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Optimal teleworking agreements vs. yearning for normality when vaccine is on the horizon

Jacek Rothert
United States Naval Academy

Abstract

During a pandemic, companies may adopt teleworking agreements even if they lower current productivity. If managers (or policymakers) want to project an image of ``return to normality'', completely orthogonal to any economic or health outcomes, the scope of teleworking agreements is lower but constant in a stationary equilibrium. In response to the news about upcoming vaccine, rational managers always increase the scope of teleworking agreements, unless the desire to project the image of ``return to normality'' is sufficiently strong, effectively creating a reopening-smoothing motive. The ``return to normality'' may be premature if managers do not understand Lucas' Critique.

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Contact: Jacek Rothert - jacek.rothert@gmail.com.
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1 Introduction

During the Covid-19 pandemic many institutions adopted teleworking agreements for their employees, with the scope varying by sector and establishment (Brynjolfsson et al., 2020; Rahman, 2020). Over time, employers implemented health-related safety protocols for workers who stayed on-site (Honein et al., 2021). As the first vaccine shots were made available, some analysts and policymakers argued it was time for life to "return to normality", both in the context of general reopening of the economy[1], and in the context of returning to pre-Covid working arrangements in some areas, most notably education [2], coinciding with the roll-out of a vaccine.

This situation yields natural questions for economists. Does the development of a vaccine affect the optimal scope of teleworking agreements? Can the observed changes in the scope of teleworking agreements tell us about the extent to which companies care about their future output vs. about projecting the *image* of "return to normality"? Is it safe to bring workers back on-site when new health safety protocols appear effective? In this note I address these questions by laying out a simple theory of task allocation between on-site and teleworking agreements during a public health emergency. The theory offers three main insights. First, if business cannot quickly replace workers who passed away or are on an extended medical leave, the management will adopt teleworking agreements for part of its workforce even if they lower current productivity. Second, upon the news about the upcoming vaccine, rational mangers should *always* expand the scope of teleworking agreements. Third, if companies base their "return to in-person work" policy on successful implementation of health safety protocols, the reduction in the scope of teleworking agreements may be excessive if managers do not understand Lucas' Critique (Lucas, 1976).

2 Model

The final product is created with a continuum of tasks, indexed by $x \in [0,1]$. They can be done on-site (in person) or via teleworking (remotely). A task done on-site yields a unit of output. A task x done teleworking yields $f(x)$ units where $f' > 0$, $f(0) = 0$, and $f(1) = 1$. Hence, the lower the x, the more important it is for the task to be done on-site: overseeing the IT network might have $f(x)$ close to one, whereas being a security guard will have $f(x)$ equal to zero. Quality of the final product is given by:

$$Y = \int_0^1 [p(x) + (1 - p(x)) \cdot f(x)]\, dx$$

where $p(x)$ indicates whether the task is done in person — $p(x) = 1$, or remotely — $p(x) = 0$. The distribution of employees across tasks is uniform, so the total number of people working on-site is:

$$p = \int_0^1 p(x) dx. \quad (2.1)$$

[1] Most notably, the governor of Texas increased the capacity of all businesses to 100% in early March quoting, among other factors, the on-going vaccine roll-out: https://www.dshs.texas.gov/GA34/

[2] On his first day in office, Joe Biden issues an Executive Order on Supporting the Reopening of Schools https://www.whitehouse.gov/briefing-room/presidential-actions/2021/01/21/executive-order-supporting-the-reopening-and-continuing-operation-of-schools-and-early-childhood-education-providers/, and GOP leaders criticized teachers' unions for their opposition to the return to in-person schooling: https://www.republicanleader.senate.gov/newsroom/research/democrats-side-with-teachers-unions-against-students.

In-person vs. remote completion of task x yields different probability of getting infected for the employee involved in the task. I will assume that probability is 0 for remote work.[3] Typically, the task that does not lose much of its quality when done remotely, will involve fewer close interactions with people. Hence, when on-site, workers involved in such task will have a relatively lower probability of getting infected. Specifically, I assume that a worker involved in task x on-site faces probability $\tilde{s}(x) \equiv p \cdot s(x)$ of getting sick, with $s'(x) < 0$ and such that $|s'(x)| \leq 2 \cdot s(x)$ for all x (e.g. a function $s(x) \equiv e^{-x}$ satisfies that restriction). The last assumption ensures that total infections are a convex function of the fraction of workers who work on-site. Additionally, I assume that $0 < s(1) < s(0) < 1$, so that the most (least) in-person intensive task has infection probability that is less than one (positive).

