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An Evolutionary Justification for Overconfidence

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Abstract

This paper provides an evolutionary justification for overconfidence. Players are pairwise matched to compete for a resource, and there is uncertainty about who will win the resource if they choose to compete. Players have different confidence levels about their chance of winning, although in reality they have the same chance of winning. Each player may or may not know her opponent's confidence level. We characterize the evolutionarily stable equilibrium, represented by players' strategies and distribution of confidence levels. Under different informational environments, a majority of players are overconfident---i.e. they overestimate their chance of winning. We also characterize the evolutionary dynamics and the rate of convergence to the equilibrium.

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1 Introduction

A person is overconfident when they overestimate their own ability. The literature provides extensive evidence of population-wide overconfidence: Oskamp (1965); Kidd (1970); Svenson (1981); Cooper et al. (1988); Bondt et al. (1989); Russo and Schoemaker (1992); Babcock and Loewenstein (1997); and Guthrie et al. (2001). This evidence suggests that overconfidence may be an evolutionarily desirable trait.

This paper provides an evolutionary justification for overconfidence. We show that in a class of games with players of heterogeneous confidence levels about their own chance of winning the game, players with overconfident beliefs survive. The key improvement is that we do not need to assume any bias or constraint in nature, in contrast to the majority of previous studies. Rather, strategic interactions themselves result in the survival of biased beliefs and, in particular, of overconfident beliefs.

The intuition for evolutionarily optimal overconfident beliefs is that rational players do not necessarily get the most payoff in strategic settings. In a game in which players compete for resources, a rational Bayesian player should not fight an overconfident player, but an overconfident player will fight a rational Bayesian player—who will back down in the face of the overconfident player's aggression. As a result, a rational player chooses not to fight with the overconfident player, and the overconfident player wins when the two compete.

Over the long run, there will be a distribution of players with heterogeneous beliefs about the state of the world. We show that generically, over the long run, the majority of players will be overconfident. We show the robustness of the result both when a player knows his opponent's type and when a player does not know his opponent's type.

After a brief literature review, the paper is organized as follows. Section 2 describes the model. Section 3 solves the model when confidence levels are observed, Section 4 solves the model when confidence levels are not observed, and Section 5 concludes.

1.1 Literature Review

1.1.1 Biased Beliefs of a Single Decision Maker

When a player departs from being a risk-neutral utility maximizer, holding a biased belief may correct other inherent biases. Zhang (2013) shows that when a decision maker is risk-averse, the evolutionarily optimal belief is positively biased. Herold and Netzer (2015) show that non-Bayesian probability weighting can correct for the S-shaped value function that arises in prospect theory, and make the general point that non-Bayesian probability weighting is a correction for biases in nonlinear utilities.[1] The common thread in these studies is that there is some constraint in nature, and evolution works to find the second-best solution. One a priori bias results in an ex post bias in another dimension (e.g., risk aversion results in an overconfident, S-shaped utility function that results in nonlinear probability weight-

[1]The natural question why Bayesian and linear utilities do not evolve to survive and dominate other suboptimal second-best probability weighting and strategies is addressed in Ely (2011). He uses Microsoft Office updates as an analogy: The system is not completely overhauled to reach the first-best solution; rather, patches and what he calls "kludges" are added over time. Similarly, evolution works by adding patches rather than completely overhauling the system.

ing). Benabou and Tirole (2002) and Compte and Postlewaite (2004) also demonstrate the desirable effects of self-perception for a single decision maker who exhibits other constraints.

Zabojnik (2004); Benoit and Dubra (2011); and Harris and Hahn (2011) attempt to rationalize observed overconfidence, but the empirical evidence for overconfidence is too overwhelming to be disregarded.

1.1.2 Biased Beliefs in Strategic Interactions

Güth and Yaari (1992) and Güth (1995) developed the so-called indirect evolutionary approach we use in this paper: Players choose actions to maximize their perceived utilities, but their actual payoffs and fitness differ from their perceived utilities. Heifetz et al. (2007) show in general that rational players are not the ones who survive in an evolutionary process. Because the games are assumed to be dominance solvable and have a unique Nash equilibrium, only one type of player survives in the limit. Instead, in this paper, we consider games with multiple equilibria and a distribution of players with different belief types will survive. Dekel et al. (2007) examine two-by-two games with potentially multiple equilibria and characterize stable distributions of preferences. The game we consider in this paper does not have a stable distribution according to Proposition 3 in Dekel et al. (2007). For our purpose of showing the limit distribution of overconfidence, we do not need to restrict our attention to any particular equilibrium. The paper most closely related to ours is Johnson and Fowler (2011). However, they treat biased estimation of winning probability as a “mistake" and also do not discuss the cases in which the players’ types are not observed at all. Other papers also highlight the advantage of being an overconfident type in Tullock contests; see Ludwig et al. (2011).

