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Price discrimination with network effects: different welfare results from identical demand functions

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Abstract

We show that the welfare effects of third-degree price discrimination depend on what kind of network effects are present—between-markets or within-market. Different combinations of parameters that determine the strength of network effects between-markets and within-market induce the same demand functions; however, measured consumer surplus and social welfare based on the demand functions vary across these parameters. This result indicates that welfare analysis of markets with network effects must be based on consumer utility functions that parameterize the network effects, and not on demand functions that, although sufficient to describe monopoly price-setting, mask the impact of network effects on consumer welfare.

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1. Introduction

In the digital era, with the development of information technologies, network effects appear in various ways such as computer software and apps for communication. We focus on the relationships among demand functions, consumer surplus, and social welfare in the presence of network effects and third-degree price discrimination.

In the literature on third-degree price discrimination, the welfare effects of price discrimination have long been studied starting with Pigou (1920). There is a well-known result that a necessary condition for price discrimination to improve social welfare is that it increases total output.[1] Against this result, Adachi (2002) demonstrates that, when markets are interdependent and demand functions exhibit cross-price symmetry, price discrimination can improve social welfare even if total output remains the same. What should be noted is that Adachi's argument starts with demand functions, and it is misleading because social welfare is calculated without sufficient explanation. In fact, in response to Adachi (2002), Bertoletti (2004) shows that when consumer surplus and social welfare are measured based on ordinal microeconomic foundations, price discrimination decreases social welfare if total output remains the same. In terms of these conflicting results, Adachi (2004) gives a *rational expectations* rationale for the micro-foundation difference between them. More precisely, the utility function and consumer surplus in the model with network effects are different from those in the model without network effects, although the same demand functions are derived under both models.[2] Consequently, Adachi succeeds in justifying his result in the presence of network effects. On the other hand, he merely shows that his result can be justified when the only network effects are *between* markets, and does not mention that there are other network structures that could support his same demand functions.

In this paper, we show that different combinations of between-markets and within-market network effects that support the same demand function give rise to different welfare effects of third-degree price discrimination. Even though the different network effects induce the same demand functions, measured consumer surplus and social welfare based on the demand functions vary across the parameters that define the two types of network effect. In addition, we show that price discrimination does not improve social welfare if the strength of network effects within-market is large enough. This result indicates that, when considering legal restrictions on price discrimination, government regulators should pay attention to the network structure in which consumers benefit and not only the demand functions in markets.

The remainder is organized as follows: Section 2 sets out our heterogeneous consumer model. We present our main results in Section 3 and conclude in Section 4. In the Appendix, we give a representative consumer model to which similar argument can be applied.

2. The model

We consider a monopolist selling a network good in two markets i $(i = 1, 2)$ without cost. In each market i, there is a continuum of mass M consumers indexed by θ_i and uniformly distributed over the interval $[0, M]$, where positive real number M is large enough to exclude a boundary solution. Consumers form expectations of the future size of each market x_i, x_j. Assuming that each consumer purchases at most one unit of the good, and the consumer of type θ_i gains utility $u(\theta_i; p_i, x_i, x_j) = a_i - b\theta_i + \zeta x_i + \eta x_j - p_i$, if he/she purchases the good and equal to zero otherwise, where $b > 0$,

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[1] See Schmalensee (1981), Varian (1985), Schwartz (1990) and Bertoletti (2004).

