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Inefficiency in alternately repeated coordination games with dynastic preferences

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Abstract

This study investigates a specific model of alternately repeated pure coordination games with overlapping generations, where the one-shot game has multiple Pareto-ranked Nash equilibria. We consider the case in which the payoff of each player is affected by the outcome after his retirement as well as the outcome during his participation. Unlike the preceding results on alternately repeated coordination games where only the Pareto-efficient outcome is obtained in equilibria, we show that an inefficient equilibrium arises in our model.

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1. Introduction

In repeated games, players can obtain payoffs that are not obtainable in one-shot games. Under the condition of full dimensionality, Fudenberg and Maskin (1986) proved that any feasible and individually rational one-shot payoffs are attainable in equilibria of repeated games. However, in many studies of repeated games, including theirs, some assumptions that may not fit many real-world situations are imposed. First, simultaneous decision-making may be unrealistic. For example, once implemented, an investment decision to build a new factory cannot be changed for a long time, which causes stakeholders' decisions to alternate. Second, players' participation for infinitely long periods seems over-idealized. Instead, it would be more natural to consider a model in which players participate for some periods and then are sequentially replaced by new ones, which is called the overlapping generations (OLG) game. In addition, it is often the case that organizations are run by overlapping generations of participants, each of whom is concerned with not only the outcome during his own participation, but also the outcome after his retirement. For example, a former president of a company may well have more interest in his successor's performance than his own.

To analyze such problems, we examine an alternately repeated OLG pure coordination game with dynastic preferences whose stage game is given in Table I. In our stage game, two players choose either A or B and receive an identical payoff given an action profile. Only when both players choose A can they obtain a positive payoff, while their payoff is zero otherwise. The stage game is repeated by players of overlapping generations with two periods of life. Each player's older opponent is replaced by a new opponent one period after his entrance. Once each player has made a decision upon entering the game, he cannot alter this decision later. We also assume that each player receives the payoff for one period after his retirement, in addition to the payoff during his own participation. We show that, unlike the preceding results of repeated coordination games stating that only the efficient result is obtained in equilibria, an inefficient outcome arises where players play B in the equilibrium path.

Table I: Pure coordination game

	A	B
A	1	0
B	0	0

The reasons for this inefficiency are as follows. Suppose first that a player's payoff is not affected by his successors' play. When his older opponent deviates one period before his entrance and plays an efficient action, he also wants to play it because he can obtain a higher payoff irrespective of what his younger opponent plays in the next period. As a result, only the efficient action is played on the path in this case. However, when he has a dynastic preference, the result changes considerably. Suppose again that his predecessor deviates. When he also deviates, he immediately obtains an efficient outcome, but his successor's punishment of playing the inefficient action can diminish his payoff not only in the next period but also after his retirement. As a result, his payoff from playing the efficient action is not very high, causing him to give up the deviation.

Our result contrasts with those of Lagunoff and Matsui (1997) and Dutta (2012), who show that only the best one-shot payoff arises in any equilibrium of alternately repeated coordination games in which patient players live forever or die in the same period. At first glance, the existence of the Pareto inferior equilibrium seems to be a negative result. However, such an equilibrium serves as a punishment device that in turn sustains a superior outcome in other games.

Our present research may be related to that of Morooka (2019), which also showed the existence of inefficient equilibria in a different class of alternately repeated coordination games. In Morooka (2019), the one-shot payoffs gradually grow with a strictly positive rate over time, a feature that represents economic growth. Each player who plays an efficient action can be punished by his successor when the latter plays an inefficient action, diminishing the former's continuation payoffs. From the results of the present study and Morooka (2019), we may conclude that inefficiency in alternately repeated coordination games arises when the weight of continuation payoffs after becoming old is high, where players cannot control payoffs by their own behavior.

Our research is also related to Anderlini et al. (2008). In their model, OLG games with simultaneous moves are played by short-run players, where each player's payoff is affected by his successors' play. They showed that players can obtain any one-shot feasible payoffs in equilibria, including those without individual rationality, which are unobtainable in the standard model of simultaneously repeated games. Our research focuses on OLG games with different structures of repetition and shows the existence of equilibria that can never be obtained in the standard model of repeated coordination games with alternating moves.

The remainder of this paper is organized as follows. In Section 2, we define the model and prove the main result. In Section 3, we provide concluding remarks.

2. Model and Results

In our stage game, two players choose one of two actions, A or B, and their payoffs are described in Table I. There exist two Nash equilibria, $(A,\ A)$ and $(B,\ B)$, with payoffs 1 and 0, respectively. Our OLG game with perfect monitoring is defined as follows: Each player $k \geq 1$ lives for two periods, k and $k+1$. The opponent of player k during his first period is $k-1$, who is then replaced by $k+1$. Exclusively, the initial player 0 lives between periods 0 and 1, and his payoff in period 0 is assumed to be 0. Once each player decides his action at the beginning of his entrance, he cannot change the realized pure action at any point in his life.