Marginal benefit and marginal cost of moving a given task on-site Figure 1 plots typical graphs of $MB(x) \equiv 1 - f(x)$ and $MC(x) \propto \kappa \cdot s(x)$ (for some given κ), which illustrate the marginal benefit and marginal cost of making any given x done on-site (keeping all other tasks unchanged). It is evident that $p(x_1) \geq p(x_2) \iff x_1 \leq x_2$ - if it is optimal to have x done on-site (remotely), then it must also be optimal to have any other task $x' < x$ ($x' > x$) done on-site (remotely).

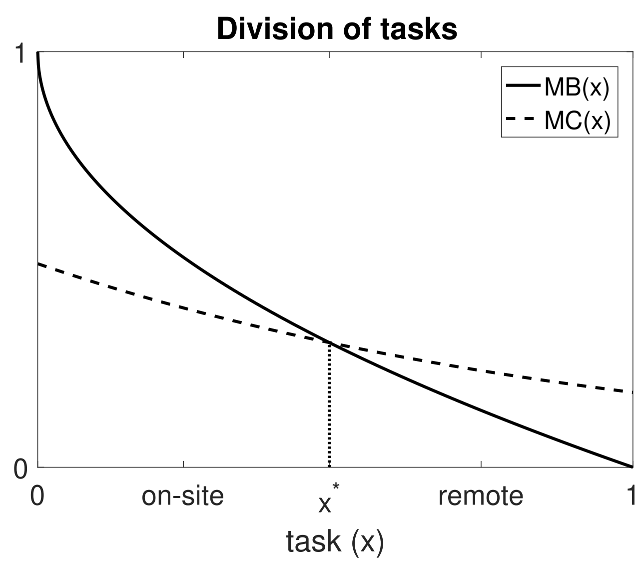

Figure 1: Allocation of tasks between on-site and remote work

Let x^* be the threshold task such that for any $x \leq x^*$ we have $p(x) = 1$ (all tasks $x \leq x^*$ are done in-person, on-site) and for any $x > x^*$ we have $p(x) = 0$ (all tasks $x > x^*$ are done remotely, off-site), which implies that $p = x^*$. Given x^*, the fraction of workforce infected is $I(x^*) = \int_0^{x^*} \tilde{s}(x)dx = x^* \cdot \int_0^{x^*} s(x)dx$. Applying the Fundamental Theorem of Calculus to $I(x)$, $I'(x)$, and $Y(x)$ yields the

[3] This is a normalization - the key is that employees' behavior outside of their workplace is uncorrelated with the nature of their job.

following properties of I',I'',Y', and Y'' (assuming interior x^*):

$$
\begin{aligned}
I'(x^*) &= \int_0^{x^*} s(x)dx + x^* \cdot s(x^*) > 0 && (2.2)\\
I''(x^*) &= 2 \cdot s(x^*) + x^* \cdot s'(x^*) > 0 && (2.3)\\
Y'(x^*) &= 1 - f(x^*) > 0 \iff x^* < 1, \qquad \Rightarrow Y''(x^*) = -f'(x^*) < 0 \text{ if } x^* < 1 && (2.4)
\end{aligned}
$$

2.1 Dynamic Optimization

Since the focus of the model is on the change in teleworking agreements in response to the news about the upcoming vaccine, the management's optimization problem has to be dynamic. I model it by considering decisions made during three distinct stages: the waiting stage (W), the jabbing stage (J), and the victory stage (V). The W-stage is an ongoing pandemic, when the world is waiting for the development of the vaccine. It can last multiple periods, potentially forever. In each period there is a constant probability ν that vaccine will be developed, and the world would enter the J-stage. The J-stage lasts one period after which the world enters the post-pandemic V-stage and stays there forever: the pandemic is over and everyone has been vaccinated. The focus is then on the change in teleworking policy between the W- and the J-stage.

V-stage The output in each period is 1, because everyone works on-site and $x^* = 1$. The value of being in the V-stage in each period $t > 1$ is simply $V^* = 1 + \beta V^*$, so $V^* = \frac{1}{1-\beta}$, where β is the discount factor between periods. The first period of the V-stage, however, features some baggage from the previous stage: fraction δ of the company's workforce cannot work (because they passed away or are recovering from the infection and cannot be quickly replaced). The value of being in the V-stage in period $t = 1$ is thus:

$$V(\delta) = 1 - \delta + \beta \cdot \frac{1}{1-\beta}. \tag{2.5}$$

J-stage The jabbing stage lasts only one period. At this stage, vaccine is being rolled out. Again, a fraction δ of the company's workforce is unable to work. The value of being in the J-stage is thus:

$$J(\delta) = \max_x (1-\delta)Y(x) + \beta V(\delta(x)), \tag{2.6}$$

where x now denotes the threshold task done on-site, so that any task $x' \leq x$ is done on-site, and any task $x' > x$ is covered by a teleworking agreement, where $\delta(x) \equiv \bar{\delta} I(x)$, and $\bar{\delta}$ is a constant parameter. Notice that $\delta(x)$ is the internalized fraction of workers that the company will be missing in the first period of the V-stage.