1.1.3 The Market Selection Hypothesis

The market selection hypothesis literature investigates the survival of players with different beliefs in a competitive market, instead of pairwise strategic interactions in evolutionary games. Blume and Easley (1992) show that when players maximize their discounted log utilities of consumption, those who have correct beliefs about the state of the world will dominate the asset market in the long run. Sandroni (2000) endogenizes the savings decision in Blume and Easley (1992) and shows that regardless of the players’ utility functions, those with correct beliefs will dominate. A crucial assumption is that markets are complete. Subsequent papers consider incomplete markets, and find that players with incorrect prior beliefs may survive: Mailath and Sandroni (2003); Blume and Easley (2009b); Blume and Easley (2009a); Blume and Easley (2010); Beker and Chattopadhyay (2010); Coury and Sciubba (2012); and Condie and Phillips (2016).

2 Model

There is a continuum of players with heterogeneous beliefs $\theta \in [0,1]$ about their chance of winning. We say a player is *rational* if $\theta = 1/2$, and *overconfident* if $\theta > 1/2$. Let μ denote the measure of players with different beliefs. The fitness payoff is illustrated in the following table.

$i \setminus j$	Fight	Not Fight
Fight	$1_i R + 1_j r - c$, $1_j R + 1_i r - c$	R, r
Not Fight	r, R	r, r

Assume (i) $r < R$, (ii) $\frac{1}{2}(R - r) < c$, and (iii) $c < R - r$. Assumption (ii) $\frac{1}{2}(R - r) < c$ implies that a rational player would not fight if the opponent fights. Without Assumption (ii), fight is a dominant strategy for a rational $\theta = 1/2$ player. Assumption (iii) $c < R - r$ implies that a player who believes he always wins would prefer to fight even if the opponent fights, i.e., a player of type $\theta = 1$ has a strictly dominant strategy of fight.

With probability q, a player observes his opponent's play. We will address the observable case in Section 3 and the unobservable case in Section 4. Note that Assumption (ii) will be critical for our result in Section 3 and Assumption (iii) will be critical for our result in Section 4.

3 Observable Confidence

Suppose for now that opponents' confidence types are always observable, i.e., $q = 1$. In this section, we solve for the pairwise static games, players' payoffs, equilibrium distribution, and the evolutionary dynamics.

3.1 Pairwise Static Games

Two players of types $\theta_i \in [0, 1]$ and $\theta_j \in [0, 1]$ play the following perceived game.

$i \setminus j$	Fight	Not Fight
Fight	$\theta_i R + (1 - \theta_i) r - c$, $\theta_j R + (1 - \theta_j) r - c$	R, r
Not Fight	r, R	r, r

Any type θ player who fights receives a minimum expected payoff of $\theta R + (1 - \theta) r - c$ (in the case in which the opposing player fights for sure). The alternative is to not fight, and the type θ player gets r. Consequently, a player definitely fight if $\theta R + (1 - \theta) r - c > r$, which rearranges to

$$\theta > \frac{c}{R - r} = \theta^*,$$

where we call θ^* the *critical type*. Since $c > (R - r)/2$ by assumption, the critical type $\theta^* > 1/2$. Any player with type $\theta < \theta^*$ is called *under-critical* and any player with type $\theta \geq \theta^*$ is called *over-critical*. Since $\theta^* > 1/2$, an over-critical player is necessarily overconfident, and a rational player is necessarily under-critical, but some under-critical players are also overconfident.

For any over-critical $\theta \geq \theta^*$ player, fighting is a dominant strategy. For any under-critical $\theta < \theta^*$ player, fighting is not a dominant strategy. In particular, for a rational $\theta = 1/2 < \theta^*$ player, fighting is never a dominant strategy.

