (15) is a finite value. Assume that the economy starts from $N_0^E < \overline{N^E}$. Then, using (14), (15), and Fig. 3, we can show the following fact.

Proposition 1. *The steady state always uniquely exists, and it is stable for all $m > 0$.*

3.2 Comparative statics

In the steady state, N_t^E and w_t are constant over time: $N_{t+1}^E = N_t^E = N^{E*}$ and $w_{t+1} = w_t = w^*$. From (12) and (14), we can obtain the steady-state value of the relative wage and the number of educated workers as follows:

$$w^* = 1 + \frac{1}{1+m(1-\alpha)/\alpha}. \tag{16}$$

$$N^{E*} = \frac{1}{1+w^*(1-\alpha)/\alpha}. \tag{17}$$

Then, from (16) and (17), we have the following result:

Proposition 2. *A higher mobility (m) reduces inequality (w^*) and increases the number of educated workers (N^{E*}) in the steady state.*

The intuition is as follows. Upward mobility in the steady state is

$$f^{U*} = \frac{m}{2+m(1-\alpha)/\alpha}.$$

Therefore, a higher m increases the share of uneducated workers' children who acquire education. This increases the relative supply of educated workers and decreases the wage gap between the two types of workers. The tradeoff between mobility and inequality is consistent with the Great Gatsby curve.

Next, we consider the growth effect of m. Using (5), (6), (9), (16) and (17), we obtain the economic growth rate on the balanced growth path as follows:

$$\begin{aligned} g &= \frac{H_{t+1}-H_t}{H_t} \\ &= Am^\beta \left(\frac{1-\alpha}{\alpha}\right)^\beta \left[\frac{w^*-1}{1+w^*(1-\alpha)/\alpha}\right] \\ &= A\frac{m^\beta[(1-\alpha)/\alpha]^\beta}{m(1-\alpha)/\alpha^2+2(1-\alpha)/\alpha+1}. \end{aligned} \tag{18}$$

Differentiating (18) with respect to m, we obtain

$$\frac{dg}{dm} = A\left(\frac{1-\alpha}{\alpha}\right)^\beta \frac{\beta[2(1-\alpha)/\alpha+1]/m^{1-\beta} - (1-\beta)m^\beta(1-\alpha)/\alpha^2}{[m(1-\alpha)/\alpha^2+2(1-\alpha)/\alpha+1]^2}. \tag{19}$$

We can easily see that (19) is strictly decreasing in m. Moreover, $dg/dm > 0$ holds when $m = 0$, and $dg/dm = 0$ holds when m is

$$m^* = \frac{\beta[2(1-\alpha)/\alpha + 1]}{(1-\beta)(1-\alpha)/\alpha^2}.$$

In addition, $dg/dm < 0$ holds in $m \geq m^*$. Therefore, we obtain the following result.

Proposition 3. *The mobility parameter m has an inverted U-shaped relationship with the growth rate.*

By combining this result with Proposition 2, we can show an inverted U-shaped relationship between inequality and growth. Fig.4 is a numerical example, and Panel (d) illustrates a clear inverted U-shaped curve. This relationship is indirectly obtained through the change in the mobility parameter m, which is a confounding factor in the model.[8]

The intuition of the nonlinearity is as follows. The mobility parameter m differentially affects the decision to acquire education between two types of workers (see Panel (c) in Fig.4). First, since a higher m enables many uneducated workers' children to acquire education, it accelerates the accumulation of knowledge capital. Second, as shown in Proposition 2, a higher m decreases the relative wage (inequality). This discourages educated workers' children from acquiring education. Although a higher m increases the number of educated workers, as shown in Proposition 2, their children are reluctant to acquire education. This retards the accumulation of knowledge capital.

4 Conclusion

In this paper, we build a simple model in which intergenerational mobility, income inequality, and economic growth are endogenously determined through the educational choices of children. In the baseline model, we investigate the relationship between inequality and growth by obtaining comparative statics. Our results can be summarized as follows. First, mobility through educational choice is negatively correlated with inequality. Second, there is an inverted U-shaped relationship between mobility and growth. Finally, as a result, we can indirectly obtain an inverted U-shaped relationship between inequality and growth.

[8] Since inequality and growth are endogenous variables, to investigate their relationship, we need some parameters that affect both variables.

