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Testing the degree of persistence of Covid-19 using Fourier quantile unit root test

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Abstract

In this paper, we test the stochastic properties of daily accumulated number of confirmed cases of Covid-19 (ANCCC) in the 20 countries using Fourier quantile unit root test. The results indicate that the negative shocks to ANCCC series have long lasting effects in all countries (except China) and big positive shocks have permanent effects in some countries. Our results have important policy implications.

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1. Introduction

Coronavirus disease 2019 or COVID-19 is an infectious disease and ongoing pandemic, which was first identified in December 2019 in Wuhan, China, and has infected across 188 countries and territories with more than 62.6 million positive cases and more than 1.5 million deaths until 1 December 2020[1]. Its spread has left businesses around the world suffering and has resulted in economic shutdown in many countries and large industries. According to the International Monetary Fund (IMF) projection, the global GDP growth rate and trade will shrink by 9% in the year 2020. Furthermore, the IMF predicted that the share of people living in extreme poverty will increase by 1.5%. The United Nations Development Program (report of May 2020) reveals that due to the negative effects of Covid-19 on the income, health, and education, the recent crisis will decrease human development in the world for the first time since 1990.

Due to the contractionary effects of COVID-19 in the world economy, the persistency degree of the disease in the countries is very important. The longer the disease persists in a country, the greater and more devastating its effects on the economy. In this paper, we test the degree of persistence of COVID-19 in 20 countries with the most positive cases using Fourier-quantile unit root (FQUR) test. Since countries implemented various policies to control the disease, e.g. social distancing, there is possibility of smooth breaks in the accumulated number of positive cases over the time. Using the FQUR test, we are able to control for smooth breaks in the deterministic parts of time trend during study period. Also using the test, we able to compare the persistent degree of disease when the positive cases are high and when they are low. To that end, we explain the FQUR test in Section 2 and describe the data in Section 3. The empirical results are reported in Section 4 followed by a summary and conclusion is Section 5.

2. Fourier Quantile Unit Root Test

Quantile Unit Root test was originally developed by Koenker and Xiao (2004). Recently, Bahmani-Oskooee et al. (2017, 2018) developed a new version of the test, namely, Fourier Quantile Unit Root test (FQUR), which allows for smooth breaks in the deterministic parts of the trend function by including the Fourier expansions. The two specific properties of the FQUR test are (1) control for asymmetric responses of time series variables (accumulated number of confirmed case of Covid-19 in our case) to positive and negative shocks and (2) allowing for time-varying intercept/slope of the trend function (smooth breaks), asymmetric mean-reverting speed across quantiles, and linear mean-reverting speed within quantiles of time series variables.

FQUR test is done in the two steps procedure. In the first step, the time series variable COV_t is de-trended by Fourier expansion:

$$COV_t = a_1 + a_2 t + a_3 \sin\left(\frac{2\pi kt}{N}\right) + a_4 \cos\left(\frac{2\pi kt}{N}\right) + \varepsilon_t, \qquad (1)$$

where COV, k, t, and ε are respectively the accumulated number of confirmed cases of Covid-19 series, the number of frequencies, time trend, and i.i.d error term. The sin(.) and cosine(.) terms are included in the model to capture the smooth breaks. To estimate the equation (1), first we select the optimal value of k (say k^*) from $k \in [0.1\,.5]$, where the sum of squared residuals of the OLS estimator is minimized. An integer value (fractional value) of k^* is related to transitory (permanent) structural breaks in the long-run steady state of COV_t.

In the second step, using the estimated residuals of OLS (i.e. $\hat{\varepsilon}_t$) from step 1, we examine the mean reverting properties of COV_t within each quantiles using following quantile-ADF type regression:

[1] . The figures were prepared by World Health Organization.

$$Q_{\hat{\varepsilon}_t}(\tau|\vartheta_{t-1}) = \theta_0(\tau) + \theta_1(\tau)\hat{\varepsilon}_{t-1} + \sum_{p=1}^{p=l} \theta_{1+p}(\tau)\Delta\hat{\varepsilon}_{t-p} + \epsilon_t, \quad (2)$$

where $Q_{\hat{\varepsilon}_t}(\tau|\vartheta_{t-1})$ is the τ^{th} quantile of residuals of equation (1), which are obtained using the OLS estimator, conditional on the past information set, ϑ_{t-1}. Here, $\theta_0(\tau)$ is the value of the τ^{th} conditional quantile of $\hat{\varepsilon}_t$ and captures the magnitude of the COV shock in each quantile. The negative (positive) values of $\theta_0(\tau)$ indicate the negative (positive) shocks to the COV series. The terms $\sum_{p=1}^{p=l}\theta_{1+p}(\tau)\Delta\hat{\varepsilon}_{t-p}$ are included in the equation (2) to control for the serial correlation of the error term ϵ_t. Bahmani-Oskooee et al. (2017, 2018) developed two test statistics including $t_n(\tau_i)$ and $Fourier - QKS$ to examine the unit root hypothesis within each quantile and over a range of quantiles, respectively:

$$t_n(\tau_i) = \frac{\hat{f}(F^{-1}(\tau_i))}{\sqrt{\tau_i(1-\tau_i)}}(W'_{-1}B_x W_{-1})^{1/2}(\widehat{\theta_1}(\tau_i) - 1), \quad (3)$$

$$Fourier - QKS = \sup_{\tau_i \in [\underline{\xi}, \overline{\xi}]} |t_n(\tau_i)| . \quad (4)$$

where W_{-1} is the vector of the first lag of $\hat{\varepsilon}_t$, $\hat{f}(F^{-1}(\tau_i))$ is a consistent estimator of $f(F^{-1}(\tau_i))$ that is discussed in more details by Koenker and Xiao (2004), and B_x is the projection matrix onto the space orthogonal to $X = (\Delta\hat{\varepsilon}_{t-1}. \dots . \Delta\hat{\varepsilon}_{t-k})$.

The $Fourier - QKS$ test statistics is a Kolmogorov–Smirnov test statistics type and is related to the maximum $|t_n(\tau_i)|$ statistics over quantiles between $\underline{\xi}$=0.1 and $\overline{\xi}$=0.9, and their critical values are computed using re-sampling procedures, as discussed in greater detail in Bahmani-Oskooee *et al.* (2018).

3. Data description

Daily accumulated number of confirmed cases of Covid-19 (ANCCC, hereafter) are collected from 20 countries with most confirmed cases, including United States, Russia, United Kingdom, Spain, Italy, Brazil, Germany, Turkey, France, Iran, China, India, Peru, Canada, Belgium, Saudi Arabia, Netherlands, Mexico, Pakistan, and Chile over the period 01/01/2020 to 14/05/2020 from "*Oxford Covid-19 Government Response Tracker*" database. In Panel A of Table 1, we present the data description. Dynamics of ANCCC series indicate that up to 14/05/2020, most confirmed cases of Covid-19 are related to United State, Russia, United Kingdom, Spain, and Italy respectively with 1390746, 242271, 229705, 229540, and 222104 cases and the least confirmed cases are related to Iran, China, India, Peru, and Belgium respectively with 112725, 84024, 78003, 76306, and 53981 confirmed cases.

[Insert Table 1 about here]

4. Empirical results

To analyze the degree of persistence of ANCCC series[2], first we apply five conventional unit root/stationary tests including augmented Dickey and Fuller (1979, ADF), Elliot et al. (1996, DF-GLS), Phillips and Perron (1988, PP), Ng-Perron (Ng and Perron, 2001, NP), and Kwiatkowski et al. (1992, KPSS)[3]. The results of the mentioned tests indicate that the null hypothesis of unit root is rejected for the ANCCC series of (1) Turkey by three tests ADF, DF-GLS, and PP, (2) Russia and Canada by ADF and DF-GLS, (3) Netherlands by ADF and PP, (4) China and Pakistan by ADF, and (5) Brazil, Iran. Saudi Arabia and Mexico by PP unit

[2] . We use the logs form of the ANCCC series to test the null hypothesis of unit root.

[3] . To save the space, we do not report the results of conventional unit root tests. The results are available upon request from authors.

root test at 5% level of significant. The unit root hypothesis is rejected for none of the ANCCC series using KPSS stationary test and NP unit root test.

It is clear that using the conventional linear unit root tests, we do not reach a consensus on the stochastic behavior of the ANCCC series. There are two potential reasons; (1) structural breaks in the growth rates of daily confirmed cases of Covid-19 due to various policies that are applied in the countries and (2) non-normal distribution of the ANCCC series. The mentioned unit root/stationary tests have low power when the time series variables exhibit structural breaks and/or non-normal distribution. The p-values of the Jarque-Bera (1980) non-normality test in Table 1 indicate that all ANCCC series except the one that belongs to Saudi Arabia are non-normally distributed at least at the 10% level of significant.

Table 1: Descriptive Statistics

Countries	Mean	Maximum	P-value of Jarque-Bera	Observations
United States	321643.2	1390746	0.000	115
Russia	33180.33	242271	0.078	104
United Kingdom	55605.7	229705	0.007	105
Spain	89395.46	229540	0.000	104
Italy	85159.82	222104	0.000	105
Brazil	36025.61	188974	0.064	79
Germany	60642.24	172239	0.000	108
Turkey	64817.32	143114	0.000	60
France	46857.67	140734	0.000	111
Iran	48528.22	112725	0.009	85
China	57963.27	84024	0.035	135
India	11382.42	78003	0.068	105
Peru	17187.16	76306	0.088	68
Canada	16412.11	72278	0.004	110
Belgium	17357.33	53981	0.001	101
Saudi Arabia	10221.03	44830	0.153	70
Netherlands	19299.99	43211	0.000	77
Mexico	9219.471	40186	0.099	68
Pakistan	8079.726	35788	0.019	73
Chile	8534.07	34381	0.009	71