W-stage The waiting stage lasts until the vaccine is developed. I assume that the probability of developing a vaccine is a constant parameter $\nu > 0$. The value of being in W-stage can therefore be written as:

$$W(\delta) = \max_x (1-\delta)Y(x) + \beta \left[\nu J(\delta(x)) + (1-\nu)W(\delta(x))\right], \tag{2.7}$$

where again δ is the fraction of workforce that cannot work this period, and $\delta(x) \equiv \bar{\delta} I(x)$ is the fraction of workforce that will be unable to work next period.

Notice that there is a very important difference between a stage (a regime) and a period. The V-stage has infinitely many periods. The J-stage lasts only one period. The W-stage can last anywhere from one to infinitely many periods - its duration is a random variable.

2.1.1 Stationary Equilibrium

I focus the theoretical analysis on the outcomes in the stationary equilibrium, assuming interior solution.

Definition 2.1 (Stationary Equilibrium)**.** A stationary equilibrium is a tuple (V, J, W, x_J^*, x_W^*) such that V, J, and W solve (3.1)-(3.3), and J and W attain maximum at x_J^* and x_W^*, respectively.

2.1.2 Characterization

The Envelope and First Order Condition, evaluated at (x_J^*, x_W^*) are as follows:

$$V'(\delta) = -1, \qquad J'(\delta) = -Y(x_J^*) > -1, \qquad W'(\delta) = -Y(x_W^*) > -1, \tag{2.8}$$

and

$$\begin{aligned} \text{J-stage:} \quad 0 &= (1-\delta(x_W^*))Y'(x_J^*) + \beta\delta'(x_J^*)V'(\delta(x_J^*)) & (2.9) \\ \text{W-stage:} \quad 0 &= (1-\delta(x_W^*))Y'(x_W^*) + \beta\delta'(x_W^*)\underbrace{\left[\nu J'(\delta(x_W^*)) + (1-\nu)W'(\delta(x_W^*))\right]}_{>-1\equiv V'(\delta(x_J^*))} & (2.10) \end{aligned}$$

The central proposition of this paper is that, in the stationary equilibrium, rational managers will always expand the scope of teleworking agreements during the jabbing period.

Proposition 2.2. *Let (V, J, W, x_J^*, x_W^*) be the stationary equilibrium such that $x_W^* \in (0,1)$. Then $x_J^* < x_W^*$.*

Proof. Define $\tilde{J}(x; \delta(x_W^*)) := (1-\delta(x_W^*))Y(x) + \beta V(\delta(x))$. Proof requires showing that $\frac{\partial \tilde{J}}{\partial x} < 0$ if $x \geq x_W^*$. See Appendix A for details. □

The intuition behind Proposition 2.2 follows directly from conditions (2.8) - (2.10): the marginal cost of increasing x is higher during the J-stage, because after the vaccinations are completed, teleworking is no longer necessary. As a result, risking workers' health today incurs higher cost in the future: the continuation value V is higher than $(1-\nu)W + \nu J$.

3 Projecting the "return to normality"

Is it possible to have a reduction in the scope of teleworking agreements during the jabbing period in the stationary equilibrium? This section will offer one such possibility - when the management of the company, for a variety of reasons, tries to project the image of "return to normality". When that is the case, the benefit of prematurely bringing part of the workforce back on site may outweigh the cost.

The need to project the image of return to normality is modeled by introducing an additional term which captures the utility from a decrease in the scope of teleworking agreements, relative to the previous period. That need is, in a non-economic sense, irrational[4]: it is not based on anything related to workers' productivity, since the impact on output has already been taken into account. Using a common notation from time series, let Lx denote the lagged value of x. Let $\phi(x - Lx)$ denote the utility from reducing the scope of teleworking agreements. Since the reduction in the scope of teleworking agreements means that $x > Lx$, I assume that $\phi' > 0$. In order to ensure that FOCs are sufficient I assume $\phi'' \leq 0$.

