Economics Bulletin

Volume 41, Issue 3

Welfare-reducing discriminatory output subsidies with mixed ownership and R&D

Jiaqi Chen
Chonnam National University

Sang-Ho Lee
Chonnam National University

Timur K. Muminov
Management Development Institute of Singapore in Tashkent

Abstract

This study examines the welfare-reducing effect of time-consistent discriminatory output subsidies under duopolistic R&D competition with a semi-public firm. Contrary to the committed case with mixed ownership, we find that the private firm invests more in R&D than the semi-public firm to induce the government to grant higher subsidy, which results in higher output production than the semi-public firm. We also show that optimal discriminatory subsidy rates are higher (lower) than uniform subsidy rates for a sufficiently high (low) degree of privatization, which could decrease (increase) social welfare.

Correspondence: Sang-Ho Lee (sangho@jnu.ac.kr). This study was financially supported by the 2020 program of Chonnam National University, the Republic of Korea. (CNU2019-3894) We thank an editor and anonymous referees for their helpful and constructive comments. All remaining errors are our responsibility.
Citation: Jiaqi Chen and Sang-Ho Lee and Timur K. Muminov, (2021) "Welfare-reducing discriminatory output subsidies with mixed ownership and R&D", *Economics Bulletin*, Vol. 41 No. 3 pp. 1592-1602
Contact: Jiaqi Chen - chenjiaqi@naver.com, Sang-Ho Lee - sangho@jnu.ac.kr, Timur K. Muminov - smtk0171@gmail.com.
Submitted: January 18, 2021. **Published:** September 17, 2021.

1. Introduction

Due to globalisation and technological improvement among innovative firms, governments have not only reformed mixed ownership, but significantly enacted various policies to encourage R&D activities. While many industries are composed of mixed ownership in which (partially privatized) semi-public firms have substantial influence in a broad range of sectors with large portions of shares,[1] many governments in both developed and developing countries offer substantial subsidies to firms for improving financial resource accessibility and resource reallocation.[2] However, it is not clear how the government should grant economic aid to different types of firms with heterogenous objectives and cost structures in the industries.

Recent studies have examined innovation activities and R&D policies in a mixed oligopoly where a public firm competes with private firms. For example, Gil-Molto et al. (2011, 2020) and Lee and Muminov (2020) considered cost-reducing innovation and investigated the R&D subsidies and privatization policies. Further, Kasevayuth and Zikos (2013), Lee et al. (2017), Haruna and Goel (2017), and Atallah (2019) compared the relative superiority between R&D subsidies and output subsidies. They found that R&D subsidies are more socially beneficial when R&D spillovers are high, whereas output subsidies are superior when R&D spillovers are low.[3]

While existing literature has only examined a uniform output subsidy to the firms in a mixed oligopoly, Hamada (2016, 2017) and Li et al. (2019) identified firm heterogeneity and examined discriminatory output subsidies without considering R&D activities. They demonstrated that if the government can credibly commit to a discriminatory subsidy rate from the introduction stage of an output subsidy policy, it poses significant implications for supporting superior welfare properties associated with a committed subsidy policy. Therefore, the discriminatory output subsidy always yields the highest welfare compared to the uniform output subsidy, irrespective of the degree of privatization. However, these works did not address an increasing topic in a mixed oligopoly about the timing of privatization and related subsidy policies.[4]

In this study, we consider a time-consistent output subsidy framework in which the government can observe the R&D investments of the firms with mixed ownership, which yields heterogenous cost structure, and set the discriminatory output subsidy rate opportunistically. We highlight the interaction between the output subsidy policy and ex-ante innovation

---

[1] For example, the Chinese government has steadily advanced mixed-ownership reform of state-owned enterprises, which resulted in the number of mixed-owned firms under government control totalling approximately 30% in 2018, while intensively competing with privately-owned firms in certain sectors such as transportation, energy, finance, education, and healthcare. See the 2019 China Statistical Yearbook published by National Bureau of Statistics of China (http://www.stats.gov.cn/english/Statisticaldata/AnnualData/) and reports from State-Owned Assets Supervision and Administration Commission of the State Council of China (http://en.sasac.gov.cn/).

[2] In OECD countries, governments finance between 8–10% of R&D spending by firms (García-Quevedo, 2004). Studwell (2013) shows that subsidies, along with other policies, played an important role in the economic development of Asian countries such as Japan, South Korea, and China.

[3] In practical policies, while governments decide to subsidise R&D directly, there is some debate on the effectiveness of R&D subsidies (Kauko, 1996; Rebolledo and Sandonis, 2012; Lee and Muminov, 2021). For example, governments might subsidise output rather than R&D either when output enhancement is politically more popular among consumers or when the administration of R&D subsidies is relatively more complicated compared to output subsidies.