(a) Under-critical type $\theta_i < \theta^*$ versus under-critical type $\theta_j < \theta^*$. Suppose two under-critical players encounter each other. Specifically, two rational $\theta = 1/2$ players encountering each other is a special case. There are three equilibria: two pure-strategy equilibria (fight, not fight) and (not fight, fight) and one mixed-strategy equilibrium $(\sigma^*_{\theta_i}(\theta_j), \sigma^*_{\theta_j}(\theta_i))$. In the two pure-strategy Nash equilibria, one player gets R and the other gets r, and the total payoff is $R + r$. The two players on average get a payoff $(R + r)/2$. In the unique mixed-strategy Nash equilibrium, each person gets r in expectation. Therefore, each $\theta_i, \theta_j < \theta^*$ player gets a payoff $x \in [r, (R + r)/2]$.

(b) Over-critical type $\theta_i \geq \theta^*$ versus under-critical type $\theta_j < \theta^*$. It is a dominant strategy to fight. Given that a type $\theta_i \geq \theta^*$ player fights, the $\theta_j < \theta^*$ player does not fight. The unique Nash equilibrium is (fight, not fight), and the type $\theta_i > \theta$ player gets R and the type $\theta_j < \theta^*$ player gets r.

(c) Over-critical $\theta_i \geq \theta^*$ versus over-critical $\theta_j \geq \theta^*$. Two overconfident players both fight and they end up with an expected payoff of $(R + r)/2 - c$.

3.2 Evolutionarily Stable Distribution of Confidence

Definition 1. An equilibrium consists of a CDF F^* with associated PDF f^* on $[0, 1]$ representing the distribution of types, and a strategy $\sigma^*_\theta : [0, 1] \cup \emptyset \to [0, 1]$ such that

1. For each $\theta_i, \theta_j \in [0, 1]$,

$$\sigma^*_{\theta_i}(\theta_j) \in \arg\max_{\sigma \in [0,1]} \sigma[\sigma^*_{\theta_j}(\theta_i)(\theta_i R + (1 - \theta_i)r - c) + (1 - \sigma^*_{\theta_j}(\theta_i))R] + (1 - \sigma)r,$$

   or more loosely, to incorporate asymmetric strategies, $(\sigma^*_{\theta_i}(\theta_j), \sigma^*_{\theta_j}(\theta_i)) \in \mathcal{N}(\theta_i, \theta_j)$, where $\mathcal{N}(\theta_i, \theta_j)$ denotes the set of Nash equilibria in the perceived game played between types θ_i and θ_j players.

2. π_θ equalizes across all θ, where

$$\pi_{\theta_i} = \int_0^1 [\sigma^*_{\theta_i}(\theta_j)\sigma^*_{\theta_j}(\theta_i)(\frac{1}{2}R + \frac{1}{2}r - c) + \sigma^*_{\theta_i}(\theta_j)(1 - \sigma^*_{\theta_j}(\theta_i))R + (1 - \sigma^*_{\theta_i}(\theta_j))r] f^*(\theta_j) d\theta_j.$$

We can show that the stable proportion of overconfident players is always above 1/2. Suppose p^* is the proportion of players with types above θ^*. Then, an overconfident $\theta \geq \theta^*$ player gets a payoff of

$$\pi_{\theta > \theta^*} = p^*[(R + r)/2 - c] + (1 - p^*)R$$

and a type $\theta < \theta^*$ player gets a payoff of

$$\pi_{\theta \leq \theta^*} = p^* r + (1 - p^*)x$$

where $x \in [r, (R + r)/2]$. In equilibrium, the two equate,

$$p^*[(R + r)/2 - c] + (1 - p^*)R = p^* r + (1 - p^*)x,$$

which simplifies to

$$p^* = \frac{R - x}{\frac{R+r}{2} + c - x}.$$

Since $c < R - r$,

$$p^* > \frac{R - x}{\frac{R+r}{2} + R - r - x} = \frac{R - x}{R + \frac{1}{2}(R - r) - x} = 1 - \frac{\frac{1}{2}(R - r)}{R + \frac{1}{2}(R - r) - x}.$$

Since $x \leq (R + r)/2$,

$$p^* > 1 - \frac{\frac{1}{2}(R - r)}{R + \frac{1}{2}(R - r) - x} \geq 1 - \frac{\frac{1}{2}(R - r)}{R + \frac{1}{2}(R - r) - \frac{1}{2}(R + r)} = \frac{1}{2}.$$

That is, in equilibrium, it always holds that more than half of the agents have a confidence level above $\theta^* = (R - r)/c > 0.5$. The exact distribution of confidence levels depends on the initial distribution one starts with, but we have shown that regardless of the initial distribution, at least a half of the population has strictly positively biased beliefs.