To control for the structural breaks in the growth rates of daily confirmed cases of Covid-19 and non-normal distribution, we apply the FQUR test and present the results in Table 2. In panel A of Table 2, we report the results of $t_n(\tau_i)$ to test the null hypothesis of unit root within quantiles 0.1, 0.3, 0.5, 0.7, and 0.9. The results indicate that almost, all ANCCC series, except China, behave like non-stationary process in the low quantiles 0.1 and 0.3 and some of ANCCC series behave like non-stationary process in the highest quantile 0.9. Whereas the low quantiles are related to negative shocks to the ANCCC series, the results indicate that negative shocks to all ANCCC series have persistence effects and very big positive shocks to ANCCC series of United States, Russia, Italy, Brazil, Germany, France, China, Canada, Belgium, Saudi Arabia, Netherlands, Pakistan, and Chile will have long-lasting effects. The results of $Fourier - QKS$ test statistics indicate the null hypothesis of unit root is rejected for the

ANCCC series of Russia, Peru, Canada, Belgium, Saudi Arabia, Netherlands, and Chile and for other series, the null hypothesis of unit root is not reject.

Table 2: Results of FQUR test

Countries	Panel A: P-value of t_n test statistics					$Fourier - QKS$	Panel B: Critical values		
	0.1	0.3	0.5	0.7	0.9		10%	5%	1%
United States	0.790	0.720	0.670	0.010	0.410	3.648	3.333	3.481	4.236
Russia	0.880	0.120	0.030	0.110	0.110	2.843	3.018	3.226	3.895
United Kingdom	0.150	0.070	0.000	0.010	0.010	5.172	3.952	4.458	6.030
Spain	0.980	0.690	0.310	0.220	0.070	3.397	2.916	3.460	4.530
Italy	0.830	0.890	0.120	0.020	0.880	2.803	2.704	2.895	3.218
Brazil	0.020	0.310	0.080	0.120	0.890	4.855	4.727	6.306	7.654
Germany	0.700	0.210	0.020	0.000	0.330	5.944	2.904	3.244	3.786
Turkey	0.170	0.190	0.100	0.060	0.060	7.626	7.475	9.698	11.123
France	0.050	0.360	0.000	0.000	0.190	4.492	2.992	3.261	4.161
Iran	0.410	0.950	0.750	0.020	0.000	7.300	4.526	5.174	5.686
China	0.030	0.020	0.010	0.000	0.680	4.028	2.785	2.927	3.717
India	0.740	0.970	0.170	0.040	0.110	2.935	2.724	2.914	4.511
Peru	0.480	0.290	0.150	0.070	0.110	6.166	6.575	8.007	10.021
Canada	0.970	0.970	0.170	0.320	0.350	1.742	3.343	3.826	4.175
Belgium	0.950	0.710	0.100	0.020	0.100	2.367	2.764	3.021	4.426
Saudi Arabia	0.330	0.300	0.360	0.070	0.360	3.162	4.238	4.572	6.123
Netherlands	0.720	0.600	0.430	0.440	0.160	3.101	5.292	6.301	7.576
Mexico	0.820	0.180	0.300	0.210	0.000	10.528	6.966	10.125	14.876
Pakistan	0.010	0.130	0.040	0.450	0.290	6.440	4.565	6.457	10.722
Chile	0.210	0.130	0.110	0.130	0.250	3.174	4.911	5.972	8.081

Notes: We use the AIC criteria to select the optimum lag(s). The critical values of $Fourier - QKS$ test statistics are computed using bootstrapping procedure and 5000 replications.

5. Conclusion

In this paper, we analyze the degree of persistence of ANCCC series of 20 countries, which have the most number of confirmed cases using FQUR test. The results indicate that all ANCCC series show asymmetric stochastic behavior. Negative shocks to all ANCCC series (except China) have long lasting effects than positive ones. The results indicate that government responses to control the disease such as containment and closure policies, school closures and restrictions in movement, income support to citizens or provision of foreign aid, health system policies such as the COVID-19 testing regime or emergency investments into healthcare, especially when the number of confirmed cases are low, may have long lasting effects. In contrast, big positive shocks, especially when the number of confirmed cases are high in the countries including United States, Russia, Italy, Brazil, Germany, France, China, Canada, Belgium, Saudi Arabia, Netherlands, Pakistan, and Chile, will have long lasting effects. The results indicate that governments have to continue their social and economic policies until the vaccine for the virus is discovered or completely eradicated. The message of this paper is more important for countries in which the null hypothesis of unit root is rejected. For this group of countries, the ANCCC series exhibit mean reverting properties and thus the negative shocks may have transitory effects.

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