Economics Bulletin

Volume 41, Issue 3

A Possible Perverse Effect of Favoritism in Asymmetric Competitions

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Abstract

In this paper, we show that in an incomplete-information all pay auction, favoring the weaker player can perversely reduce his/her winning chance, compared to the case of standard all pay auction without favoritism. Thus, favoritism towards the weaker may not increase the winner diversity.

The author is grateful to Professor Sergio Parreiras for his guidance. All errors are my own.
Citation: David Lu, (2021) "A Possible Perverse Effect of Favoritism in Asymmetric Competitions", *Economics Bulletin* Vol. 41 No. 3 pp. 2089-2098
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Submitted: July 10, 2021. **Published:** September 18, 2021.

1 Introduction

In many competitions such as R&D races, government procurements, job promotions, political campaigns, school admissions and athletic events, the players are often asymmetric in their technological capacities and expertise. Heterogeneity in contestants' competitiveness can generate an undesirable discouragement effect on total effort supply, as confirmed by the "superstar effect" discovered empirically by Brown (2011).

Leveling the battle field by favoring the weaker player can mitigate the discouragement effect by encouraging the weaker and disciplining the stronger, as revealed by many studies, including McAfee and McMillan (1989), Che and Gale (2003), Nti (2004), Sahuguet (2006), Fu (2006), Tsoulouhas et al. (2007), Frank (2012), Epstein et al. (2011), Li and Yu (2012), Kirkegaard (2012), Frank et al. (2013), Lee (2013), Seel (2014), Seel and Wasser (2014), Segev and Sela (2014), Siegel (2014), Frank et al. (2018), and Zhu (2021) among many others.[1]

In this paper, we study a two-player all-pay auction with incomplete information and investigate whether favoring the weaker player always enhances his/her winning chance.[2] Note that if we focus on linear favoritism rules such as head start, handicap or a combination of the two instruments, we can answer this question relying on the analysis of Kirkegaard (2012), whose result reveals that the favored weaker player would win with a higher chance compared to the case without favoritism.[3] A natural question thus arises. Does this intuitively expected finding necessarily extend to nonlinear favoritism rules? In this paper, we provide an answer by constructing an example, which shows that if we allow nonlinear favoritism rules, favoring the weaker player can perversely reduce his/her winning chance.

2 Model setup

There are two players $i = 1, 2$ in the competition. Player i's value of winning the competition is v_i, which is private information of player i. Player i's value distribution is $G_i(\cdot)$ with continuous density $g_i(\cdot) > 0$ on $[0, \bar{v}]$. The players are asymmetric in terms of their value distributions.

Assumption 1. *$G_1(\cdot) \leq G_2(\cdot)$, i.e., player 1 is stronger than player 2 in terms of first order stochastic dominance (FOSD).*

Assumption 1 holds under the following hazard rate dominance condition $\frac{g_2(v)}{1-G_2(v)} \geq$

[1] Please refer to Chowdhury et al. (2020) for a comprehensive review.

[2] Parreiras and Rubinchik (2010) and Minchuk (2014) studied weak contestants' behavior in all pay auctions with incomplete information.

[3] According to eqn. (2) in Kirkegaard (2012), a linear favoritism towards the weaker player entails a higher winning value-threshold $\tilde{k}(\cdot)$ for the stronger player.

$\frac{g_1(v)}{1-G_1(v)}$. For example, consider $G_1(v_1) = (v_1)^4$ on $[0, 1]$ and $G_2(v_2) = (v_2)^2$ on $[0, 1]$. One can verify that $\frac{g_2(v)}{1-G_2(v)} \geq \frac{g_1(v)}{1-G_1(v)}$.

Moreover, we assume the following assumptions.

Assumption 2. *(i) Virtual value functions* $J_i(v_i) = v_i - \frac{1-G_i(v_i)}{g_i(v_i)}$ *are increasing in* v_i *on* $[0, \bar{v}]$. *(ii)* $J_1(v) \leq J_2(v)$, $\forall v \in [0, \bar{v}]$.