Suppose Assumption (ii) does not hold, that is, $\frac{1}{2} \cdot (R - r) \geq c$ instead. The derivation above continues to hold. We will have the result that more than half of the agents have a confidence level above $\theta^* = (R - r)/c$, but $\theta^* < \frac{1}{2}$. Overconfident individuals will continue to survive in the market, but they will not be predominant.

We can also look at the change in the evolutionarily stable proportion of overconfident players. The stable proportion of overconfident players increases when the winning payoff R increases, when the losing payoff r decreases, and/or when the cost of fighting c decreases.

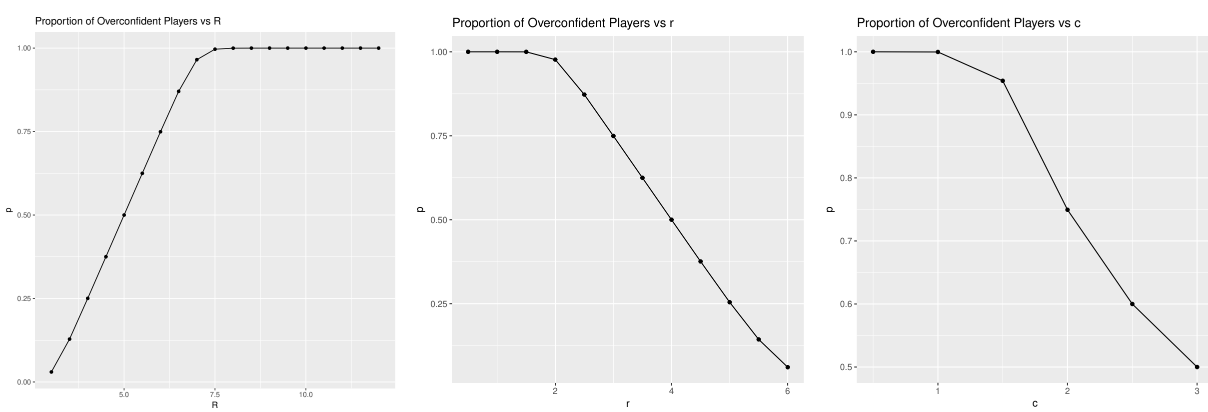

Figure 1: Evolutionarily stable proportion of overconfident players as R, r, and c vary individually, when $R = 6$, $r = 3$, and $c = 2$.

Our analysis also allows variation on various parameters within the model. Figure 1 notes the differences in the evolutionarily stable proportion of overconfident players for different values of R, holding constant the values of r and c. Indeed, we note that with a value R less than 5, the long-run proportion of overconfident players is less than the initial proportion 0.5. At $R = 5$, the value of p remains constant at 0.5, and any value of R greater than 5 yields an increase in p. We note that around roughly $R = 7.5$, the proportion of overconfident players begins to approach 1 in the long run.

Figure 2 illustrates the change in the stable proportion of overconfident players for different values of winning payoff R, losing payoff r, cost c, and the initial proportion of overconfident players.

3.3 Evolutionary Dynamics

Figure 3 plots the number of time periods necessary for the population to reach within 0.01 of the evolutionarily stable distribution. Somewhat surprisingly, the number of generations to reach within 0.01 of the equilibrium proportion is not monotonic in R, r, c, and p_0. The variations of convergence rates merit attention in future research.

4 Unobservable Confidence

Suppose that players cannot observe their opponent's confidence type. We are interested in characterizing the equilibrium distribution of confidence levels and also the evolutionary dynamics.