[4] Recent research have examined the time-consistent policy in a mixed oligopoly where the government is not able to commit to privatization policy or subsidy/tax (Xu et al., 2017; Leal et al., 2018; Garcia et al., 2018; Chen et al., 2019) For some discussions on the practical evidences of contract and commitment in mixed oligopolies, see also Lee et al. (2018), Chen et al. (2020) and Ino and Miyaoka (2020).

activities, and show that strategic incentive for innovation among firms do not necessarily result in welfare improvement. We find contrary results to the committed case where the government can credibly commit to the subsidy rate. We show that the private firm invests more in R&D than the semi-public firm to induce the government to grant a higher subsidy, resulting in the private firm producing a higher output, regardless of the degree of privatization. It happens because the private firm intentionally and strategically invests aggressively in R&D to earn a higher subsidy, which increases the firm's output and profit. Since R&D investments are strategic substitutes irrespective of the degree of privatization, when the private firm increases R&D and output, the semi-public firm reduces R&D and output. Subsequently, a discriminatory output subsidy redistributes output from the public firm with high marginal costs to the private firm with low marginal costs. Hence, the government grants a lower output subsidy to the public firm than the private firm, while the difference decreases as the degree of privatization increases. Finally, we compare with the time-consistent uniform output subsidy and show that optimal discriminatory subsidy rates are higher (lower) than those of uniform subsidy rates for a sufficiently high (low) degree of privatization, which results in a decrease (increase) of social welfare. This finding is contrast to the committed case in Li et al. (2019) where the discriminatory subsidy always yields higher welfare than the uniform subsidy irrespective of the degree of privatization.[5]

2. The Model

We consider a duopoly market which produces homogeneous goods. The inverse demand function is $P(Q) = a - Q$, where P is market price, $Q(= q_0 + q_1)$ is market output, and q_i is the output of firm $i = 0,1$, respectively. Then, consumer surplus is $\mathrm{CS} = Q^2/2$.

Following Lee et al. (2017), we consider a quadratic cost function of output production where marginal costs increase with respect to output:[6] $C(q_i, x_i) = (c - x_i)q_i + q_i^2$, where c is the initial cost level with $a > c > 0$ and x_i denotes the amount of R&D investment required for firm i to reduce the cost level. We also assume that the firm has to spend x_i^2 to implement cost-reducing R&D, x_i, which exhibits decreasing returns to scale. Subsequently, the R&D cost function is given as $\Gamma(x_i) = x_i^2$ where $i = 0,1$.

We consider that each firm receives an output subsidy, $s_i q_i$, where s_i denotes the per-unit (discriminatory) subsidy to output quantity, q_i. Then, the profit function of the firm and social welfare, defined as the sum of consumer surplus, industry profits, and net subsidy, are as follows:[7]

$$\pi_i = (a - q_0 - q_1)q_i - (c - x_i)q_i - q_i^2 - x_i^2 + s_i\, q_i, \tag{1}$$

$$W = CS + \pi_0 + \pi_1 - s_0\, q_0 - s_1 q_1. \tag{2}$$

---

[5] In Appendix B, we examine the committed case where the government can commit a discriminatory (or uniform) output subsidy before firms choose R&D investments, and show that the committed discriminatory output subsidy policy always yields the highest welfare for society than the uniform subsidy. It confirms the result in Li et al. (2019) in which commitment can eliminate the strategic incentive of the firms to invest more in R&D after observing the output subsidy policy. This result comes from the revelation principle where the government can achieve at least the same welfare level with the uniform subsidy under the discriminatory subsidy because it can choose the same subsidy rate from the first stage if the discriminatory subsidy worsens the welfare.

[6] It is necessary in the analysis of mixed markets for ruling out the uninteresting case of a public monopoly.

[7] We assume that subsidies are financed by taxpayers in the form of lump-sum payments, to avoid influencing welfare directly and to be cancelled out when aggregated.

We assume that firm 1 is a private firm that maximizes profit while firm 0 is a (partially privatized) public firm. That is, we allow the government to sell shares of firm 0 to private investors who are seeking profit maximization. As a result, firm 0 is under mixed ownership of government control and jointly owned by the government and private investors. Following Matsumura (1998), let $\theta \in [0,1)$ refer to the shares of firm 0 held by private investors, while firm 0 maximizes the convex combination of profit and welfare:

$$V = (1-\theta)W + \theta\pi_0. \tag{3}$$

The timing of the game is as follows. In the first stage, both firms decide to invest in R&D simultaneously. In the second stage, the government sets discriminatory output subsidies, s_i, $i = 0,1$, after both firms' R&D decisions are realized. In the last stage, firms compete in quantities in a Cournot fashion. We solve the subgame perfect Nash equilibrium by backward induction.