Assumption 2(i) is a standard regularity condition on the virtual value functions (Myerson, 1981). Assumption 2(ii) requires that the weak player has a higher virtual value function, which holds under the hazard rate dominance condition $\frac{g_2(v)}{1-G_2(v)} \geq \frac{g_1(v)}{1-G_1(v)}$. Clearly, we always have $J_1(\bar{v}) = J_2(\bar{v}) = \bar{v}$.

Everyone is risk neutral. The players' effort costs are sunk regardless of the outcome of the competition. A player's expected payoff is his/her value multiplied by his/her winning probability minus his/her effort cost. We normalize the marginal effort cost as 1 for both players. The contest organizer's expected payoff is the expected total effort supply of the players.

In a standard all-pay auction, the two players simultaneously exert their effort, the one exerting a higher effort wins the competition. In an all-pay auction with a favoritism towards the weaker player, the winner is determined by the two players' effort profile based on the favoritism rule. A favoritism rule is specified by player 2's winning threshold $B(e_1) \in [0, \bar{v}]$ in terms of his/her effort, which is an increasing function of player 1's effort $e_1 \in [0, \bar{v}]$. This rule means that player 1 exerting effort e_1 wins if and only if player 2's effort e_2 is less than $B(e_1)$.[4] If $B(e_1) < e_1, \forall e_1 \in (0, \bar{v})$, then the rule $B(\cdot)$ favors player 2; if $B(e_1) = e_1, \forall e_1 \in [0, \bar{v}]$, then the rule is neutral; if $B(e_1) > e_1, \forall e_1 \in [0, \bar{v})$, then the rule favors player 1.[5]

3 The analysis

For any (v_1, v_2), define

$$p_1^*(v_1, v_2) = \begin{cases} 1, & \text{if } J_1(v_1) - J_2(v_2) > 0, \\ 0, & \text{if } J_1(v_1) - J_2(v_2) \leq 0, \end{cases} \text{ and } p_2^*(v_1, v_2) = 1 - p_1^*(v_1, v_2). \tag{1}$$

Let $P_i^*(t) = \int_0^{\bar{v}} p_i^*(t, v_j) g_j(v_j) dv_j$, $\forall t \in [0, \bar{v}]$, and

$$e_i^*(v_i) = v_i P_i^*(v_i) - \int_0^{v_i} P_i^*(t) dt. \tag{2}$$

[4] When $e_2 = B(e_1)$, we assume that the weaker player wins.

[5] $B(e_1)$ admits a discontinuity at $e_1 = \bar{v}$.

We have $J_i(0) = -\frac{1}{g_i(0)}$. Note $G_1(v) \le G_2(v)$ and the density functions are continuous. Thus $g_1(0) \le g_2(0)$, which implies $J_1(0) \le J_2(0)$. Recall that $J_1(\bar{v}) = J_2(\bar{v})$ and $J_i(\cdot)$ is strictly increasing. Define $\hat{v}_1(0) \in [0, \bar{v})$ such that $J_1(\hat{v}_1(0)) = J_2(0)$.

Define $\hat{v}_1(v_2) \in [\hat{v}_1(0), \bar{v}], \forall v_2 \in [0, \bar{v}]$ by $J_1(\hat{v}_1(v_2)) = J_2(v_2)$. Define $\hat{v}_2(v_1) \in [0, 1], \forall v_1 \in [\hat{v}_1(0), \bar{v}]$ by $J_2(\hat{v}_2(v_1)) = J_1(v_1)$. Note $\hat{v}_j(v_i)$ increases with v_i. Therefore, $P_1^*(v_1) = 0, \forall v_1 \in [0, \hat{v}_1(0)]$; $P_1^*(v_1) = \int_0^{\hat{v}_2(v_1)} g_2(v_2) dv_2 = G_2(\hat{v}_2(v_1)), \forall v_1 \in [\hat{v}_1(0), \bar{v}]$, which strictly increases with v_1. Similarly, $P_2^*(v_2) = G_1(\hat{v}_1(v_2)), \forall v_2 \in [0, \bar{v}]$, which strictly increases with v_2. We summarize these results in the following lemma.