Definition 2. An equilibrium consists of an equilibrium distribution of types represented by a CDF F^* and PDF f^* on $[0,1]$ and an equilibrium strategy $\sigma^*_\theta \in [0,1]$ such that

1. For each $\theta_i \in [0,1]$,

$$\sigma^*_{\theta_i} \in \arg\max_{\sigma\in[0,1]} \int_0^1 \left\{\sigma[\sigma^*_{\theta_j}(\theta_i R + (1-\theta_i)r - c) + (1-\sigma^*_{\theta_j})R] + (1-\sigma)r\right\} f^*(\theta_j)d\theta_j.$$

2. π_θ equalizes across all $\theta \in [0,1]$, where

$$\pi_{\theta_i} = \int_0^1 [\sigma^*_{\theta_i}\sigma^*_{\theta_j}(\frac{1}{2}R + \frac{1}{2}r - c) + \sigma^*_{\theta_i}(1-\sigma^*_{\theta_j})R + (1-\sigma^*_{\theta_i}(\theta_j))r]f^*(\theta_j)d\theta_j.$$

By the following argument, we will see that overconfident players will always make up more than half of the population in the stable equilibrium. There is some cutoff type who is indifferent between fighting and not fighting. Let's call this type τ^*. Any type $\theta > \tau^*$ player fights and any type $\theta < \tau^*$ player does not fight. Let p^* denote the proportion of type $\theta > \tau^*$ players, Δ^* the proportion of type τ^* players, and $1-\Delta^*-p^*$ the proportion of type $\theta < \tau^*$ players.

When a type θ player chooses to fight, his expected utility is

$$u_\theta = (p^* + \Delta^*\sigma^*)[\theta R + (1-\theta)r - c] + (1-p^*-\Delta^*\sigma^*)R.$$

This must hold for τ^*, $u_{\tau^*} = r$:

$$(p^* + \Delta^*\sigma^*)[\tau^* R + (1-\tau^*)r - c] + (1-p^*-\Delta^*\sigma^*)R = r.$$

Second, the fitness level of all surviving confidence types must be the same, so we must have

$$(p^* + \Delta^*\sigma^*)(\frac{1}{2}R + \frac{1}{2}r - c) + (1 - p^* - \Delta^*\sigma^*)R = r.$$

Therefore, $\tau^* = 1/2$.

The equation simplifies to

$$p^* + \Delta^*\sigma^* = \frac{R - r}{\frac{1}{2}(R - r) + c}.$$

Since $c < R - r$, $p^* + \Delta^*\sigma^* > 2/3$. If there is no atom at $\tau^* = 1/2$, then at least 2/3 of the population have overconfident beliefs.

We derive τ^* without assuming any relations between R, r, and c, and we only need Assumption (iii) $c < R - r$ to show the result that overconfident types prevail. In particular, we do not need Assumption (ii) $c/(R - r) < 1/2$ as in the previous sections. In comparison, the critical type in the previous sections is $\theta^* = c/(R - r)$, which is above 1/2 only when Assumption (ii) holds.

Evolutionary dynamics are straightforward when confidence levels are not observable. Rational players all best respond by varying between fighting and not fighting. Rational players and overconfident players get the same expected payoffs, because rational players vary their strategies. Extremely underconfident players who do not fight at all are driven out of the market over time, but anyone who mixes fighting and not fighting survives the market.

5 Conclusion

This paper provides an evolutionary justification for overconfident beliefs. We show that in a class of resource-fighting games, under different settings of the observability of players' beliefs, players with overconfident beliefs are more likely to survive and thrive.

A word about the simplicity of our model. First, we only consider players with the same ability in the model. One justification for this is that players of lower abilities will be driven out of the market and competition prior to any belief heterogeneity. Belief heterogeneity only plays a significant role when players of the same ability compete. Second, we also strive to be agnostic about equilibrium selection and restrictions on evolutionary stability. Although we have multiple equilibria and unidentified distributions of confidence in the model, the main point—that the majority of players are overconfident in equilibrium—holds with minimal assumptions.

The paper considers the case when $(R-r)/2 < c < R-r$, and we find that overconfident players prevail in equilibrium. When the condition does not hold, overconfident players survive at the same rate as rational players. (i) When $c \leq (R - r)/2$, the cost of fighting is sufficiently low that it is a dominant strategy for a rational player and overconfident players to fight, and they get the same payoff of $(R + r)/2 - c$. (ii) When $c \geq R - r$, the cost of fighting is so high that fighting is a dominated strategy for even the most confident players. As a result, no player fights, and the payoff for everyone is r. Hence, overconfident players continue to survive for different costs of fighting.

