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## Performance measure aggregation – two action levels

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## Abstract

In practice, effort is often measured in two levels due to the simplicity. When employees' actions are restricted to two levels, this study shows that the basic measures can be aggregated into a one-dimensional sufficient aggregate without losing any information from the basic measures. The format of the one-dimensional sufficient aggregate is determined only by the statistical properties of the basic measures and the one-dimensional sufficient aggregate can be uniformly used regardless of organizational particulars. By using the one-dimensional sufficient aggregate, complexity of performance evaluation systems can be reduced and efficiency of motivating employees through performance evaluation can be substantially improved.

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# 1. Introduction

Aggregation of performance measures is an important factor in designing a performance evaluation system. With aggregate measures that summarize the information from the basic measures, performance evaluation systems can become simpler and more efficient.

We analyze sufficient aggregation of performance measures when the agent's action is restricted to two levels. In practice, effort is often measured in two levels due to the simplicity. For example, students attending class or not, employees going to work place or not, government officers giving permission or not, etc. On the other hand, the literature on performance measure aggregation seldom uses the agent's action restricted to two levels. This study contributes to the literature by first analyzing sufficient aggregation of performance measures when the agent's action is restricted to two levels.

In designing a performance evaluation system, two important questions arise in regards to performance measure aggregation. The first question is whether an aggregate measure can carry all information from the basic measures. Statistically sufficient aggregates satisfy this requirement and incur no loss of information in the aggregation process for all agency particulars (Holmstrom 1979, Amershi and Huges 1989, Amershi, Banker, and Datar 1990, Sabac and Yoo 2018). The second question is whether an aggregate measure is good enough for the principal to substitute for the basic measures in the optimal contract. Economically sufficient aggregates satisfy this requirement and incur no economic loss to the principal in inducing the optimal action from the agent.

A statistically sufficient aggregation is always an economically sufficient aggregation. When an aggregate of the basic measures incurs no loss of information in the aggregation process, the principal can always substitute, with no economic loss, the aggregate for the basic measures in the optimal contract.

On the other hand, an economically sufficient aggregation is generally not statistically sufficient. A special condition under which an economically sufficient aggregation is also statistically sufficient is given by Holmstrom (1979) and Amershi et al. (1990) as the "all $a$ or no $a$" condition. In discussing the informativeness condition, Holmstrom (1979) assumes that the likelihood ratio $\frac{\partial\varphi(y,z;a)/\partial a}{\varphi(y,z;a)}$ of a probability density function $\varphi(y,z;a)$ exists for all (i.e., not some) levels of the agent's action (see footnotes 18 and 21 in his paper).

Amershi et al. (1990) extends the analysis and presents that when the likelihood ratio $\frac{\partial\varphi(y,z;a)/\partial a}{\varphi(y,z;a)}$ is represented as a function $h(\cdot)$ of the same sufficient statistic $T(y,z)$ of the basic measures for all (i.e., not some) levels of the agent's action

$$\frac{\partial\varphi(y,z;a)/\partial a}{\varphi(y,z;a)} = h\big(T(y,z);a\big), \tag{1}$$

the "all $a$ or no $a$" condition is satisfied. When the "all $a$ or no $a$" condition (1) is satisfied, an economically sufficient aggregation is also statistically sufficient such that all sufficient aggregates are statistically sufficient.

Both Holmstrom (1979) and Amershi et al. (1990) assume that the agent's action space is continuous such that the probability density function $\varphi(y,z;a)$ is differentiated by the action $a$. As the agent's action is restricted to two levels, the criterion (1) for satisfying the "all $a$ or no

$a$" condition cannot be used in this study. Instead, we explicitly analyze sufficient aggregation of performance measures that follow a joint normal distribution and show that all sufficient aggregates are statistically sufficient and the result is virtually equivalent to satisfying the "all $a$ or no $a$" condition.