Lemma 1. $P_1^*(v_1) = 0, \forall v_1 \in [0, \hat{v}_1(0)]; dP_1^*(v_1)/dv_1 > 0, \forall v_1 \in [\hat{v}_1(0), \bar{v}]$; $dP_2^*(v_2)/dv_2 > 0, \forall v_2 \in [0, \bar{v}]$.

Recall (2), we have

$$e_i^{*\prime}(v_i) = P_i^*(v_i) + v_i P_i^{*\prime}(v_i) - P_i^*(v_i) = v_i P_i^{*\prime}(v_i). \tag{3}$$

It is clear we have the following results.

Lemma 2. *$e_i^*(0) = 0$, $e_i^*(\bar{v}) = \bar{e}_i^* = \bar{v} - \int_0^{\bar{v}} P_i^*(t) dt \in (0, \bar{v})$, $\forall i$; $e_1^*(v_1) = 0, \forall v_1 \in [0, \hat{v}_1(0)]$; $e_1^{*\prime}(v_1) > 0, \forall v_1 > \hat{v}_1(0)$; $e_2^{*\prime}(v_2) > 0, \forall v_2 > 0$.*

Let $e_1^{*-1}(0) = \hat{v}_1(0)$. Based on Lemma 2, we are now ready to define the following favoritism rule B^*: Given $(e_1, e_2) \in [0, \bar{e}_1^*] \times [0, \bar{e}_2^*]$, player 1 wins if and only if

$$J_1(e_1^{*-1}(e_1)) > J_2(e_2^{*-1}(e_2)).$$

In other words, the favoritism rule B^* is defined as:

$$B^*(e_1) = e_2^* \circ (J_2^{-1}) \circ J_1 \circ e_1^{*-1}(e_1), \text{ and } (B^*)^{-1}(e_2) = e_1^* \circ (J_1^{-1}) \circ J_2 \circ e_2^{*-1}(e_2). \tag{4}$$

Note that $B^*(0) = 0$. The domain of function $B^*(e_1)$ is $[0, \bar{e}_1^*]$ and its range is $[0, \bar{e}_2^*]$, where $\bar{e}_i^* = e_i^*(\bar{v}) < \bar{v}$ by Lemma 2. For $e_1 \in (\bar{e}_1^*, \bar{v}]$, we let $B^*(e_1) = \bar{e}_2^* + \frac{\bar{v} - \bar{e}_2^*}{\bar{v} - \bar{e}_1^*}(e_1 - \bar{e}_1^*)$ by interpolation. We thus have $B^*(\bar{v}) = \bar{v}$. Note that there is no need to consider $e_i > \bar{v}$, since such a bid definitely leads to a negative payoff for the player.

Such defined function $B^*(e_1)$ specifies a favoritism rule in an all pay auction in the sense explained at the end of Section 2. For any bids $e_1, e_2 \in [0, \bar{v}]$, player 2 wins if and only if $e_2 \ge B^*(e_1)$. If $B^*(e_1) < e_1, \forall e_1 \in (0, \bar{v})$, the weaker player, i.e. player 2, is favored in the all pay auction under favoritism rule $B^*(\cdot)$. As a result, the favored weaker player can slack off, while the discriminated stronger player might be forced to compete more aggressively.

We next establish the following result.

