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A comment on "salaries or piece rates: on the endogenous matching of harvest workers and crops"

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Abstract

In Kandilov and Vukina (2016), the authors conclude that -when agents differ in their ability and principals in the riskiness of their projects- negative assortative matching (NAM) always ensues in equilibrium: good-type (high-ability) agents always match with bad-type (high-risk) principals and vice-versa (p. 78 and 82). We prove that this conclusion is incorrect. We revisit their model and show that positive assortative matching (PAM) always holds in equilibrium by applying standard literature results.

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1. Introduction

A firm's incentive scheme impacts both its hiring policy and the productivity of the agents hired. Kandilov and Vukina (2016) modify the canonical linear-Constant Absolute Risk Aversion (CARA) model (Holmstrom and Milgrom (1987)) by allowing agents to differ in their abilities. Their main objective is to explore the relationship between agents' abilities and the power of incentives. In their model, the production function is multiplicatively separable in ability and effort. The authors claim that the equilibrium matching on ability is always NAM, meaning that good-type (high-ability) agents will always match with bad-type (high-risk task) principals and bad-type (low-ability) agents will always match with good-type (low-risk task) principals. In this note, we prove that this prediction is incorrect. In fact, in their model, efficiency dictates PAM: in equilibrium, good-type (high-ability) agents always match with good-type (low-risk) principals and vice-versa. The discrepancy in the findings relies on Kandilov and Vukina (2016)'s misinterpretation of the submodularity of the expected joint surplus function. Furthermore, we show that PAM results in high-ability agents sorting themselves into high-powered contracts in equilibrium, a result that Kandilov and Vukina (2016) also (but serendipitously) obtain.

2. The Model

The matching on ability model by Kandilov and Vukina (2016) (p. 81-82) is a modified version of the canonical linear-CARA agent-principal model. There is a continuum of risk-neutral principals, characterized by the risk (variance) of their projects (σ_p^2), who are uniformly distributed on the interval $[\sigma_L^2, \sigma_H^2]$. Each principal wants to hire a risk-averse agent, characterized by his inherent ability level ($\theta_a > 0$), to perform a task in exchange for a compensation (w). There exists a continuum of agents who are uniformly distributed on the interval $[\theta_L, \theta_H]$. Each agent's ability is assumed public information to abstract from hidden information issues, that is, there is no adverse selection. All agents are homogeneous with respect to their degree of risk aversion, having the same Arrow-Pratt measure of absolute risk aversion (λ).

Each principal has to be matched with exactly one agent in order to produce output. The output of any given match ($y_{p,a}$) is informative about the hidden effort exerted by the agent (e_a). Specifically, $y_{p,a} = \theta_a e_a + \varepsilon_p$ where $\varepsilon_p \sim N(0, \sigma_p^2)$ is the i.i.d. productivity shock. The principal's expected profit function is given by $\pi_{p,a} = y_{p,a} - w_{p,a}$. The agent's cost of effort is a quadratic function given by $C_a = (c/2)e_a^2$. He has CARA preferences with his utility function given by $V_{p,a} = 1 - \exp[-\lambda(w_{p,a} - (c/2)e_a^2)]$. All players have zero reservation utilities.

The optimal compensation scheme in the above set-up is linear in output (refer to Holmstrom and Milgrom (1987)) : $w_{p,a} = \alpha_{p,a} + \beta_{p,a} y_{p,a}$ where $\alpha_{p,a}$ is the fixed salary and $\beta_{p,a}$ is the power of the incentive scheme. In this case, it is convenient to solve the program by using the agent's certainty income given by $\alpha_{p,a} + \beta_{p,a}\theta_a e_a - (c/2)e_a^2 - (1/2)\lambda\beta_{p,a}^2\sigma_p^2$. Given any match (p, a), the agent's incentive compatibility constraint determines his optimal level of effort which is directly proportional to the power of the incentive scheme and to his ability:

$$e_a^* = \frac{\theta_a}{c}\beta_{p,a} \tag{1}$$

and, together with the agent's individual rationality constraint, it implies the following optimal power of incentives:

$$\beta^*_{p,a} = \frac{\theta_a^2}{\theta_a^2 + c\lambda\sigma_p^2} \tag{2}$$

and expected total surplus:

$$\Pi^*_{p,a} = \frac{1}{2c}\theta_a^2\beta^*_{p,a} = \frac{1}{2c}\left(\frac{\theta_a^4}{\theta_a^2 + c\lambda\sigma_p^2}\right) \tag{3}$$