The rest of the article is structured as follows. Section 2 presents the model. Sections 3 and 4 analyze statistically sufficient aggregation and economically sufficient aggregation, respectively. Section 5 concludes this study.

# 2. Model

A risk neutral principal owns a production technology which requires an action $a_i$, $i = H, L$ from a risk averse agent. The agent's unobservable personal action choice is either a high level $a_H$ or a low level $a_L$. The economic outcome from the agency is not contractible and the compensation $C(y,z)$ for the agent is based on two contractible performance measures $y$ and $z$

$$y = m\ a_i + \varepsilon,\ i = H, L,\ \varepsilon \sim N\left(0, \sigma_\varepsilon^2\right) \tag{2}$$

$$z = k\ a_i + \delta,\ i = H, L,\ \delta \sim N\left(0, \sigma_\delta^2\right) \tag{3}$$

Two performance measures $y$ and $z$ reflect the agent's action. The agent exerts a high level effort $a_H$ or low level effort $a_L$ to affect the basic measures $y$ and $z$ with the respective sensitivity coefficients $m$ and $k$. The agent's utility function $U^a(\cdot)$ is additively separable for the personal action cost $v(a_i)$

$$U^a\left(C(y,z), a_i\right) = U\left(C(y,z)\right) g(a_i) - v(a_i),\ i = H, L, \tag{4}$$

where $U(\cdot)$ is a strictly concave consumption function( $U'(\cdot) > 0,\ U''(\cdot) < 0$ ) and $g(a_i)$ is a positive coefficient( $g(a_i) > 0$ ) depending on the agent's action level. The high level action $a_H$ is more costly than the low level action $a_L$ to the agent( $v(a_H) > v(a_L)$ ).

The basic measures $y$ and $z$ are also affected by the respective random factors $\varepsilon$ and $\delta$. The random factor $\varepsilon$ is normally distributed with the mean $0$ and variance $\sigma_\varepsilon^2$. The random factor $\delta$ is also normally distributed with the mean $0$ and variance $\sigma_\delta^2$. The basic measures $y$ and $z$ may be correlated with the covariance $Cov(y,z) = \rho_{\varepsilon\delta}\sigma_\varepsilon\sigma_\delta$. The basic measures $y$ and $z$, conditional on the agent's action $a_i$, follow a joint normal distribution with the density function

$$f(y,z;a_i) = \frac{1}{2\pi\sqrt{1-\rho_{\varepsilon\delta}^2}\,\sigma_\varepsilon\sigma_\delta} \exp\left[-\frac{1}{2(1-\rho_{\varepsilon\delta}^2)}\Psi(a_i)\right], \tag{5}$$

$$\Psi(a_i) = \frac{(y - m\ a_i)^2}{\sigma_\varepsilon^2} - 2\rho_{\varepsilon\delta}\frac{(y - m\ a_i)(z - k\ a_i)}{\sigma_\varepsilon\sigma_\delta} + \frac{(z - k\ a_i)^2}{\sigma_\delta^2}$$

# 3. Statistically sufficient aggregation

Statistical sufficiency of aggregation is characterized by no loss of information in the aggregation process for all agency particulars. Statistical sufficiency is defined in terms of an aggregate measure being a sufficient statistic for the basic measures $y$ and $z$ with respect to the agent's unobservable action $a_i$. A sufficient statistic for the basic measures $y$ and $z$ can be found by analyzing the joint normal density function $f(y,z;a_i)$ (5). The existence of a sufficient statistic is shown by the factorization criterion (Degroot, 1986; 1970).

**Definition 1**. (Sufficient statistic; factorization criterion)

A statistic $T(y,z)$ is statistically sufficient for the basic measures $y$ and $z$ if, and only if, the joint normal density function $f(y,z;a_i)$ (5) is factorized for all action levels $a_i$, $i=H,L$

$$f(y,z;a_i)=p(y,z)\ q\left(T(y,z);a_i\right), \tag{6}$$

where $p(\cdot)$ and $q(\cdot)$ are positive functions. The function $q(\cdot)$ depends on the basic measures $y$ and $z$ only through the aggregate $T(y,z)$.