3. The Analysis

In the third stage of output choice, the first-order conditions provide the following equilibrium output:

$$q_0(x_0,x_1,s_0,s_1) = \frac{3(a-c)+4x_0-x_1+4\theta s_0-s_1}{11+4\theta},$$
$$q_1(x_0,x_1,s_0,s_1) = \frac{(a-c)(2+\theta)-x_0+(3+\theta)(x_1+s_1)-\theta s_0}{11+4\theta},$$
$$Q(x_0,x_1,s_0,s_1) = \frac{(a-c)(5+\theta)+3x_0+3\theta s_0+(2+\theta)(x_1+s_1)}{11+4\theta}. \tag{4}$$

This shows that an increase in R&D increases the output of the firms, but the increasing rate of a public firm is higher than a private firm, i.e., $\frac{\partial q_0(x_0,x_1,s_0,s_1)}{\partial x_0} > \frac{\partial q_1(x_0,x_1,s_0,s_1)}{\partial x_1} > 0$. However, the firm's R&D decreases the output of the rival firm, i.e., $\frac{\partial q_1(x_0,x_1,s_0,s_1)}{\partial x_0} = \frac{\partial q_0(x_0,x_1,s_0,s_1)}{\partial x_1} < 0$. In sum, an R&D investment will increase total industry output. It also shows that the output subsidy increases the output of the firm, but the increasing rate of the private firm is higher than that of the public firm, i.e., $\frac{\partial q_1(x_0,x_1,s_0,s_1)}{\partial s_1} > \frac{\partial q_0(x_0,x_1,s_0,s_1)}{\partial s_0} > 0$. However, the output subsidy of a firm decreases the output of the rival firm, i.e., $\frac{\partial q_0(x_0,x_1,s_0,s_1)}{\partial s_1} \leq \frac{\partial q_1(x_0,x_1,s_0,s_1)}{\partial s_0} < 0$, where the equality holds only if $\theta = 1$. Note that the output subsidy of the public firm has no effect on the output of either of the firms if the public firm is fully nationalised, that is, $\theta = 0$. Therefore, the output subsidy is insignificant with regards to welfare in the presence of a nationalised public firm.

In the second stage, the government determines the welfare-maximizing output subsidy rate, by considering the firm's R&D investment. The first order condition yields the following optimal output subsidies:[8]

[8] Note that given R&D investments, the optimal output subsidy is independent of the degree of privatization. However, once the R&D stage is introduced in the below, the degree of privatization affects the equilibrium R&D investments and thus output subsidy depends on the degree of privatization. See Lee et al. (2017) and Lee and Tomaru (2017).

$$s_0^D(x_0,x_1)=\frac{2(a-c)+3x_0-x_1}{8} \qquad \text{and} \qquad s_1^D(x_0,x_1)=\frac{2(a-c)-x_0+3x_1}{8}. \qquad (5)$$

where the superscript *D* denotes the equilibrium under the discriminatory output subsidy. This shows that the output subsidy increases in the firm's R&D, i.e., $\frac{\partial s_i^D(x_0,x_1)}{\partial x_i}>0$, while it decreases in the rival firm's R&D, i.e., $\frac{\partial s_i^D(x_0,x_1)}{\partial x_j}<0$.

In the first stage of R&D investment, the first-order conditions present the following reaction functions:

$$x_0(x_1)=\frac{(2+\theta)(2(a-c)-x_1)}{26-3\theta} \qquad \text{and} \qquad x_1(x_0)=\frac{3(2(a-c)-x_0)}{23}. \qquad (6)$$

Then, R&D decisions are strategic substitutes where the slope of the reaction function of each firm is negative, but that of the public firm is higher than the private firm, i.e., $\frac{\partial x_1^D(x_0)}{\partial x_0}<\frac{\partial x_0^D(x_1)}{\partial x_1}<0$.

Solving the reaction function yields the following equilibrium R&D investments:

$$x_0^D=\frac{5(a-c)(2+\theta)}{74-9\theta} \qquad \text{and} \qquad x_1^D=\frac{3(a-c)(6-\theta)}{74-9\theta}. \qquad (7)$$

Lemma 1. $x_0^D<x_1^D$ for $\theta\in[0,1)$.

The private firm aggressively invests in R&D under the discriminatory output subsidy. The public (private) firm's R&D increases (decreases) as the degree of privatization increases, i.e., $\frac{\partial x_0^D}{\partial\theta}>0>\frac{\partial x_1^D}{\partial\theta}$.

By substituting (7) with (5), we obtain the following optimal output subsidy rates:

$$s_0^D=\frac{20(a-c)}{74-9\theta} \qquad \text{and} \qquad s_1^D=\frac{4(a-c)(6-\theta)}{74-9\theta}. \qquad (8)$$

Proposition 1. $s_0^D<s_1^D$ for $\theta\in[0,1)$.