Thus, the expected joint surplus is increasing in the agent's ability level ($\partial\Pi^*_{p,a}/\partial\theta_a > 0$) and it is decreasing in the principal's project risk ($\partial\Pi^*_{p,a}/\partial\sigma_p^2 < 0$). The cross partial derivative of the expected joint surplus is unambiguously negative for all parameter values:

$$\frac{\partial^2\Pi^*_{p,a}}{\partial\theta_a\partial\sigma_p^2} = (-2)c\lambda^2\sigma_p^2\left(\frac{\theta_a}{\theta_a^2 + c\lambda\sigma_p^2}\right)^3 < 0 \tag{4}$$

Since utility is transferable, the equilibrium match will maximize the expected joint surplus. On page 78, Kandilov and Vukina (2016) define positive assortative matching (PAM) as a situation in which: (i) high-ability agents are efficiently matched with principals who own low-risk projects; and (ii) low-ability agents are efficiently matched with high-risk tasks principals. Negative assortative matching (NAM) is characterized by exactly the opposite situation. Based on the work of Legros and Newman (2002), Kandilov and Vukina (2016) mistakenly concluded that the submodularity of the surplus function is a sufficient condition for negative assortative matching (NAM). This conclusion would be correct if the surplus function were increasing in both arguments (as in Legros and Newman (2002)) or decreasing in both arguments. However, Kandilov and Vukina (2016)'s expected total surplus function is crucially increasing in one argument and decreasing in the other argument, which implies that NAM is unstable. Therefore, Kandilov and Vukina (2016)'s claim is incorrect. In fact, we show that positive assortative matching (PAM) ensues in equilibrium for all parameter values, meaning that high-ability agents are efficiently matched with low-risk tasks principals and vice-versa.

Proposition 1. *In Kandilov and Vukina (2016)'s heterogenous-ability model: (i) NAM is unstable; and (ii) PAM ensues in equilibrium.*

Proof. In equilibrium, all matches must be stable. This means that no pair (p, a) made of a principal and an agent who are currently not matched with each other, can increase their payoffs by leaving their current partners and signing a contract, with a power of incentives characterized by condition (2), with each other. Consider two agents, agent 1 and agent 2, and two principals, principal 1 and principal 2. Assume that agent 2 is more talented than agent 1 ($\theta_2 > \theta_1$) and that principal 2 owns a lower risk project than principal 1 ($\sigma_2^2 < \sigma_1^2$). Agent 2 is more desirable than agent 1 for both principals since the expected joint surplus of any match is increasing in the agent's ability level. Suppose that as established by NAM, principal 2 is matched with agent 1 and principal 1 is matched with agent 2. Then, by condition (3),

$$\Pi^*_{2,2} - \Pi^*_{2,1} = \frac{1}{2c}\left(\frac{\theta_2^4}{\theta_2^2 + c\lambda\sigma_2^2}\right) - \frac{1}{2c}\left(\frac{\theta_1^4}{\theta_1^2 + c\lambda\sigma_2^2}\right) = \frac{1}{2c}(\theta_2^2 - \theta_1^2)\left(\frac{\theta_1^2\theta_2^2 + c\lambda\sigma_2^2(\theta_1^2 + \theta_2^2)}{(\theta_2^2 + c\lambda\sigma_2^2)(\theta_1^2 + c\lambda\sigma_2^2)}\right)$$

$$\Pi^*_{1,2} - \Pi^*_{1,1} = \frac{1}{2c}\left(\frac{\theta_2^4}{\theta_2^2 + c\lambda\sigma_1^2}\right) - \frac{1}{2c}\left(\frac{\theta_1^4}{\theta_1^2 + c\lambda\sigma_1^2}\right) = \frac{1}{2c}(\theta_2^2 - \theta_1^2)\left(\frac{\theta_1^2\theta_2^2 + c\lambda\sigma_1^2(\theta_1^2 + \theta_2^2)}{(\theta_2^2 + c\lambda\sigma_1^2)(\theta_1^2 + c\lambda\sigma_1^2)}\right)$$