The three value functions V, J, and W now take the following form:

$$
\begin{aligned}
V(\delta, Lx) &= 1 - \delta + \phi(1 - Lx) + \frac{\beta}{1-\beta} && (3.1)\\
J(\delta, Lx) &= \max_x (1-\delta)Y(x) + \phi(x - Lx) + \beta V(\delta(x), x) && (3.2)\\
W(\delta, Lx) &= \max_x (1-\delta)Y(x) + \phi(x - Lx) + \beta\left[\nu J(\delta(x), x) + (1-\nu)W(\delta(x), x)\right] && (3.3)
\end{aligned}
$$

The definition of a stationary equilibrium is very similar to the one in the previous section, the only difference being in the objective functions.

Definition 3.1 (Stationary Equilibrium)**.** A stationary equilibrium is a tuple (V, J, W, x_J^*, x_W^*) such that V, J, and W satisfy (3.1)-(3.3), and , and J and W attain maximum at x_J^* and x_W^*, respectively.

3.1 Characterization

Relative to the previous section, in addition to V_δ, J_δ, and W_δ, we have the following Envelope Conditions w.r.t. Lx:

$$
V_{Lx} = -\phi'(1 - x_J^*) \qquad J_{Lx} = -\phi'(x_J^* - x_W^*) \qquad W_{Lx} = -\phi'(0)
$$

The First Order Conditions now read:

$$
\begin{aligned}
0 &= (1 - \delta(x_W^*))Y'(x_J^*) + \phi'(x_J^* - x_W^*) + \beta\delta'(x_J^*)V_\delta + \beta V_{Lx}\\
0 &= (1 - \delta(x_W^*))Y'(x_W^*) + \phi'(0) \qquad + \beta\delta'(x_W^*)\left[\nu J_\delta + (1-\nu)W_\delta\right] + \beta\left[\nu J_{Lx} + (1-\nu)W_{Lx}\right]
\end{aligned}
$$

The two first order conditions above are very similar to conditions (2.9)-(2.10). However, they both have extra terms associated with ϕ'. It then follows that a necessary condition to rationalize a reduction in teleworking agreements during the J-stage has to do with the shape of ϕ': it has to be decreasing. The following proposition formalizes it.

Proposition 3.2. *Let (V, J, W, x_J^*, x_W^*) be the stationary equilibrium as in Definition 3.1. Then $x_J^* \geq x_W^*$ only if $\phi'' < 0$.*

---

[4] In decision theory, *rational* is a property of preference relation, which means "complete and transitive". That property, of course, holds in this framework.

Proof. Define $\tilde{J}(x;x_W^*) := (1-\delta(x_W^*))Y(x) + \phi(x-x_W^*) + \beta V(\delta(x),x)$. Proof requires showing that
$\frac{\partial \tilde{J}}{\partial x}|_{x=x_W^*} \geq 0$ only if $\phi'' < 0$. See Appendix B for details. □

In words, the only justification to reduce the scope of teleworking agreements (i.e., $x_J^* > x_W^*$) when vaccine is on the horizon is when marginal utility from such reduction is positive and decreasing. Notice that this result is more subtle than it appears. After all, we always have $\phi'(0) > 0$, even during the W-stage, when the teleworking agreements remain constant. Their extent is smaller than in the model without the extra term $\phi(\cdot)$, but it does not change over time. Yet, the existence of that term can incentivize managers to act in a manner which, on face value, appears irrational (that term is independent of anything related to actual productivity and output Y). Mechanically, the concavity of ϕ means that the managers want to smooth the reduction in teleworking agreements over time (in this case, over two periods). One way to interpret $\phi'' < 0$ would be as existence of an irrational, impatient need of projecting a "return to normality".

Of course, Proposition 3.2 provides only a necessary condition required to rationalize a potential reduction in the scope of teleworking agreements when vaccine is on the horizon: strict concavity of $\phi(\cdot)$ is a necessary condition but may not be sufficient. In fact, one can show (see Appendix B) that the degree of concavity of $\phi(\cdot)$ must be quite large, so that at $x^* = x_W^*$ we would have:

$$\phi'(0) \geq \phi'(1-x_W^*) + \delta'(x_W^*)[1-Y(x_W^*)]$$

The left-hand side of the inequality is the marginal benefit of projecting the "return to normality" today. The right-hand side is the marginal cost of doing so. The first term on the right-hand side is the marginal benefit of "returning to normality" in the first period of the V-stage. The second term is the marginal cost of having a smaller workforce that would produce future output in a normal, more efficient, in-person setting.