The government grants a higher subsidy to the private firm than the public firm under the discriminatory output subsidy policy. Since R&D investments are strategic substitutes irrespective of the degree of privatization, when the private firm increases R&D and output, the semi-public firm reduces R&D and output. Subsequently, the discriminatory output subsidy redistributes output from the public firm with higher marginal costs to the private firm with lower marginal costs. Hence, the government grants a lower output subsidy to the public firm than the private firm, while the difference decreases as the degree of privatization increases. Note that output subsidy rates are always positive, but that to the public (private) firm increases according to the degree of privatization, i.e., $\frac{\partial s_0^D}{\partial\theta}>0>\frac{\partial s_1^D}{\partial\theta}$. Thus, a higher degree of privatization can encourage the semi-public firm to invest more in R&D and output production. The equilibrium outputs are as follows:

$$q_0^D = \frac{20(a-c)}{74-9\theta} \qquad \text{and} \qquad q_1^D = \frac{4(a-c)(6-\theta)}{74-9\theta} \tag{9}$$

Lemma 2. $q_0^D < q_1^D$ for $\theta \in [0,1)$.

The private firm aggressively produce under the discriminatory output subsidy.[9] The public (private) firm's output is positive and increases (decreases) in the degree of privatization, that is, $\frac{\partial q_0^D}{\partial \theta} > 0 > \frac{\partial q_1^D}{\partial \theta}$.

Finally, we obtain the resulting profit of the private firm and social welfare as follows:

$$\pi_1^D = \frac{23(a-c)^2(6-\theta)^2}{(74-9\theta)^2} \qquad \text{and} \qquad W^D = \frac{10(a-c)^2(152-\theta(36+\theta))}{(74-9\theta)^2} \tag{10}$$

The profit of the private firm decreases in the degree of privatization, that is, $\frac{\partial \pi_1^D}{\partial \theta} < 0$. This is because the (partially privatized) rival firm invests more in R&D and produces more output as the degree of privatization increases. If the degree of privatization is sufficiently high, the profit incentive of the semi-public firm is stronger, which encourages investment by the semi-public firm. Then, the distribution of production costs across the firms is more symmetric and efficient while the firms produce a lower industry output. Hence, although a sufficiently high degree of privatization can remove cost inefficiency, underproduction has distorting effects on welfare. Contrarily, if the degree of privatization is sufficiently low, a high level of asymmetry exists between the firms, which results in inefficient distribution of production costs across the firms, while the semi-public firm can produce more output to increase total industry output. Consequently, although a sufficiently low degree of privatization can increase cost inefficiency, there is a welfare distortion of underproduction distortion. Therefore, social welfare decreases (increases) when the degree of privatization is high (low), that is, $\frac{\partial W^D}{\partial \theta} \gtreqless 0$ if $\theta \lesseqgtr 0.153$. It implies that the optimal degree of privatization is $\theta^D = 0.153$.

Proposition 2. Mixed ownership with partial privatization policy is optimal under the discriminatory output subsidy policy.

4. Discussion and Comparisons with Non-discriminatory Uniform Output Subsidy

We now compare with uniform output subsidy of which equilibrium outcomes are provided in Appendix A.

Proposition 3. $s_0^D \gtreqless s_q^N$ if $\theta \gtreqless 0.699$ and $s_1^D \gtreqless s_q^N$ if $\theta \gtreqless 0.162$.

The government is more flexible in granting output subsidies under discriminatory output subsidies, but that the rate depends on the degree of privatization. This is because the firm's strategic profit incentive for making R&D decisions in a time-consistent framework in which the output subsidy rate is affected by the cost asymmetry between the firms, significantly depends on the degree of privatization. In particular, the discriminatory output subsidy rate is

[9] Haruna and Goel (2017) and Lee and Muminov (2021) considered R&D spillovers and showed that the public firm produces less (more) output than the private firm with higher (lower) R&D spillovers.

always higher (lower) than the uniform output subsidy for a sufficiently higher (lower) degree of privatization. When the degree of privatization is high (low) where the semi-public firm is dominantly owned by private investors (government), the government can set a lower (higher) uniform subsidy rate. As a result, welfare could possibly be reduced by aiding a less efficient public firm under the discriminatory output subsidy. However, when the degree of privatization is intermediate, $s_0^D < s_q^N < s_1^D$ if $0.162 < \theta < 0.699$. For instance, if the semi-public firm is almost equally owned by the government and private investors, the government will set an intermediate subsidy rate under the uniform output subsidy.

Proposition 4. $W^D \gtreqless W^N$ if $\theta \lesseqgtr 0.033$.

In a time-consistent regulatory framework, when the government determines the output subsidy policy between discriminatory and uniform subsidy rates after observing the firms' R&D decisions, it crucially depends on mixed ownership. In most cases, the government prefers the non-discriminatory output subsidy to the discriminatory output subsidy to reduce strategic welfare-distorting R&D decisions of the firms that initiate profit. However, the opposite is true for a sufficiently lower degree of privatization in which the welfare loss from strategic R&D can be minimized. Furthermore, if the government chooses the optimal degree of privatization, the welfare effect of the output subsidy presents that $W^D(\theta = 0.153) < W^N(\theta = 0.933)$. Therefore, the discriminatory output subsidy could further distort welfare under the optimal privatization policy.