An aggregate measure $T(y,z)$ is a sufficient statistic for the basic measures $y$ and $z$ with respect to the agent's unobservable action $a_i$ if observing the aggregate measure without observing the basic measures is as useful as observing the basic measures in estimating all levels of the agent's action. All information relevant to estimating the agent's unobservable action is preserved and passed on the sufficient statistic $T(y,z)$ from the basic measures $y$ and $z$. When an aggregation is statistically sufficient, the aggregate is determined only by the statistical properties of the basic measures.

With Definition 1, we explicitly derive a statistically sufficient aggregate of the basic measures $y$ and $z$.

**Proposition 1.** (Statistically sufficient aggregate)

When the basic measures $y$ and $z$ are joint normally distributed by the density function $f(y,z;a_i)$ (5), statistical sufficiency of aggregation is achieved by an aggregate $T(y,z)$ of the basic measures $y$ and $z$

$$\begin{aligned} T(y,z) &= \left\{ \frac{m-\left(\dfrac{\rho_{\varepsilon\delta}\sigma_\varepsilon\sigma_\delta}{\sigma_\delta{}^2}\right)k}{\sigma_\varepsilon^2} \right\} y + \left\{ \frac{k-\left(\dfrac{\rho_{\varepsilon\delta}\sigma_\varepsilon\sigma_\delta}{\sigma_\varepsilon{}^2}\right)m}{\sigma_\delta^2} \right\} z \\ &= \left\{ \frac{m-\dfrac{Cov(y,z)}{Var(z)}k}{Var(y)} \right\} y + \left\{ \frac{k-\dfrac{Cov(y,z)}{Var(y)}m}{Var(z)} \right\} z \end{aligned} \tag{7}$$

(All proofs are in Appendix.)

The aggregate $T(y,z)$ (7) is statistically sufficient and loses no information from the basic measures $y$ and $z$ in the aggregation process. The format of the statistically sufficient aggregate $T(y,z)$ (7) is determined only by the statistical properties of the basic measures $y$ and $z$.

# 4. Economically sufficient aggregation

While statistical sufficiency of aggregation requires no loss of information in the aggregation process, economic sufficiency of aggregation requires no economic loss to the principal although there can be some loss of information in the aggregation process. When an aggregation of performance measures incurs no economic loss to the principal in inducing the optimal action level, the aggregate is economically sufficient. An economically sufficient aggregate can depend on the agent's utility function $U^a(\cdot)$ and the action level $a_i$.

**Definition 2**. (Economic sufficiency)

An aggregation of performance measures is economically sufficient if there exist an aggregate $\zeta\left(y,z;U^a(\cdot),a_i^\dagger\right)$ and a function $l(\cdot)$ such that the aggregate $\zeta\left(y,z;U^a(\cdot),a_i^\dagger\right)$ can be substituted for the basic measures $y$ and $z$ in the optimal contract to induce the optimal action level $a_i^\dagger$

$$C^\dagger(y,z)=l\left(\zeta\left(y,z;U^a(\cdot),a_i^\dagger\right)\right), \tag{8}$$

where the function $l(\cdot)$ depends on the basic measures $y$ and $z$ only through the aggregate $\zeta\left(y,z;U^a(\cdot),a_i^\dagger\right)$.

For inducing the optimal action level, the principal is as well off with an economically sufficient aggregate $\zeta\left(y,z;U^a(\cdot),a_i^\dagger\right)$ as with the basic measures $y$ and $z$. In inducing the optimal action level, an economically sufficient aggregate can vary depending on the agent's utility function $U^a(\cdot)$.