[2] See Katz and Shapiro (1985) for the definition of consumer surplus in the presence of network effects.

and ζ represents the strength of symmetric network effects within-market, η the strength of symmetric network effects between markets, and p_i is the price of the good *i*. Only consumers whose type belongs to $[0, q_i]$ buy the good in market *i*, where $a - bq_i + \zeta x_i + \eta x_j = p_i$. Based on the consideration of Katz and Shapiro (1985, Appendix) followed by Adachi (2004), we assume that the monopoly firm pre-commits to its price, and consumers then form *self-fulfilling* expectations. In this case, for given market prices, consumers correctly anticipate the actual network size and form rational expectations under which total demands match expectations; $q_i = x_i, q_j = x_j$. Then, by letting $b = 1 + \zeta$ and using this rational expectations condition, we can obtain the same (inverse) demand functions used by Adachi (2002, 2004):

$$p_i = a_i - q_i + \eta q_j; \qquad i, j = 1, 2, i \neq j, \tag{1}$$

Note that ζ must be larger than –1 to satisfy $b > 0$. We assume that $-1 < \eta < 1$, and no arbitrage occurs between consumers. Without loss of generality, let $a_1 > a_2 > 0$, and define $\alpha \equiv a_2/a_1 < 1$. Here, in our model with network effects, we must pay attention to how to measure consumer surplus. For realized output (q_1^*, q_2^*), the price of market *i* is $p_i^* = a_i - q_i^* + \eta q_j^*$ from (1), and then consumer surplus in market *i* is given by

$$CS_i = \int_0^{q_i^*} [a_i - (1 + \zeta)\theta_i + \zeta q_i^* + \eta q_j^* - p_i^*] d\theta_i = (1 + \zeta)(q_i^*)^2/2. \tag{2}$$

Note that the strength of symmetric network effects within-market, ζ, does not influence the form of demand functions, but does influence the sizes of consumer surplus. Alternatively, consumer surplus differs across combinations of parameters that induce the same demand functions.

So far, we have considered a heterogeneous consumer model to derive demand functions and consumer surplus given by (1) and (2) respectively, but we can duplicate them using a representative consumer model based on Hoernig (2012) (see the Appendix)[3].

3. The analysis

In what follows, we consider the welfare effect of a regime change from uniform pricing to discriminatory pricing while retaining the shapes of the demand functions, by letting $b = 1 + \zeta$. Under uniform pricing, the monopoly firm maximizes its profit subject to the constraint, $p_i = p_j$. On the other hand, the monopolist earns unconstrained maximum profit under discriminatory pricing. Because demand functions facing the monopolist coincide with Adachi's (2002, 2004) ones and cost functions are also the same, the behavior of the monopolist is no different from under his model and summarized as follows.

Results 1: (Adachi, 2002).
(i) *Both markets are open under either price regime if and only if* $\eta(\alpha) < \eta < 1$ *where* $\eta(\alpha) = -\alpha + (1 - \alpha^2)/2$.
(ii) *If* $\eta(\alpha) < \eta < 1$, *each equilibrium outcome is as follows.*[4]
Uniform pricing:

[3] In the literature on network effects, Hoernig's (2012) representative consumer approach is followed by some research (Chirco and Scrimitore, 2013; Lee and Choi, 2018; Lee et al., 2018; Hashizume and Nariu, 2020; Naskar and Pal, 2020).
[4] Superscript *u* and *d* denote uniform and discriminatory pricing, respectively.

$$p^u = (a_1 + a_2)/4, \quad (3)$$
$$q_i^u = [(3-\eta)a_i + (3\eta - 1)a_j]/4(1-\eta^2). \quad (4)$$

Discriminatory pricing:

$$p_i^d = a_i/2, \quad (5)$$
$$q_i^d = (a_i + \eta a_j)/[2(1-\eta^2)]. \quad (6)$$

Then, total output does not change: $q_1^u + q_2^u = q_1^d + q_2^d$.

Henceforth, we focus on the situation where both markets are open under uniform pricing; i.e., $\eta(\alpha) < \eta < 1$.

Based on this result, we will analyze the welfare effect of price discrimination. First, consider the change of consumer surplus in market *i*, which is given by

$$\Delta CS_i = CS_i^d - CS_i^u = (1+\zeta)[(q_i^d)^2 - (q_i^u)^2]/2. \quad (7)$$

From $\zeta > -1$ and $q_i^d + q_i^u > 0$, the sign of ΔCS_i equals that of $q_i^d - q_i^u = (a_j - a_i)/[4(1+\eta)]$ by (4) and (6). Considering that $a_1 > a_2$, price discrimination decreases consumer surplus in market 1 and increases consumer surplus in market 2.