Economic explanations of the findings are as follows: the optimal subsidy rates for both firms are higher (lower) than those of the uniform subsidy rate for a high (low) degree of privatization, which results in a decrease (increase) of social welfare. On the one hand, if the degree of privatization is high, social concern of the semi-public firm is weak with a much stronger profit incentive. Subsequently, once the firms choose R&D investments, the distribution of production costs across the firms is more symmetric and efficient. Thus, the government can provide higher output subsidies for both firms to increase total industry output. That is, for a high degree of privatization, underproduction distortion from the private firms becomes more serious and is thus remedied by the higher uniform output subsidy.

On the other hand, if the degree of privatization is sufficiently low, social concern of the semi-public firm is much stronger, which discourages overinvestment between the firms, compared to the higher degree of privatization. Due to the larger asymmetry between the firms, the distribution of production costs across the firms is inefficient, however, the semi-public firm can produce more output to increase total industry output. That is, although the (efficient) private firm receives an output subsidy for a low degree of privatization, so does the (inefficient) public firm under the uniform output subsidy. Therefore, welfare could decrease when less efficient firms are aided. Further, these uniform subsidies distort competition further according to the fact that firm's R&D choices occur before they are set. Thus, the firms' strategic behaviour that influences subsidies, further reduces the welfare level compared to the discriminatory subsidy.

References

Atallah, G. (2019) "Subsidizing Innovation and Production" *Economía* **42**, 9-35.

Chen, J. Lee, S.H. and Muminov, T. (2019) "Time-Inconsistent Output Subsidy/Tax Policies in Free-Entry Mixed Markets" *Hitotsubashi Journal of Economics* **60**, 61-77.

Chen, J. Lee, S.H. and Muminov, T. (2020) "R&D spillovers, output subsidies and privatization in a mixed duopoly: Flexible versus irreversible R&D investments" *Working paper* MPRA_paper_91452.

Garcia, A. Leal, M. and Lee, S.H. (2018) "Time-inconsistent environmental policies with a consumer-friendly firm: Tradable permits versus emission tax" *International Review of Economics & Finance* **58**, 523-537.

Garcia-Quevedo, J. (2004) "Do public subsidies complement business R&D? A meta-analysis of the econometric evidence" *Kyklos* **57**, 87-102.

Gil-Molto, M. Poyago-Theotoky, J. and Zikos, V. (2011) "R&D subsidies, spillovers and privatization in mixed markets" *Southern Economic Journal* **78**, 233-255.

Gil-Molto, M. Poyago-Theotoky, J. Neto J. and Zikos, V. (2020) "Mixed oligopoly, cost-reducing research and development, and privatization" *European Journal of Operational Research* **283**, 1094-1106.

Hamada, K. (2016) "Privatization Neutrality Theorem and Discriminatory Subsidy Policy" *Springer Series in Game Theory*, *Equilibrium Theory for Cournot Oligopolies and Related Games,* 133-153.

Hamada, K. (2017) "Privatization Neutrality Theorem: When a Public Firm Pursues General Objectives" *Japanese Economic Review* **69**, 59-68.

Haruna S. and Goel, R.K. (2017) "Output subsidies in mixed oligopoly with research spillovers" *Journal of Economics and Finance* **41**, 235-256.

Ino, H. and Miyaoka, A. (2020) "Government-induced production commitment vs. Import-tariff under endogenous entry of foreign firms" *Journal of International Trade and Economic Development* **29**, 797-820.

Kauko K. (1996) "The ineffectiveness of R&D subsidies – an oligopoly theoretic insight" *Review of Industrial Organization* **11**, 401-412.

Kesavayuth, D. and Zikos, V. (2013) "R&D versus output subsidies in mixed markets" *Economics Letters* **118**, 293–296.

Leal, M. Garcia, A. and Lee, S.H. (2018) "The timing of environmental tax policy with a consumer-friendly firm" *Hitotsubashi Journal of Economics* **59**, 25-43.

Lee S.H. Matsumura T. and Sato S. (2018) "An analysis of entry-then-privatization model: welfare and policy implications" *Journal of Economics* **123**, 71-88.

Lee, S.H. and Muminov, T. (2020) "R&D information sharing in a mixed duopoly and incentive subsidy for RJV competition" *Bulletin of Economic Research* **72**, 154-170.

Lee, S.H. and Muminov, T. (2021) "Endogenous timing of R&D decisions and privatization policy with research spillovers" *Journal of Industry, Competition and Trade* DOI:10.1007/s10842-021-00365-5

Lee, S.H. Muminov, T. and Tomaru, Y. (2017) "Partial privatization and subsidization in a mixed duopoly: R&D versus output subsidies" *Hitotsubashi Journal of Economics* **58**, 163-177.

Lee, S.H. and Tomaru, Y. (2017) "Output and R&D subsidies in mixed oligopoly" *Operations Research Letters* **45**, 238-241.