Proposition 1. *In an all pay auction with favoritism rule $B^*(e_1)$ or equivalently $(B^*)^{-1}(e_2)$, it is a Bayesian Nash equilibrium for player i to adopt bidding strategy $e_i^*(v_i)$.*

Proof: Suppose that player 2 adopts strategy $e_2^*(v_2)$. Note that player 1 has no incentive to bid above $e_1^*(\bar{v})$, since bidding $e_1^*(\bar{v})$ makes sure s/he wins. We consider player 1's expected payoff if s/he bids $e_1 = e_1^*(v_1'), v_1' \in [\hat{v}_1(0), \bar{v}]$, and his/her value is v_1:

$$\begin{aligned}\pi_1(e_1; v_1) &= v_1 \Pr(v_2 | e_2^*(v_2) < B^*(e_1)) - e_1 = v_1 \Pr(v_2 | v_2 < e_2^{*-1} \circ B^*(e_1)) - e_1 \\ &= v_1 \Pr(v_2 | v_2 < (J_2^{-1}) \circ J_1 \circ e_1^{*-1}(e_1)) - e_1 = v_1 \Pr(v_2 | J_2(v_2) < J_1(v_1')) - e_1^*(v_1') \\ &= v_1 P_1^*(v_1') - e_1^*(v_1').\end{aligned}$$

Let

$$\tilde{\pi}_1(v_1'; v_1) = v_1 P_1^*(v_1') - e_1^*(v_1').$$

We want to show for given $v_1 \in [\hat{v}_1(0), \bar{v}]$, $\tilde{\pi}_1(v_1'; v_1)$ is maximized at $v_1' = v_1$. For this purpose, we want to show the following results:

$$\frac{\partial \tilde{\pi}_1(v_1'; v_1)}{\partial v_1'}|_{v_1' = v_1} = 0; \ \frac{\partial \tilde{\pi}_1(v_1'; v_1)}{\partial v_1'}|_{v_1' > v_1} < 0; \ \frac{\partial \tilde{\pi}_1(v_1'; v_1)}{\partial v_1'}|_{v_1' < v_1} > 0. \quad (5)$$

Note

$$\frac{\partial \tilde{\pi}_1(v_1'; v_1)}{\partial v_1'} = v_1 P_1^{*\prime}(v_1') - e_1^{*\prime}(v_1').$$

Using (3), we have

$$\frac{\partial \tilde{\pi}_1(v_1'; v_1)}{\partial v_1'} = [v_1 - v_1'] \, P_1^{*\prime}(v_1').$$

Thus (5) holds, which means $\tilde{\pi}_1(v_1'; v_1)$ is maximized at $v_1' = v_1$. In other words, bidding $e_1 = e_1^*(v_1)$ is optimal for player 1, if $v_1 \in [\hat{v}_1(0), \bar{v}]$. For $v_1 < \hat{v}_1(0)$, bidding $e_1^*(v_1')$ with $v_1' > \hat{v}_1(0)$ would generate a lower payoff than $\pi_1(e_1^*(v_1'); \hat{v}_1(0))$, which is type $\hat{v}_1(0)$'s payoff if s/he bids $e_1^*(v_1')$. Given it is optimal for type $\hat{v}_1(0)$ to bid $e_1^*(\hat{v}_1(0)) = 0$ and get zero expected payoff, it must be optimal for type $v_1 (< \hat{v}_1(0))$ to bid $e_1^*(\hat{v}_1(0)) = 0$ and get zero payoff.

Similarly, we can show that given player 1 adopts bidding strategy $e_1^*(v_1)$, it is optimal for player 2 to bid $e_2 = e_2^*(v_2)$ if his/her value is v_2. □

We next investigate the implications of favoritism rule $B^*(e_1)$ and equilibrium bidding strategy $e_i^*(v_i)$. Suppose bidder 1's value is v_1 and bidder 2's value is v_2. Then at equilibrium, bidder 1 bids $e_1^*(v_1)$ and bidder 2 bids $e_2^*(v_2)$. Bidder 1 wins if and only if $e_2^*(v_2) < B^*(e_1^*(v_1))$, which is $J_2(v_2) < J_1(v_1)$ or $v_1 > \hat{v}_1(v_2)$.