With Definition 2, we explicitly derive an economically sufficient aggregate of the basic measures $y$ and $z$ by analyzing the aggregation of performance measures in the optimal contract. We assume that the high level action $a_H$ is the optimal action for the principal to induce. The first reason for this assumption is that the principal may benefit more from the high level action $a_H$ than the low level action $a_L$ although we do not formally include this in the model. The second reason is that when the principal induces the low level action $a_L$ from the agent, the basic measures $y$ and $z$ are not used for the compensation contract in equilibrium and we do not observe the way of aggregation.

When the principal induces the low level action $a_L$, the principal offers in equilibrium to pay a fixed amount of compensation regardless of the values of the basic measures $y$ and $z$. Because the agent's action is not observable and the low level action $a_L$ is less costly than the high level action $a_H$ to the agent( $v(a_L)<v(a_H)$ ), the agent rationally chooses the low level action $a_L$ and the principal does not have to provide an incentive for the agent to choose the low level action $a_L$. As there is no need for providing an incentive, the optimal contract is only for risk sharing and it is efficient for the risk neutral principal to take all risk. In inducing the low level action $a_L$, the principal offers the risk averse agent a fixed payment contract in equilibrium and consequently, we observe no aggregation of the basic measures $y$ and $z$.

With the agent's additively separable utility function $U^a(\cdot)$ (4), the agent's expected utility is, for both action levels $i=H,L$

$$
\begin{aligned}
& E\left[U^a\left(C\left(y,z\right),a_i\right)\right] \\
& = E\left[U\left(C\left(y,z\right)\right)\right]g\left(a_i\right) - v\left(a_i\right) \\
& = \iint U\left(C\left(y,z\right)\right) f\left(y,z;a_i\right) \text{ dydz } \mathrm{g}\left(a_i\right) - v\left(a_i\right),
\end{aligned} \tag{9}
$$

where $f(y,z;a_i)$ is the joint normal density function (5).

The participation (individual rationality) constraint for the agent's high level action $a_H$ is

$$
\text{(PC)} \quad E\left[U^a\left(C\left(y,z\right),a_H\right)\right] \geq C_0, \tag{10}
$$

where $C_0$ is the agent's minimum required utility level which is available from an alternative employment opportunity in the labor market.

The participation constraint (10) binds in equilibrium. Otherwise, suppose the principal offers a contract $C^A\left(y,z\right)$ such that the participation constraint does not bind. In this case, the principal will be better off by offering another contract $C^B\left(y,z\right)$ such that the principal pays the agent a compensation reduced from $C^A\left(y,z\right)$ by the same small amount $\Delta$ for all values of $y$ and $z$ ($C^B\left(y,z\right) = C^A\left(y,z\right) - \Delta$). The principal is better off by the small amount $\Delta$ with the new contract $C^B\left(y,z\right)$ and the agent accepts the new contract $C^B\left(y,z\right)$. Therefore, offering the contract $C^A\left(y,z\right)$ is not in equilibrium. In fact, any contract resulting in the agent's expected utility from the contract( $E\left[U^a\left(C\left(y,z\right),a_H\right)\right]$ ) more than the minimum required utility level( $C_0$ ) is not in equilibrium and the participation constraint (10) binds in equilibrium.

With the agent's action being restricted to two levels, the first order approach (Rogerson 1985) cannot be used for the incentive compatibility constraint. The incentive compatibility constraint for the agent's high level action $a_H$ is

$$
\text{(IC)} \quad E\left[U^a\left(C\left(y,z\right),a_H\right)\right] \geq E\left[U^a\left(C\left(y,z\right),a_L\right)\right] \tag{11}
$$

The incentive compatibility constraint (11) binds in equilibrium (see proof of Proposition 2).