Li, M. Lien, J.W. and Zheng, J. (2019) "Optimal subsidies in the comparison between private and state-owned enterprises" *International Review of Economics and Finance* DOI:10.1016/j.iref.2019.11.011.

Matsumura, T. (1998) "Partial privatization in mixed duopoly" *Journal of Public Economics* **70**, 473-483.

Rebolledo, M. and J. Sandonis (2012) "The Effectiveness of R&D Subsidies, Review of Industrial Organization" *Economics of Innovation and New Technology* **21**, 815-825.

Studwell, J. (2013) "How Asia Works: Success and Failure in the World's Most Dynamic Region" *London: Profile Books*.

Xu, L. Lee, S.H. and Matsumura, T. (2017) "Ex-ante versus Ex-post Privatization Policies with Foreign Penetration in Free-Entry Mixed Markets" *International Review of Economics and Finance* **50**, 1-7.

Appendix A.: Non-discriminatory uniform output subsidy

Under uniform output subsidy, the equilibrium outcomes in the last stage are similar but $s_0 = s_1 = s_q$. In the second stage of the welfare-maximizing output subsidy, the first order condition yields:

$$s_q^N(x_0, x_1) = \frac{6(a-c)(1+2\theta^2) - (3+4\theta(1-4\theta))x_0 + (9+4(1-\theta)\theta)x_1}{24+48\theta^2}. \qquad (A1)$$

where the superscript *"N"* denotes the non-discriminatory output subsidy.
In the first stage of R&D, the first-order condition yields the equilibrium R&D investments:

$$x_0^N = \frac{(a-c)(360+81\theta+23430\theta^2+516\theta^3+50760\theta^4+1376\theta^5+35840\theta^6+1280\theta^7)}{2664+351\theta+175290\theta^2+1052\theta^3+382520\theta^4-224\theta^5+277760\theta^6-1792\theta^7},$$

$$x_1^N = \frac{(a-c)(3+4\theta^2)(216+27\theta+9090\theta^2+8\theta^3+9920\theta^4-64\theta^5)}{2664+351\theta+175290\theta^2+1052\theta^3+382520\theta^4-224\theta^5+277760\theta^6-1792\theta^7}. \qquad (A2)$$

Lemma A1. $x_0^N < x_1^N$ for $\theta \in [0,1)$.

Similar to the discriminatory output subsidy, the private firm invests more aggressively in R&D under the output subsidy policy. Then, we obtain the optimal production subsidy level:

$$s_q^N = \frac{12(a-c)(1+2\theta^2)(72+13\theta+2870\theta^2+4\theta^3+3200\theta^4)}{2664+351\theta+175290\theta^2+1052\theta^3+382520\theta^4-224\theta^5+277760\theta^6-1792\theta^7}. \qquad (A3)$$

The equilibrium output of both firms are obtained:

$$q_0^N = \frac{12(a-c)(1+2\theta^2)(60+13\theta+283\theta^2+20\theta^3+320\theta^4)}{2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7},$$

$$q_1^N = \frac{4(a-c)(1+2\theta^2)(216+27\theta+909\theta^2+8\theta^3+992\theta^4-64\theta^5)}{2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7}. \quad (A4)$$

Lemma A2. $q_0^N < q_1^N$ for $\theta \in [0,1)$.

Similar to the discriminatory output subsidy, the private firm more aggressively produces output under the output subsidy policy. We can compare discriminatory and uniform output subsidies:[10]

Lemma A3. (i) $x_0^D > x_0^N$ for any $\theta \in [0,1)$; $x_1^D \gtrless x_1^N$ if $\theta \gtrless 0.037$ and $X^D > X^N$ for any $\theta \in [0.1)$; (ii) $q_0^D > q_0^N$ for any $\theta \in [0,1)$; $q_1^D \gtrless q_1^N$ if $\theta \gtrless 0.108$ and $Q^D \gtrless Q^N$ if $\theta \gtrless 0.028$; (iii) $\pi_1^D < \pi_1^N$ for any $\theta \in [0,1)$.

Finally, the resulting social welfare is as follows:

$$W^N = \frac{\left\{\begin{matrix} 2(a-c)^2(984960 + 2604960\theta + 131223510\theta^2 + 25375860\theta^3 + \\ 723676230\theta^4 + 89851200\theta^5 + 2117114480\theta^6 + 1214300080\theta^7 + \\ 3468122272\theta^8 - 23395840\theta^9 + 3016573440\theta^{10} - 204963840\theta^{11} + \\ 1084866560\theta^{12} - 137297920\theta^{13} - 5242880\theta^{14}) \end{matrix}\right\}}{\left\{\begin{matrix} 2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + \\ 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7 \end{matrix}\right\}^2}. \quad (A5)$$

Appendix B.: Committed output subsidy

Under the committed case in which the government credibly commits to discriminatory output subsidies before the firms choose R&D decisions, as like in Li et al. (2019). The timing of this game between the R&D stage and subsidy stage is reversed: the government chooses output subsidy in the first stage whereas both firms choose R&D investment in the second stage and output in the third stage.