Next, we consider aggregate consumer surplus. From (4) and (6) - (7), it is given by

$$\Delta CS = \Delta CS_1 + \Delta CS_2 = -3(1+\zeta)(a_1 - a_2)^2/[16(1+\eta)]^2. \quad (8)$$

Then, $a_1 > a_2$ implies $\Delta CS < 0$, and price discrimination decreases aggregate consumer surplus.

Lemma 1: *Given α, assume that η(α) < η < 1 and both markets are open.*
For any $\zeta > -1$, price discrimination decreases consumer surplus in market 1, *increases consumer surplus in market* 2, *and decreases aggregate consumer surplus.*

Even if ζ changes and the network structure changes, the signs of ΔCS_i and ΔCS do not change, and they are the same as in the Adachi (2002) case (ζ = 0). This is explained as follows. Changing ζ changes CS_i but does not change the equilibrium prices and outputs because the monopolist's demand functions remain as in (1). Consumers' gains from network effects increase by $\zeta q_i^2/2$, which is the triangle generated by the changes in the vertical axis intercept and slope of the demand function in market *i* (see Figure 1). However, this change only strengthens the increase or decrease of ΔCS_i and does not affect the sign. In addition, the sign of ΔCS is unchanged because consumer surplus in markets 1 and 2 each changes by the multiplicative factor $(1+\zeta)$. That is, $|\Delta CS|$ becomes larger as ζ increases.

Now, let us consider the effect on social welfare, which is the sum of the monopolist's profit and aggregate consumer surplus. For any network structures that duplicate (1), the change in the monopolist's profit by price discrimination is

$$\Delta\pi = \pi^d - \pi^u = (a_1 - a_2)^2/[8(1+\eta)]. \quad (9)$$

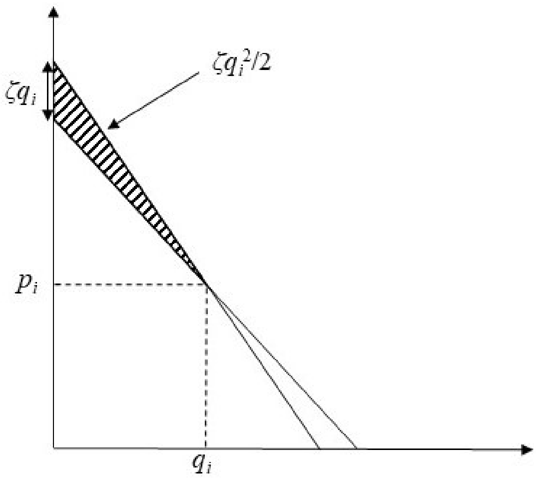

Figure 1: The change of consumer surplus in market i due to the change in ζ.

Thus, from (8) - (9), the difference of social welfare is

$$\Delta SW = \Delta\pi + \Delta CS = (2\eta - 1 - 3\zeta)(a_1 - a_2)^2/[16(1+\eta)]^2. \tag{10}$$

Based on $\eta < 1$ and $\eta(\alpha) = -\alpha + (1-\alpha^2)/2 > -1$ for any $\alpha < 1$, we obtain the following results.