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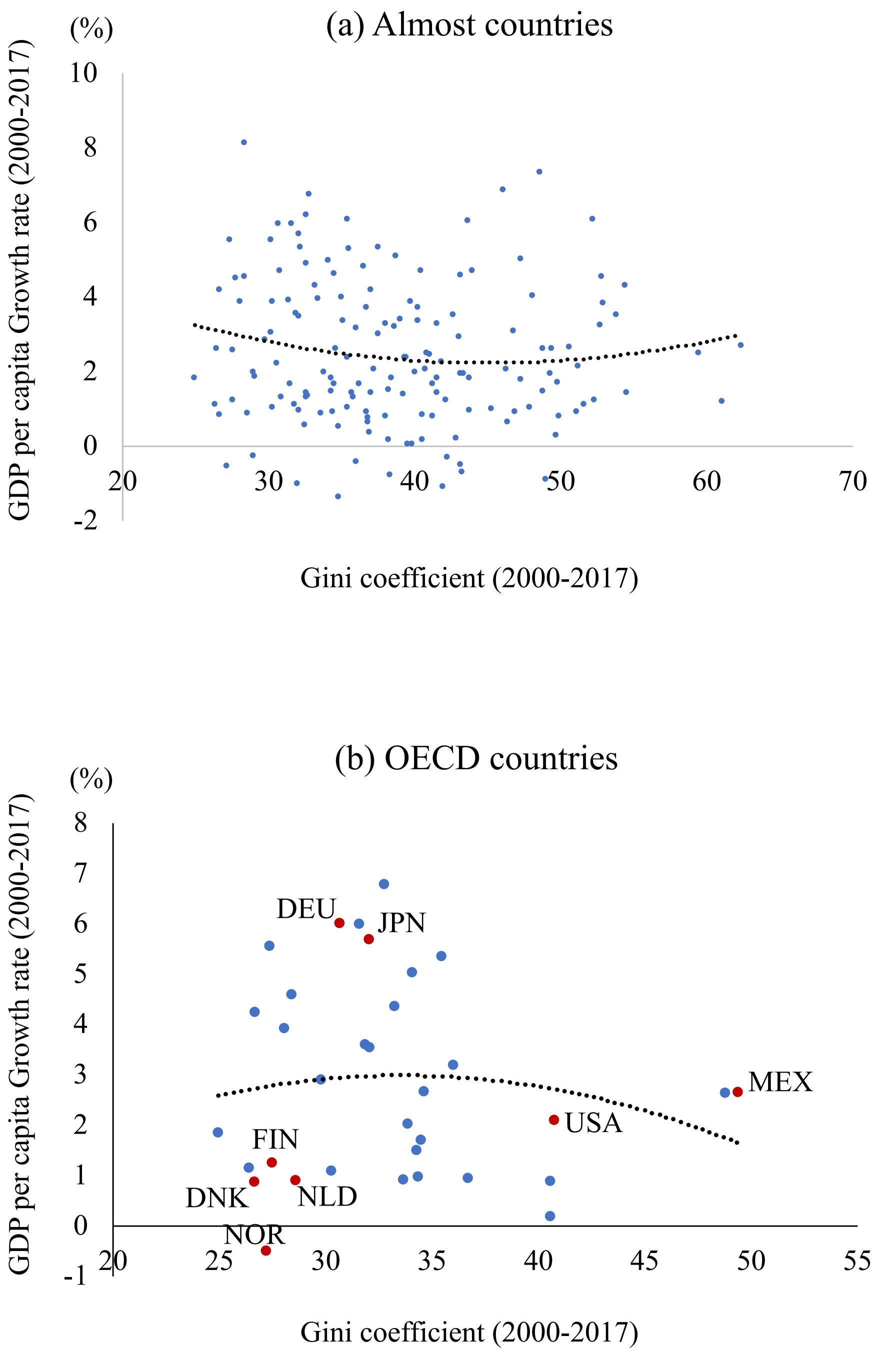