However, going beyond the current game structure, we are able to find cases in which overconfident players are wiped out in favor of rational players. One case is the situation in which players' payoffs depend on their accuracy of own evaluation of ability. For example, if a confidence level θ player's payoff from fight is $R/2 + r/2 - (\theta - 0.5)^2 c/2$, (or simply, $K - (\theta - 0.5)^2 c/2$ for some constant K) that is, their cost of fighting increases quadratically in their inaccuracy of evaluation of own ability, then overconfident players have their payoffs diminished. The demise of overconfident (and under-confident) players holds regardless of confidence level.

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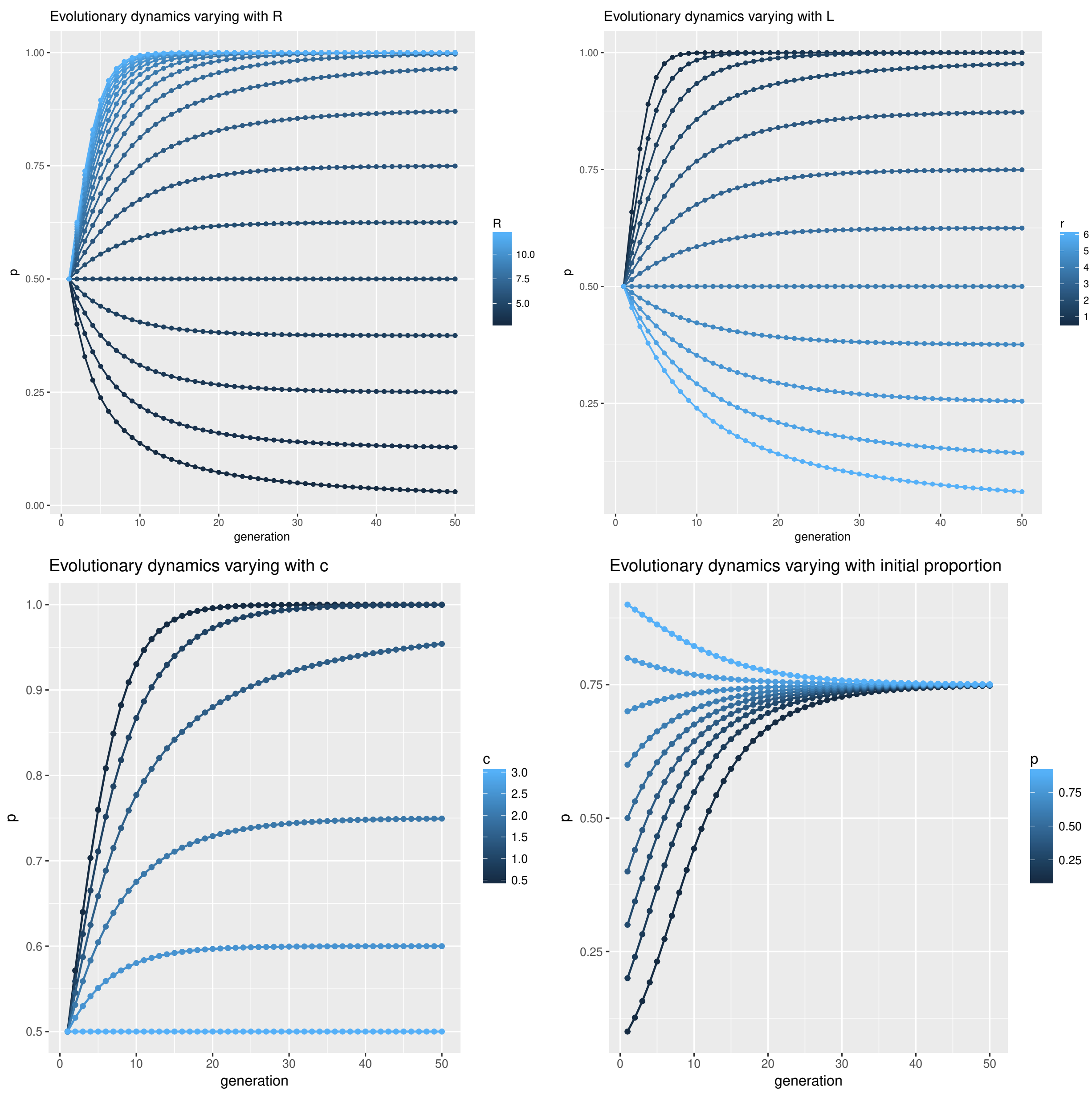

Figure 2: Evolutionary dynamics, as R, r, c, and p_0 vary individually, when $R = 6$, $r = 3$, $c = 2$, and $p_0 = 0.5$.