In the third stage, output choice is similar to the non-committed output subsidy in (4). In the second stage, the first-order conditions provide the following equilibrium R&D investments and outputs:

$$x_0^{CD}(s_0, s_1) = \frac{(a-c)\big(275+\theta(248+\theta(65+4\theta))\big)+\theta(429+\theta(337+66\theta))s_0-(3+\theta)(17+\theta(33+10\theta))s_1}{1837+\theta(2159+2\theta(425+56\theta))},$$

$$x_1^{CD}(s_0, s_1) = \frac{2(3+\theta)\big((a-c)(33+\theta(33+8\theta))-2\theta(11+4\theta)s_0+2(3+\theta)(9+4\theta)s_1\big)}{1837+\theta\big(2159+2\theta(425+56\theta)\big)}$$

---

[10] (i) $x_0^D - x_0^N = \frac{4(a-c)\theta(3519+1098\theta+20267\theta^2+4200\theta^3+33220\theta^4+5952\theta^5+14624\theta^6+640\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} > 0$ for any $\theta \in [0.1)$,
$x_1^D - x_1^N = \frac{4(a-c)\theta(-459+\theta(12366+\theta(-2775+4\theta(12330+\theta(-2965+16\theta(803-2\theta(119-6\theta)))))))}{(-74+9\theta)(-2664+\theta(-351+\theta(-17529+4\theta(-263+\theta(-9563+56\theta(1-4\theta(31-2\theta)))))))} \gtrless 0$ if $\theta \gtrless 0.037$ and
$X^D > X^N = \frac{16(a-c)\theta(765+3366\theta+4373\theta^2+13380\theta^3+5340\theta^4+14336\theta^5-152\theta^6+352\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} > 0$ for any $\theta \in [0.1)$.
(ii) $q_0^D - q_0^N = \frac{4(a-c)\theta(489-1470\theta+5929\theta^2-4190\theta^3+13922\theta^4-2120\theta^5+8320\theta^6)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} > 0$ for any $\theta \in [0.1)$,
$q_1^D - q_1^N = \frac{8\theta(-306+2916\theta-1868\theta^2+10539\theta^3-5377\theta^4+9816\theta^5-5600\theta^6+320\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \gtrless 0$ if $\theta \gtrless 0.108$ and
$Q^D - Q^N = \frac{4\theta(-123+4362\theta+2193\theta^2+16888\theta^3+3168\theta^4+17512\theta^5-2880\theta^6+640\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \gtrless 0$ if $\theta \gtrless 0.028$.

$$q_0^{CD}(s_0, s_1) = \frac{(11+4\theta)\big((a-c)(53+21\theta)+4\theta(19+7\theta)s_0\big)-\big(215+\theta(193+42\theta)\big)s_1}{1837+\theta\big(2159+2\theta(425+56\theta)\big)},$$
$$q_1^{CD}(s_0, s_1) = \frac{(11+4\theta)\Big((a-c)\big(33+\theta(33+8\theta)\big)-2\theta(11+4\theta)s_0+2(3+\theta)(9+4\theta)s_1\Big)}{1837+\theta\big(2159+2\theta(425+56\theta)\big)}.$$

where the superscript *"CD"* denotes the equilibrium under the committed discriminatory output subsidy.

In the first stage, the government chooses the optimal discriminatory output subsidy:

$$s_0^{CD} = \frac{4(a-c)(54 + 211310\theta + 30529\theta^2 + 16941\theta^3 + 4277\theta^4 + 414\theta^5)}{\theta(311129 + 443822\theta + 243553\theta^2 + 60996\theta^3 + 5876\theta^4)},$$
$$s_1^{CD} = \frac{(a-c)(84381 + 120800\theta + 68317\theta^2 + 18030\theta^3 + 1856\theta^4)}{311129 + 443822\theta + 243553\theta^2 + 60996\theta^3 + 5876\theta^4}.$$

It shows that $s_0^{CD} > s_1^{CD} > 0$ for $\theta \in [0,1)$ while $\frac{\partial s_1^{CD}}{\partial \theta} > 0$ and $\frac{\partial s_0^{CD}}{\partial \theta} \gtrless 0$ if $\theta \gtrless 0.408$. Then, we obtain the R&D investments and outputs:

$$x_0^{CD} = \frac{2(a-c)(22142 + \theta(33747 + \theta(19592 + \theta(5129 + 510\theta))))}{311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta)))},$$
$$x_1^{CD} = \frac{2(a-c)(3+\theta)(8067 + \theta(8548 + \theta(3231 + 434\theta)))}{311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta)))},$$
$$q_0^{CD} = \frac{4(a-c)(22241 + \theta(31715 + \theta(17355 + \theta(4325 + 414\theta))))}{311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta)))},$$
$$q_1^{CD} = \frac{(a-c)(11+4\theta)(8067 + \theta(8548 + \theta(3231 + 434\theta)))}{311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta)))}.$$