For convenience of analysis, define

$$
C\left(y,z\right) = w\Big(U\left(C\left(y,z\right)\right)\Big), \tag{12}
$$

where $w(\cdot)$ is the inverse function of the consumption function $U\left(\cdot\right)$, $w\left(\cdot\right) = U^{-1}\left(\cdot\right)$. Because the consumption function is monotone increasing( $U'\left(\cdot\right) > 0$ ), the inverse function is also monotone increasing( $w'\left(\cdot\right) > 0$ ). With the definition (12), the expected compensation from the contract when the agent takes the high level action $a_H$ is

$$
\begin{aligned}
E\left[C\left(y,z\right) \mid a_H\right] &= E\left[w\Big(U\left(C\left(y,z\right)\right)\Big) \mid a_H\right] \\
&= \iint w\Big(U\left(C\left(y,z\right)\right)\Big) f\left(y,z;a_H\right) \text{ dydz}
\end{aligned} \tag{13}
$$

Grossman and Hart (1983) show that the optimal contract is a special case of an efficient contract that induces an arbitrary action level with the minimum cost. In our case, the optimal contract must be a minimum cost contract that induces the high level action $a_H$. In the following optimization problem, the choice variable is the compensation $C(y,z)$ when the values of the basic measures $y$ and $z$ are given.

$$\begin{aligned}\underset{C(y,z),\lambda,\mu}{\text{Max}}\ L &= -E\left[C(y,z)\,|\,a_H\right] + \lambda\ \text{(PC)} + \mu\ \text{(IC)} \qquad (14)\\ &= -\iint w\left(U\left(C(y,z)\right)\right) f(y,z;a_H)\ \text{dydz}\\ &+\lambda\left[\iint U\left(C(y,z)\right) f(y,z;a_H)\ \text{dydz}\ \text{g}(a_H) - v(a_H) - C_0\right]\\ &+\mu\begin{bmatrix}\iint U\left(C(y,z)\right) f(y,z;a_H)\ \text{dydz}\ \text{g}(a_H) - v(a_H)\\ -\iint U\left(C(y,z)\right) f(y,z;a_L)\ \text{dydz}\ \text{g}(a_L) + v(a_L)\end{bmatrix},\end{aligned}$$

where $\lambda$ is the Lagrange multiplier for the participation constraint (10) and $\mu$ is the Lagrange multiplier for the incentive compatibility constraint (11).

Solving the optimization problem (14) results in the optimal contract. Economic sufficiency of aggregation is achieved by the aggregation of the basic measures $y$ and $z$ in the optimal contract.

**Proposition 2.** (Economically sufficient aggregate)

When the basic measures $y$ and $z$ are joint normally distributed by the density function $f(y,z;a_i)$ (5), the optimal contract $C^{\dagger}(y,z)$ in inducing the high level action $a_H$ is characterized by

$$w'\left(U\left(C^{\dagger}(y,z)\right)\right) = \left\{\lambda + \mu\left[1 - \frac{f(y,z;a_L)}{f(y,z;a_H)}\frac{g(a_L)}{g(a_H)}\right]\right\} g(a_H) \qquad (15)$$

Economic sufficiency of aggregation is achieved by the aggregate $T(y,z)$ (7) which is a statistically sufficient aggregate of the basic measures $y$ and $z$.

Proposition 2 shows that in inducing the high level action $a_H$ from the agent, the basic measures $y$ and $z$ can be aggregated into a one-dimensional measure $T(y,z)$ (7) with no loss of information on the agent's unobservable action. The sufficient aggregate $T(y,z)$ (7) of the basic measures $y$ and $z$ achieves both economic sufficiency and statistical sufficiency of aggregation. By aggregating the basic measures $y$ and $z$ into the one-dimensional sufficient aggregate $T(y,z)$ (7), the principal is not economically harmed in inducing the optimal action level from the agent and also the principal does not lose any information on the agent's unobservable action.

Proposition 2 implies that although our analysis is restricted to two action levels and lacks technical tractability, we obtain a result virtually equivalent to satisfying Amershi et al.'s (1990) "all $a$ or no $a$" condition. Although Amershi et al.'s (1990) "all $a$ or no $a$" condition (1) is not defined due to the restriction of the agent's action to two levels, Proposition 2 shows that the economically sufficient aggregate in the optimal contract also attains statistical sufficiency such that all sufficient aggregates are statistically sufficient.