Since $\partial^2\Pi^*_{p,a}/\partial\theta_a\partial\sigma^2_p < 0$, a safer principal is more harmed by being matched with a less able agent than a riskier one and thus, principal 2's willingness to pay for agent 2 rather than agent 1 ($\Pi^*_{2,2} - \Pi^*_{2,1}$) is higher than principal 1's willingness to pay for agent 2 rather than agent 1 ($\Pi^*_{1,2} - \Pi^*_{1,1}$), that is, $\Pi^*_{2,2} - \Pi^*_{2,1} > \Pi^*_{1,2} - \Pi^*_{1,1}$, implying $\Pi^*_{2,2} + \Pi^*_{1,1} > \Pi^*_{1,2} + \Pi^*_{2,1}$. In other words, the expected total surplus generated by the negatively assorted matches ($\Pi^*_{1,2} + \Pi^*_{2,1}$) is lower than the expected total surplus that would generated if the principals swapped agents ($\Pi^*_{2,2} + \Pi^*_{1,1}$). Therefore, NAM cannot be stable: efficiency dictates PAM in equilibrium. □

The relationship between the agents' abilities and the power of incentives turns out to be positive for all parameter values in equilibrium, as Kandilov and Vukina (2016) claim, so that high ability agents sort themselves into high-powered contracts for all parameter values.

Proposition 2. *In Kandilov and Vukina (2016)'s heterogenous-ability model, the equilibrium relationship between ability and incentives is positive for all parameter values: high-ability agents sort themselves into high powered incentives in equilibrium.*

Proof. A pure matching is a function $\mu : [\theta_L, \theta_H] \to [\sigma^2_L, \sigma^2_H]$ that is measure preserving. Substituting the matching function into condition (2), we have that:

$$\beta^*_a = \frac{\theta^2_a}{\theta^2_a + c\lambda\mu(\theta_a)}$$

As a result,

$$\frac{\partial\beta^*_a}{\partial\theta_a} = \frac{c\lambda\theta_a\mu(\theta_a)}{\left(\theta^2_a + c\lambda\mu(\theta_a)\right)^2}(2 - \epsilon_{\mu,\theta_a})$$

where ϵ_{μ,θ_a} denotes the elasticity of the matching function with respect to the agent's ability θ_a. By Proposition 1, in equilibrium, high-ability agents are matched with low-risk principals and low-ability agents are matched with high-risk principals, implying $\mu'(\theta_a) < 0$, where $\mu'(\theta_a)$ is the derivative of the matching function.[1] In turn, $\mu'(\theta_a) < 0$ implies $\epsilon_{\mu,\theta_a} < 0$ and hence, $\partial\beta^*_a/\partial\theta_a > 0$. □

An immediate implication of Proposition 2 is that high-ability agents work harder (exert higher effort levels) than low-ability agents. Therefore, the total effect of ability on productivity is positive due to the reinforcement of the direct and indirect effects of ability on productivity via the market assignment.

The results provided in Proposition 1 and Proposition 2 are robust to any specification of the production function $y_{p,a} = f(\theta_a)e_a + \varepsilon_p$ such that $f(\cdot)$ is increasing. Martinez-Gorricho and Sanchez Villalba (2021) set up a model in which ability does not enter into the production function but it parametrizes the agent's cost of effort. In their more general set-up, the authors find that NAM can indeed ensue in equilibrium under certain conditions. In addition, their model allows for scenarios in which high-ability agents accept low-powered contracts in equilibrium.

[1]Notice that when matching on agents' risk aversion, NAM implies $\mu'(\lambda) < 0$ since the surplus function is decreasing in both arguments and hence, low-risk averse agents and low-risk principals are considered "high-types". However, when matching on agents' ability, PAM implies $\mu'(\theta) < 0$ since the surplus function is no longer decreasing in both arguments and hence, high-ability agents and low-risk principals are considered "high-types". Instead, NAM would have implied $\mu'(\theta) > 0$ in this model.

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