In our model, each player obtains one-shot payoffs from the realized pure actions with no discount at the end of each period during his participation, along with one period after his retirement. When the sequence of one-shot payoffs $\{u(t)\}_{t=1}^{\infty}$ is realized, the payoff of player $k \geq 1$ (resp. player 0) is $u(k) + u(k+1) + u(k+2)$ (resp. $u(1) + u(2)$), where $u(t)$ is either 1 or 0. It should be noted that no player k can control $u(k+2)$ by his own action.[1] We prove the following inefficiency:

Theorem 1. There exists an equilibrium in which players play the inefficient action B on the path. Each player's payoff is 0.

Proof. Consider the following strategy consisting of three states: (Path), (Punishment), and (Reward). The initial state in period 0 is (Path).

·(Path): The entrant plays B. If B is played, the state in the next period remains (Path). If A is played, the state transfers to (Punishment).

·(Punishment): The entrant plays B. If B is played, the state transfers to (Reward). If A is played, the state remains (Punishment).

[1] If players do not have dynastic preferences, only the efficient outcome arises in equilibrium. When player $k-1$ is playing A, player k's total payoff during his life is at least 1 if he plays A, whereas it is 0 if he plays B.

·(Reward): The entrant plays A, and irrespective of what is played, the state remains (Reward).

When each player follows this strategy, the state remains (Path) over time, and his payoff is 0. The transition of states when player t deviates is shown in Table II.

Table II: Transition of states when player t deviates

<table>
<tr><td>Period</td><td>$\cdots$</td><td>$t-2$</td><td>$t-1$</td><td>t</td><td>$t+1$</td><td>$t+2$</td><td>$t+3$</td><td>$t+4$</td><td>$\cdots$</td></tr>
<tr><td>State</td><td>$\cdots$</td><td>Path</td><td>Path</td><td>Path</td><td>Punishment</td><td>Reward</td><td>Reward</td><td>Reward</td><td>$\cdots$</td></tr>
<tr><td rowspan="2">Actions</td><td colspan="2">$\cdots$ B</td><td colspan="2">B</td><td colspan="2">B</td><td colspan="2">A</td><td>$\cdots$</td></tr>
<tr><td>$\cdots$</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">A</td><td colspan="2">A $\cdots$</td></tr>
<tr><td>Payoff</td><td>$\cdots$</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>$\cdots$</td></tr>
</table>

The optimality of this strategy is shown as follows.

·The deviation from (Path): Suppose a player $k \geq 0$ deviates and plays A. The payoff in period k is 0. In the following two periods, player $k+1$ plays B in (Punishment) state, and player $k+2$ plays A in (Reward) state, as a result of which k's total payoff is also 0. Therefore, playing B in this state satisfies the best-response property.

·The deviation from (Punishment): Suppose a player $k \geq 1$ deviates and plays A. The payoff in period k is 1. In the following two periods, player $k+1$ plays B in (Punishment) state, and player $k+2$ plays A in (Reward) state, as a result of which k's total payoff is 1. In contrast, if k does not deviate, the payoff in period k is 0, but players $k+1$ and $k+2$ play A in (Reward) state, as a result of which k's total payoff is also 1. Therefore, playing B in this state satisfies the best-response property.

·The deviation from (Reward): When a player deviates, his total payoff is at most 1. When he does not deviate, his total payoff is at least 2 and is optimal, completing the proof of Theorem 1.□

3. Concluding Remarks

We analyzed a specific model of alternately repeated pure-coordination games played by overlapping generations of players with dynastic preferences. We observed that, unlike the results of previous studies of alternately repeated coordination games without overlapping generations, players may obtain bad payoffs in subgame perfect equilibria. In the future, we will focus on more general models of one-shot games and investigate whether an analogous result applies. We also intend to develop a class of games in which such an equilibrium can be used as a penalty, which in turn allows players to obtain better payoffs on the path outside the set of one-shot feasible and individually rational payoffs.

References

Anderlini, L., D. Gerardi, and R. Lagunoff (2008) "A 'super' folk theorem for dynastic repeated games" *Economic Theory* **37**, 357-394.
DOI: https://doi.org/10.1007/s00199-007-0293-9

Dutta, P. (2012) "Coordination need not be a problem" *Games and Economic Behavior* **76**, 519-534.
DOI: https://doi.org/10.1016/j.geb.2012.07.002

Fudenberg, D. and E. Maskin (1986) "The folk theorem in repeated games with discounting or with incomplete information" *Econometrica* **54**, 533-554.
DOI: https://doi.org/10.2307/1911307

Lagunoff, R. and A. Matsui (1997) "Asynchronous choice in repeated coordination games" *Econometrica* **65**, 1467-1477.
DOI: https://doi.org/10.2307/2171745

Morooka, C. (2019) "Inefficiency in alternately repeated games with overlapping generations" *Economics Letters* **184**, 108632.
DOI: https://doi.org/10.1016/j.econlet.2019.108632