# 5. Conclusion

Aggregation of performance measures is widely practiced and has important implications for efficiently evaluating and effectively motivating employees. In practice, effort is often measured in two levels due to the simplicity. When employees' actions are restricted to two levels, our analysis shows that the basic measures can be aggregated into a one-dimensional sufficient aggregate without losing any information from the basic measures. The format of the one-dimensional sufficient aggregate is determined only by the statistical properties of the basic measures and the one-dimensional sufficient aggregate can be uniformly used regardless of organizational particulars.

By using the one-dimensional sufficient aggregate, complexity of performance evaluation systems can be reduced and efficiency of motivating employees through performance evaluation can be substantially improved.

# Appendix: Proofs

## *Proof of Proposition 1*

In the joint normal density function $f(y,z;a_i)$ (5), $\Psi(a_i)$ is expanded as the sum of $A_1$, $A_2$, and $A_3$, for both action levels $a_i$, $i=H,L$

$$\Psi(a_i) = A_1 + A_2 + A_3, \tag{16}$$

$$A_1 = \frac{y^2}{\sigma_\varepsilon^2} + \frac{z^2}{\sigma_\delta^2} - 2\frac{\rho_{\varepsilon\delta}\sigma_\varepsilon\sigma_\delta}{\sigma_\varepsilon{}^2\sigma_\delta{}^2} yz,$$

$$A_2 = a_i^2\left[\frac{m^2}{\sigma_\varepsilon^2} + \frac{k^2}{\sigma_\delta^2} - 2\frac{\rho_{\varepsilon\delta}\sigma_\varepsilon\sigma_\delta}{\sigma_\varepsilon{}^2\sigma_\delta{}^2} mk\right],$$

$$A_3 = -2a_i\left[\left\{\frac{m - \left(\frac{\rho_{\varepsilon\delta}\sigma_\varepsilon\sigma_\delta}{\sigma_\delta{}^2}\right)k}{\sigma_\varepsilon^2}\right\}y + \left\{\frac{k - \left(\frac{\rho_{\varepsilon\delta}\sigma_\varepsilon\sigma_\delta}{\sigma_\varepsilon{}^2}\right)m}{\sigma_\delta^2}\right\}z\right]$$

First, $A_1$ includes the basic measures $y$ and $z$ without the agent's action $a_i$. $\exp[A_1]$ belongs to $p(y,z)$ in the factorization criterion (6). Second, $A_2$ does not include the basic measures $y$ and $z$. $\exp[A_2]$ is not relevant to the factorization criterion (6). Third, $A_3$ includes the basic measures $y$ and $z$ together with the agent's action $a_i$. Defining an aggregate $T(y,z)$ of the basic measures $y$ and $z$ as in (7), $\exp[A_3]$ is equivalent to $q(T(y,z);a_i)$ in the factorization criterion (6). Note that $A_3$ depends on the basic measures $y$ and $z$ only through the aggregate $T(y,z)$ (7).

## *Proof of Proposition 2*

In solving the optimization problem (14), the first order Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial C(y,z)} \le 0,\ C(y,z) \ge 0,\ C(y,z)\frac{\partial L}{\partial C(y,z)} = 0 \tag{17}$$

$$\frac{\partial L}{\partial \lambda} \ge 0, \lambda \ge 0,\ \lambda\frac{\partial L}{\partial \lambda} = 0 \tag{18}$$

$$\frac{\partial L}{\partial \mu} \ge 0,\ \mu \ge 0,\ \mu\frac{\partial L}{\partial \mu} = 0 \tag{19}$$

The optimal compensation in equilibrium is obviously positive( $C(y,z)>0$ ). From the complementary slackness condition $C(y,z)\frac{\partial L}{\partial C(y,z)}=0$ in (17), $\frac{\partial L}{\partial C(y,z)}=0$ is required.