Proposition 2. *Under favoritism rule $B^*(\cdot)$ and equilibrium bidding strategy $e_i^*(v_i)$, bidder 1 wins if $J_1(v_1) > J_2(v_2)$; otherwise, bidder 2 wins.*

We first study under which conditions we have $B^*(e_1) < e_1, \forall e_1 \in (0, e_1^*(\bar{v})]$, i.e. the weaker player (i.e. player 2) is always favored in terms of winning chances, provided the two players have the same bid. Note that condition $B^*(e_1) < e_1$, $\forall e_1 \in (0, e_1^*(\bar{v})]$ is equivalent to $e_1^*(v_1) > e_2^*(\hat{v}_2(v_1)), \forall v_1 > \hat{v}_1(0)$.

Note that $\forall v_1 > \hat{v}_1(0)$, we have

$$\begin{aligned} e_1^*(v_1) &= v_1 G_2(\hat{v}_2(v_1)) - \int_{\hat{v}_1(0)}^{v_1} G_2(\hat{v}_2(t))dt; \\ e_2^*(\hat{v}_2(v_1)) &= \hat{v}_2(v_1) G_1(v_1) - \int_0^{\hat{v}_2(v_1)} G_1(\hat{v}_1(t))dt. \end{aligned} \tag{6}$$

When $v_1 = \hat{v}_1(0)$, we have $e_1^*(v_1) = e_2^*(\hat{v}_2(v_1)) = 0$. One can verify that $\forall v_1 > \hat{v}_1(0)$,

$$\begin{aligned} \frac{de_1^*(v_1)}{dv_1} &= v_1 g_2(\hat{v}_2(v_1)) \frac{d\hat{v}_2(v_1)}{dv_1} = v_1 g_2(\hat{v}_2(v_1)) \frac{J_1'(v_1)}{J_2'(\hat{v}_2(v_1))}; \\ \frac{de_2^*(\hat{v}_2(v_1))}{dv_1} &= \hat{v}_2(v_1) g_1(v_1). \end{aligned}$$

We thus have the following sufficient conditions for favoring the weaker player based on bids under the favoritism rule $B^*(\cdot)$.

Proposition 3. *The favoritism rule $B^*(\cdot)$ favors the weaker player in terms of bids, i.e. $e_1^*(v_1) > e_2^*(\hat{v}_2(v_1)), \forall v_1 > \hat{v}_1(0)$ if the following two conditions hold: (i) $\exists \check{v}_1 \in [\hat{v}_1(0), \bar{v}]$ such that on $[\hat{v}_1(0), \bar{v}]$, $v_1 g_2(\hat{v}_2(v_1)) J_1'(v_1) < \hat{v}_2(v_1) g_1(v_1) J_2'(\hat{v}_2(v_1))$ if and only if $v_1 \in [\check{v}_1, \bar{v}]$; (ii) $e_1^*(\bar{v}) > e_2^*(\bar{v})$.*

Note that replacing v_1 by $\hat{v}_1(v_2)$, then the above Condition (i) can be alternatively written as $\exists \check{v}_2 \in [0, \bar{v}]$ such that on $[0, \bar{v}]$, $\hat{v}_1(v_2) g_2(v_2) J_1'(\hat{v}_1(v_2)) < v_2 g_1(\hat{v}_1(v_2)) J_2'(v_2)$ if and only if $v_2 \in [\check{v}_2, \bar{v}]$.

3.1 An example

We next provide an example, in which the weaker player is always favored based on bids under the favoritism rule $B^*(\cdot)$. Consider

$$G_1(v_1) = (v_1)^2, \forall v_1 \in [0,1] \text{ and } G_2(v_2) = v_2, \forall v_2 \in [0,1].$$

We thus have

$$g_1(v_1) = 2v_1, \forall v_1 \in [0,1] \text{ and } g_2(v_2) = 1, \forall v_2 \in [0,1].$$