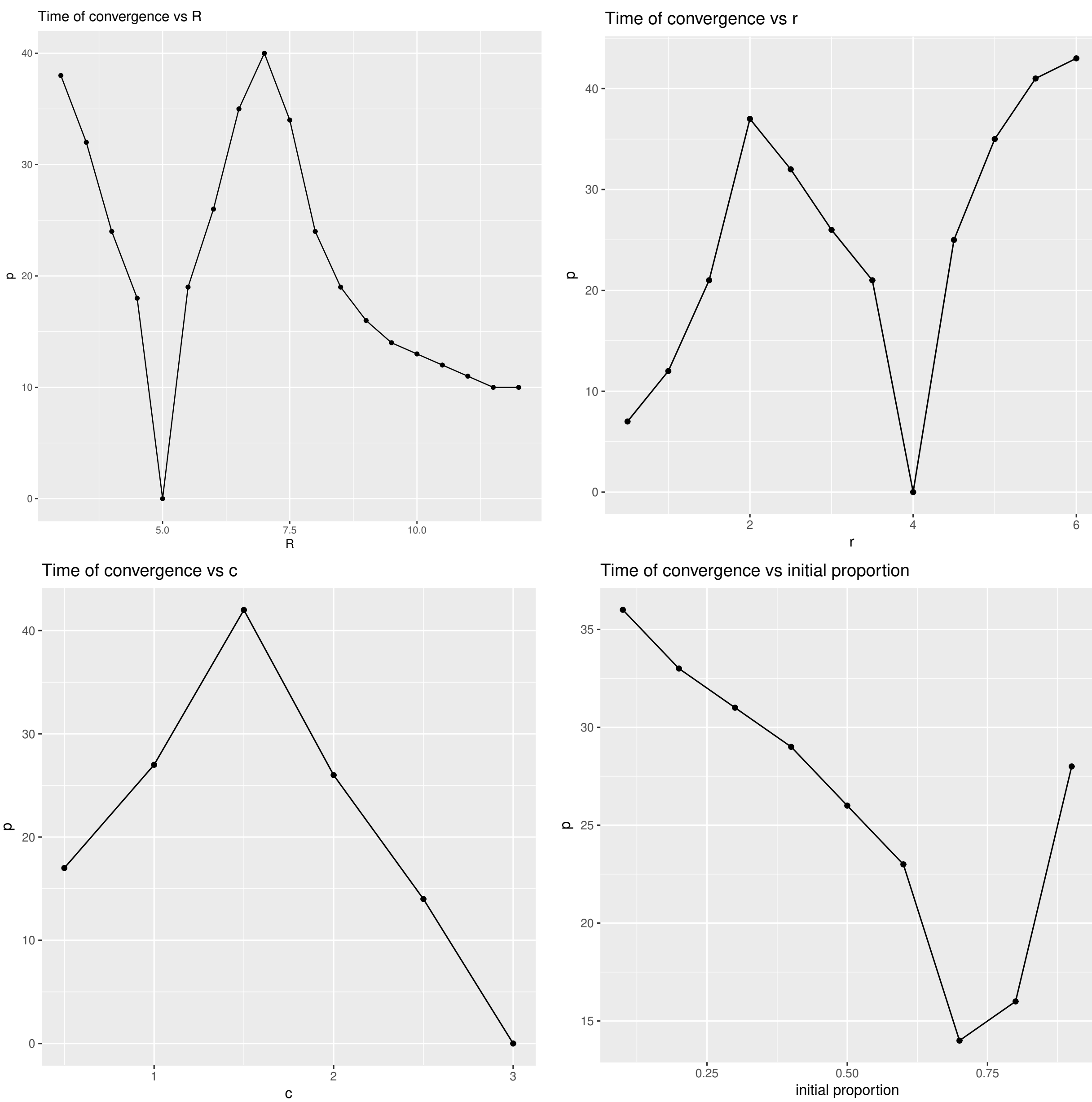

Figure 3: Number of generations to reach equilibrium, as R, r, c and p_0 vary individually, when $R = 6$, $r = 3$, $c = 2$, and $p_0 = 0.5$.