Figure 1: Descriptive data: Inequality and growth. *Data: World Bank Open Data*

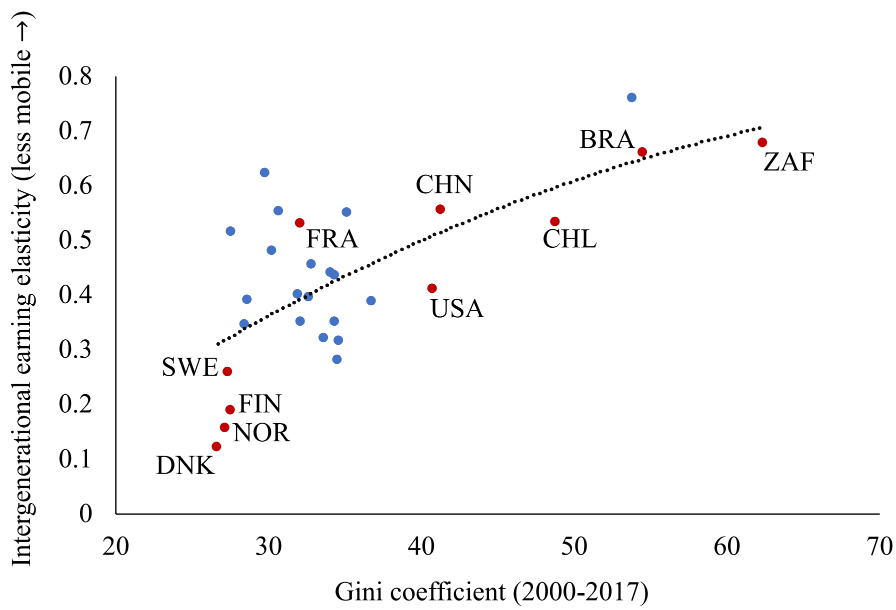

Figure 2: The Great Gatsby Curve. *Data: OECD (2018)*

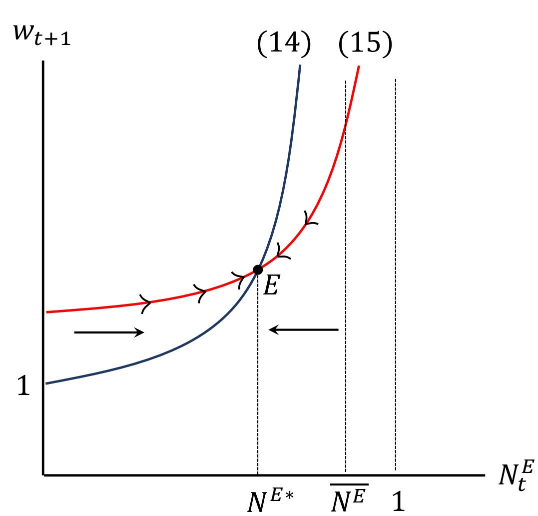

Figure 3: The phase diagram.

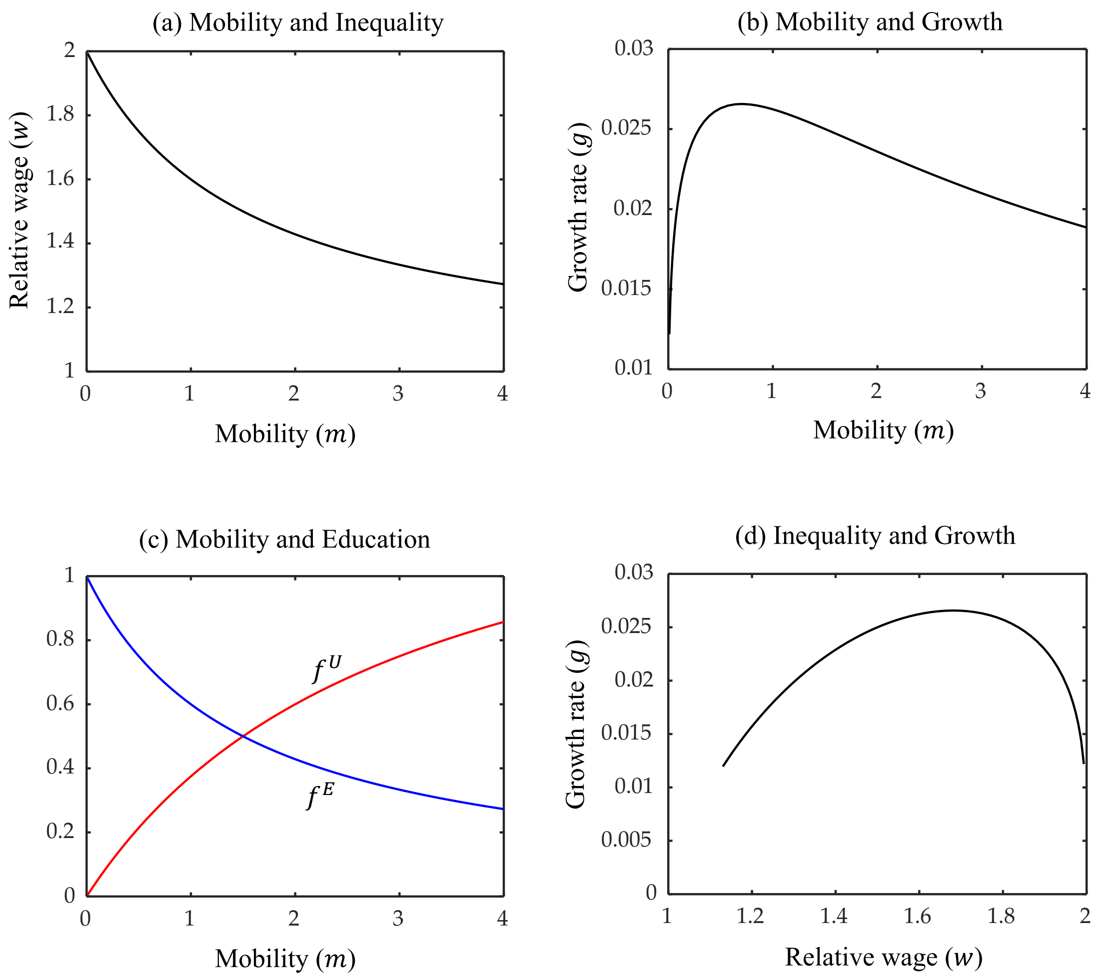

Figure 4: A numerical example ($\alpha = 0.6, \beta = 0.25, A = 0.1$).