Note that we have $G_1(v)$ stochastically dominates $G_2(v)$ in the sense of hazard rate, i.e. $\frac{g_2(v)}{1-G_2(v)} \geq \frac{g_1(v)}{1-G_1(v)}$, since $\frac{1-G_1(v)}{g_1(v)} = \frac{1-v^2}{2v} = (1-v)\frac{1+v}{2v} \geq 1-v = \frac{1-G_2(v)}{g_2(v)}$. We thus have Assumption 1 holds. One can easily verify Assumptions 2(i) and 2(ii) also hold, since

$$J_1(v_1) = v_1 - \frac{1-v_1^2}{2v_1}, \forall v_1 \in [0,1] \text{ and } J_2(v_2) = 2v_2 - 1, \forall v_2 \in [0,1].$$

Relying on Proposition 3, we next show that the weaker player, i.e. player 2, always wins when both players have the same bid. The following results can be verified by direct calculations:

$$\begin{aligned} v_1 g_2(\hat{v}_2(v_1)) J_1'(v_1) &= \frac{1}{2v_1}\left(3v_1^2+1\right) = \frac{3}{2}v_1 + \frac{1}{2v_1}; \\ \hat{v}_2(v_1) g_1(v_1) J_2'(\hat{v}_2(v_1)) &= \left(\frac{3(v_1)^2-1}{4v_1} + \frac{1}{2}\right) 4v_1 = 3v_1^2 + 2v_1 - 1. \end{aligned}$$

Note that $\left(\frac{3}{2}v_1 + \frac{1}{2v_1}\right) - (3v_1^2 + 2v_1 - 1) = \frac{1}{2v_1} - \left(3v_1^2 + \frac{1}{2}v_1 - 1\right)$, which decreases with v_1. Clearly, Condition (i) in Proposition 3 holds. Using (6), we have[6]

$$\begin{aligned} e_1^*(1) &= 1 - \int_{\frac{1}{3}}^{1} \left(\frac{3t^2-1}{4t} + \frac{1}{2}\right) dt = \frac{1}{4}\ln 3 + \frac{1}{3} \approx 0.608; \\ e_2^*(1) &= 1 - \int_0^1 \left(\sqrt{\frac{1}{3} + \left(\frac{1-2t}{3}\right)^2} - \frac{1-2t}{3}\right)^2 dt = \frac{16}{27} \approx 0.593. \end{aligned}$$

We thus verified that Condition (ii) also holds, i.e. $e_1^*(1) > e_2^*(1)$. Therefore, Proposition 3 applies: the weaker player, i.e. player 2, always wins when both players have the same bid.

3.2 Does favoritism necessarily enhance the weaker's winning chance?

One question naturally arises: Compared to a standard all pay auction without favoritism,

[6] The calculation details are available from the author upon request.

does the favoritism $B^*(\cdot)$ necessarily render a higher expected winning chance to the favored weaker player? Surprisingly, the answer to this question is negative. We illustrate this possibility using the same example with

$$G_1(v_1) = (v_1)^2, \forall v_1 \in [0,1] \text{ and } G_2(v_2) = v_2, \forall v_2 \in [0,1].$$

We have shown that in this example, the weaker player, i.e. player 2, always wins when both players have the same bid under favoritism rule $B^*(\cdot)$. By Proposition 2, we have that under $B^*(\cdot)$, players' winning rule can be described by (1) at equilibrium:

$$p_1^*(v_1, v_2) = \begin{cases} 1, & \text{if } v_2 < \frac{3(v_1)^2-1}{4v_1} + \frac{1}{2} \text{ and } v_1 \in [\frac{1}{3}, 1], \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } p_2^*(v_1, v_2) = 1 - p_1^*(v_1, v_2).$$

Thus, player 1's expected winning probability is:

$$\begin{aligned} P_1^* &= \int_{\frac{1}{3}}^{1} G_2\left(\frac{3(v_1)^2-1}{4v_1} + \frac{1}{2}\right)(2v_1)dv_1 = \int_{\frac{1}{3}}^{1} \left(\frac{3(v_1)^2-1}{4v_1} + \frac{1}{2}\right)(2v_1)dv_1 \\ &= \int_{\frac{1}{3}}^{1} \left(\frac{3(v_1)^2-1}{2} + v_1\right)dv_1 = \left\{\frac{1}{2}\left[(v_1)^3 - v_1\right] + \frac{(v_1)^2}{2}\right\}|_{1/3}^{1} = \frac{16}{27}. \end{aligned}$$

It follows that player 2's expected winning chance in the all pay auction with favoritism rule $B^*(\cdot)$ is $P_2^* = \frac{11}{27}$.