Proposition 1: *Given α, assume that $\eta(\alpha) < \eta < 1$ and both markets are open.*
(i) *If the degree of network effects within-market is sufficiently large, $\zeta > 1/3$, price discrimination worsens social welfare.*
(ii) *When the degree of network effects within-market is not so large, and $-1 < \zeta < 1/3$, price discrimination improves social welfare if and only if $(1+3\zeta)/2 < \eta < 1$.*

We can explain these results as follows: First, let us introduce Adachi's (2002) explanation of the effect of the change in η. Comparing the denominators (8) and (9), an increase in η has a first-power effect on $\Delta\pi$, and a second-power effect on ΔCS. This is because prices are not influenced by η from (3) and (5), but outputs are influenced by η from (4) and (6), and because the monopolist's profit depends on both price and output, and aggregate consumer surplus given by (7) only depends on the output. As a result, if η is large, $|\Delta CS|$ becomes smaller than $|\Delta\pi|$, so social welfare will improve. Combining Adachi's (2002) explanation and our above explanation about the change in ζ, we reach Proposition 1-(i). If ζ is sufficiently large and $\zeta > 1/3$, then $|\Delta CS|$ is sufficiently large, then $|\Delta\pi|$ does not dominate $|\Delta CS|$ for any $\eta < 1$. Moreover, if ζ is sufficiently small and $\zeta < 1/3$, then $|\Delta CS|$ is sufficiently small, then social welfare can be improved by price discrimination even if η is negative.

To gain a more intuitive understanding, we first explain why uniform pricing does not improve social welfare compared to discriminatory pricing, although total output remains the same. Pigou (1920), Robinson (1933), and Schwartz (1990) demonstrate that, in the absence of network effects, for a given level of total output, social welfare is maximized under uniform pricing because the price reflects the marginal utility of the consumer. However, in the presence of network effects, equalization of marginal utility is not the requirement for social welfare to be maximized. The efficient distribution of (q_1, q_2) is achieved when the sums of the marginal utility, $a_i - q_i + \eta q_j (= p_i)$, and the marginal network effects, $\zeta q_i + \eta q_j$, are equal between markets. Therefore, considering that the marginal utility is the same between markets under uniform pricing, the output transfer from market 1 to market 2 improves social welfare if the marginal network effect in market 2 is larger than that in market 1, which holds when η is sufficiently larger than ζ, as described in Proposition 1. This welfare improving direction of output transfer is identical to the direction of output transferred by price discrimination. Alternatively, if ζ is sufficiently large, output transferred by price discrimination never improves social welfare.

4. Conclusion

We show that different network structures could induce the same demand functions and support the same equilibrium prices and outputs under either uniform pricing or discriminatory pricing. However, measured consumer surplus and social welfare differ as network structure changes. Consequently, price discrimination has different effects on social welfare depending on the relative strengths of between-markets and within-market network effects.

This result indicates that for welfare analysis we should model the consumers' utility setting, which clarifies how network effects work, and avoid beginning with demand setting. In addition, in empirical studies, it is not enough to estimate demand functions; it is also necessary to estimate the network structure in which consumers benefit.

Appendix: an alternative model

Here, we present a representative consumer model with network effects, to which a similar argument to ours can be applied. Assume that a representative consumer's utility is given by $U = a_1q_1 + a_2q_2 - (1+\zeta)[q_1^2 + q_2^2]/2 + \zeta(q_1x_1 + q_2x_2) + \eta(q_1x_2 + q_2x_1)$. The consumer maximizes his/her utility for given prices and expectations, and thus the inverse demand functions under the expectations (x_i, x_j) are given by $p_i(x_i, x_j) = a_i - (1+\zeta)q_i + \zeta x_i + \eta x_j$. Imposing the rational expectations condition, we obtain (1). In addition, for realized output (q_1^*, q_2^*), the price of market *i* is $p_i^* = a_i - q_i^* + \eta q_j^*$ from (1), and then the aggregate consumer surplus is given by $CS = CS(q_1^*, q_2^*) = U - p_1^*q_1^* + p_2^*q_2^* = (1+\zeta)\sum_i (q_i^*)^2/2$. Letting $q_j^* = 0$, we have consumer surplus in market *i* given by (2). That is, the following equation holds: $CS = CS(q_1^*, 0) + CS(0, q_2^*) = CS_1 + CS_2$.

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