$$\begin{aligned}\frac{\partial L}{\partial C(y,z)} &= -\iint w'\left(U\left(C(y,z)\right)\right)U'\left(C(y,z)\right)f\left(y,z;a_H\right)\text{ dydz} \\ &+\lambda \iint U'\left(C(y,z)\right)f\left(y,z;a_H\right)\text{ dydz g}\left(a_H\right) \\ &+\mu \begin{bmatrix}\iint U'\left(C(y,z)\right)f\left(y,z;a_H\right)\text{ dydz g}\left(a_H\right) \\ -\iint U'\left(C(y,z)\right)f\left(y,z;a_L\right)\text{ dydz g}\left(a_L\right)\end{bmatrix} \\ &= 0\end{aligned} \tag{20}$$

After dividing both sides of (20) with $U'\left(C(y,z)\right)f\left(y,z;a_H\right)$ and taking out the integrals, we obtain the equation (15) that characterizes the optimal contract $C^{\dagger}(y,z)$.

As the participation constraint (10) binds in equilibrium,

$$\frac{\partial L}{\partial \lambda}=\iint U\left(C(y,z)\right)f\left(y,z;a_H\right)\text{ dydz g}\left(a_H\right)-v\left(a_H\right)-C_0=0 \tag{21}$$

From the complementary slackness condition $\lambda\frac{\partial L}{\partial \lambda}=0$ in (18), the Lagrange multiplier $\lambda$ for the participation constraint (10) is positive( $\lambda>0$ ).

The Lagrange multiplier $\mu$ for the incentive compatibility constraint (11) is positive in equilibrium( $\mu>0$ ). Otherwise, suppose the Lagrange multiplier $\mu$ is zero( $\mu=0$ ). Then the optimal contract $C^{\dagger}(y,z)$ is characterized by $w'\left(U\left(C^{\dagger}(y,z)\right)\right)=\lambda\text{ g}\left(a_H\right)$ such that the low level action $a_L$ as opposed to the high level action $a_H$ is induced by the fixed amount of compensation. From the complementary slackness condition $\mu\frac{\partial L}{\partial \mu}=0$ in (19), it follows that $\frac{\partial L}{\partial \mu}=0$ and the incentive compatibility constraint (11) binds in equilibrium.

The term $\frac{f(y,z;a_L)}{f(y,z;a_H)}$ in (15) summarizes the information from the basic measures $y$ and $z$ in the optimal contract $C^{\dagger}(y,z)$. By explicitly examining the term $\frac{f(y,z;a_L)}{f(y,z;a_H)}$ in (15), we can observe the way the basic measures $y$ and $z$ are aggregated in the optimal contract $C^{\dagger}(y,z)$. From the joint normal density function (5),

$$\frac{f(y,z;a_L)}{f(y,z;a_H)}=\exp\left[-\frac{1}{2\left(1-\rho_{\varepsilon\delta}^2\right)}\left\{\Psi\left(a_L\right)-\Psi\left(a_H\right)\right\}\right] \tag{22}$$

$$= \exp\left[-\frac{1}{2\left(1-\rho_{\varepsilon\delta}^2\right)}\left\{\begin{array}{l}-2\left(a_L - a_H\right)\, T\left(y,z\right) \\ +\left(a_L^2 - a_H^2\right)\left[\frac{m^2}{\sigma_\varepsilon^2} + \frac{k^2}{\sigma_\delta^2} - 2\frac{\rho_{\varepsilon\delta}\sigma_\varepsilon\sigma_\delta}{\sigma_\varepsilon{}^2\sigma_\delta{}^2} mk\right]\end{array}\right\}\right]$$

where $T\left(y,z\right)$ is as in (7). In (22), we observe that the basic measures $y$ and $z$ are aggregated into the statistically sufficient aggregate $T\left(y,z\right)$ (7) in the optimal contract $C^\dagger\left(y,z\right)$.

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