Computing the expected winning chances of the players in a standard all pay auction with no favoritism does not require us to explicitly solve for the equilibrium bidding strategy. What we need to pin down is player 2's winning value threshold $k(v_1)$ for each value of player 1. Using eqn. (2) in Kirkegaard (2012), $k(\cdot)$ is determined by[7]

$$\int_{v_1}^{1} \frac{g_1(v)}{v}dv = \int_{k(v_1)}^{1} \frac{g_2(v)}{v}dv, \text{ i.e.} \int_{v_1}^{1} 2dv = \int_{k(v_1)}^{1} \frac{1}{v}dv.$$

Therefore, $k(v_1) = \exp(2v_1 - 2)$. We now are ready to calculate player 1's expected winning

[7] We can also rely on the procedure of Amann and Leininger (1996) to pin down $k(v_1)$.

probability in a standard all pay auction:

$$\begin{aligned}
\check{P}_1^* &= \int_0^1 G_2(\exp(2v_1-2))(2v_1)dv_1 = \int_0^1 (\exp(2v_1-2))(2v_1)dv_1 \\
&= \frac{1}{e^2}\int_0^1 v_1 d\exp(2v_1) = \frac{1}{e^2}\left\{v_1\exp(2v_1)|_0^1 - \int_0^1 \exp(2v_1)dv_1\right\} \\
&= \frac{1}{2}\left(1+\frac{1}{e^2}\right).
\end{aligned}$$

It follows that player 2's expected winning chance in a standard all pay auction is:

$$\check{P}_2^* = \frac{1}{2}\left(1+\frac{1}{e^2}\right) \approx 0.43 > P_2^* \approx 0.407.$$

We thus have that in our example, the winning chance of the weaker player (player 2) is lower under the favoritism $B^*(\cdot)$, i.e. $P_2^* < \check{P}_2^*$. This result indicates that the favoritism toward the weaker player does not necessarily generate a higher winning chance for him/her. The discriminatory policy forces the stronger player to bid more aggressively, which may make him/her win with a higher chance at equilibrium.[8]

Using salary data, Fang and Norman (2006) provide empirical evidence for the discriminated Chinese group's surged incentive on human capital investment after an affirmative action program called the New Economic Policy (NEP) was introduced in Malaysia in 1970s.[9] Ashkenas, Park, and Pearce (2017) find that African American and Hispanic students are more underrepresented at the nation's top colleges and universities in US even after decades of affirmative action. While the share of college-aged African Americans has increased from 13 percent to 15 percent, African American students continue to represent only 6 percent of freshmen at elite schools. Similarly, this phenomenon is seen when analyzing the situation of Hispanic students. These empirical findings are consistent with the enhanced incentive of the students in other groups to improve their credentials for college admissions.

[8] As mentioned in the introduction of the paper, linear favoritism rules however always increase the favored weaker player's winning chance according to the analysis of Kirkegaard (2012).

[9] Despite of the enhanced skills of the ethnic Chinese group, the ethnic Malay group's employment shares in both public and private sectors have increased significantly according to Tables 6a, 6b and 7 in Jomo (2004). There are several reasons for the increased Malay employment shares in private sectors. First, the NEP affected the education policy, which gives the Malay group a favor to enter public universities. Second, the ownership of enterprise among Malay increased, which tends to increase the employment rate of Malay in private sectors. Third, Malay's population grows, while Chinese's population shrinks due to a brain drain among Chinese.

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