It shows that $x_0^{CD} < x_1^{CD}$ for $\theta \in [0,1)$; $q_0^{CD} > q_1^{CD}$ for $\theta \in [0,1)$. Finally, we obtain the welfare under the committed discriminatory output subsidy.

$$W^{CD} = \frac{(a-c)^2(177683 + 253550\theta + 139047\theta^2 + 34776\theta^3 + 3344\theta^4)}{2(311129 + 443822\theta + 243553\theta^2 + 60996\theta^3 + 5876\theta^4)}.$$

Under the committed uniform output subsidy, $s_q = s_0 = s_1$ in the second and third stages. In the first stage, the government chooses the optimal uniform output subsidy:

$$s_q^{CN} = \frac{(a-c)(203247+3217690\theta+638527\theta^2+723819\theta^3+384974\theta^4+95288\theta^5+8960\theta^6)}{750075+1179258\theta+2335027\theta^2+2619900\theta^3+1377436\theta^4+337120\theta^5+31360\theta^6}.$$

where the superscript *"CN"* to denote the committed non-discriminatory output subsidy. It shows that $s_q^{CN} > 0$, $s_0^{CD} > s_q^{CN}$, and $\frac{\partial s_q^{CN}}{\partial \theta} > 0$ for any $\theta \in [0,1)$ while $s_1^{CD} \gtrless s_q^{CN}$ if $\theta \lessgtr 0.175$. The equilibrium R&D investments and outputs are obtained:

$$x_0^{CN} = \frac{2(a-c)(53322+\theta(89079+\theta(171990+\theta(200237+4\theta(27760+\theta(7163+700\theta))))))}{750075+1179258\theta+2335027\theta^2+2619900\theta^3+1377436\theta^4+337120\theta^5+31360\theta^6},$$
$$x_1^{CN} = \frac{2(a-c)(3+\theta)(19449+\theta(23472+\theta(52085+2\theta(24557+1120\theta(8+\theta)))))}{750075+1179258\theta+2335027\theta^2+2619900\theta^3+1377436\theta^4+337120\theta^5+31360\theta^6},$$
$$q_0^{CN} = \frac{(a-c)(214260+\theta(336805+\theta(665716+\theta(748197+393854\theta+96408\theta^2+8960\theta^3))))}{750075+1179258\theta+2335027\theta^2+2619900\theta^3+1377436\theta^4+337120\theta^5+31360\theta^6},$$
$$q_1^{CN} = \frac{(a-c)(11+4\theta)(19449+\theta(23472+\theta(52085+2\theta(24557+1120\theta(8+\theta)))))}{750075+1179258\theta+2335027\theta^2+2619900\theta^3+1377436\theta^4+337120\theta^5+31360\theta^6}.$$

It shows that $x_0^{CN} < x_1^{CN}$ and $q_0^{CN} > q_1^{CN}$ for $\theta \in [0,1)$. Finally, we obtain the welfare under the committed uniform output subsidy.

$$W^{CN} = \frac{(a-c)^2(428361+673674\theta+1333436\theta^2+1496494\theta^3+786395\theta^4+192200\theta^5+17840\theta^6)}{2(750075+1179258\theta+2335027\theta^2+2619900\theta^3+1377436\theta^4+337120\theta^5+31360\theta^6)}.$$

Proposition B1. $W^{CD} > W^{CN}$ for any $\theta \in [0,1]$.

Appendix C. Proofs.

Proof of Proposition 3:

$$s_0^D - s_q^N = \frac{4(a-c)(-2664+813\theta-7686\theta^2+10237\theta^3-6398\theta^4+21242\theta^5-2984\theta^6+8320\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \gtreqless 0 \text{ if } \theta \gtreqless 0.699$$

and $$s_1^D - s_q^N = \frac{8(a-c)\theta(-750+47467\theta-3120\theta^2+15401\theta^3-8617\theta^4+12508\theta^5-10624\theta^6+896\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \gtreqless 0 \text{ if } \theta \gtreqless 0.162.$$

Proof of Proposition 4:

$$W^D - W^N = \frac{\left\{\begin{array}{c} 8(a-c)^2\theta(7336656-2223602016\theta+828565520\theta^2-21707629938\theta^3+5752051200\theta^4 \\ -86645216290\theta^5+22572094720\theta^6-179092433360\theta^7+46668561600\theta^8-198081664160\theta^9 \\ +47168759040\theta^{10}-106357025920\theta^{11}+18558673920\theta^{12}-19266580480\theta^{13} \\ +833699840\theta^{14}+66027520\theta^{15}) \end{array}\right\}}{(74-9\theta)^2(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)^2} \gtreqless 0 \text{ if } \theta \lesseqgtr 0.033.$$