It is important to note that the key behind the result above is again the change in the intertemporal trade-off as we move from the W- to the J-stage. If the only thing that links two period is the fraction δ of workers that cannot work, then $x_J^* < x_W^*$ (as in Section 2). In this section, I introduced another link between the periods to show that this result could be reversed. There are certainly other links between the periods that the model abstracts from, that should and could be analyzed. They are left for further research.

4 New safety protocols and the Lucas' Critique

An important feature of dealing with the Covid-19 pandemic in workplaces was development of new health safety protocols - introducing social distancing measures, new cleaning practices, rules about areas where mask-wearing was mandatory and where it could be done away with, etc. The success of such protocols is necessarily evaluated with a given fraction of workers on-site. The model in this paper can quickly illustrate that such evaluation suffers from the Lucas' Critique - a point that should be obvious to economists, though maybe less so to managers and policymakers.

Suppose the management is learning about the effectiveness of the new health safety protocols. They understand that different tasks yield different probability of getting infected, and they know $s(x)$. They also understand that $\tilde{s}(x) = p \cdot s(x)$. However, they believe that p is an unknown but fixed parameter, rather than a variable that depends on the fraction of workers on-site. In other words, they do not understand the Lucas' Critique.

Let $\hat{s}_{x^*}$ be the unbiased estimate of $\tilde{s}(x)$ in the early months of the pandemic, obtained in the environment with fraction x^* working in-person, implying that for every x we have $\hat{s}_{x^*}(x) = \tilde{s}(x) = x^* \cdot s(x)$. After observing I_{x^*} number of infections at workplace in the early months of the pandemic, the management estimates p to be:

$$\hat{p} = \frac{I_{x^*}}{\int_0^{x^*} s(x)dx} = x^*$$

They now believe that

$$\hat{I}(x^*) = \hat{p} \cdot \int_0^{x^*} s(x)dx, \qquad \Rightarrow \hat{I}'(x^*) = \hat{p} \cdot s(x^*) < \int_0^{x^*} s(x)dx + x^* \cdot s(x^*) = I'(x^*)$$

This, in turn, implies that:

$$\hat{\delta}\,(x^*) = \delta \cdot \hat{I}\,(x^*)\,, \qquad \text{and} \qquad \hat{\delta}'\,(x^*) < \delta'\,(x^*)$$

Given the above, the first order conditions (2.9)-(2.10) now imply that the scopes of teleworking agreements in the stationary equilibrium (during either stage) will be lower than optimal (x^* will be too high).

5 Conclusions

Economic theory tells us that upon the news of upcoming vaccine, policymakers should impose stricter lockdowns rather than ease them (Rothert, 2021). This paper brought that intuition to a micro-level of a single establishment and its decision of allocating workers between those who can and cannot complete their work remotely. A rational management always extends teleworking agreements when vaccine is on the horizon. However, the theory also offers a way to rationalize partial termination of teleworking agreements when vaccine is on the horizon, which relies on an "irrational" (unrelated to fundamentals) desire to project an image of "return-to-normality".

Until all workforce is vaccinated, the return to office space should be slow and gradual, even if the employer developed special health safety protocols that appear effective. That effectiveness is necessarily evaluated with a given fraction of workforce teleworking. When that fraction is reduced, the effectiveness of such protocols may be reduced as well, as we know from Lucas' Critique. Managers who do not take that into account will end up bringing people, who should keep teleworking, back on-site.

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A Proof of Proposition 2.2

Defining $\tilde{J}(x;\delta(x_W^*)) = (1-\delta(x_W^*))Y(x) + \beta V(\delta(x))$, we get the following:

$$\left.\frac{\partial \tilde{J}}{\partial x}\right|_{x=x_J^*} = (1-\delta(x_W^*))Y'(x_J^*) + \delta'(x_J^*)\beta V_\delta(\delta(x_J^*))$$

Suppose $x_J^* \geq x_W^*$. Then, since $Y(\cdot)$ is strictly concave, $\delta(\cdot)$ is convex, and $V_\delta = -1$ we get:

$$(1-\delta(x_W^*))Y'(x_J^*) + \delta'(x_J^*)\beta V_\delta(\delta(x_J^*)) \leq (1-\delta(x_W^*))Y'(x_W^*) - \beta\delta'(x_W^*)$$

Next, since $\nu J_\delta(\delta(x_W^*)) + (1-\nu)W_\delta(\delta(x_W^*)) > -1$, we get:

$$-\beta\delta'(x_W^*) < -\beta\delta'(x_W^*)\left[\nu J_\delta(\delta(x_W^*)) + (1-\nu)W_\delta(\delta(x_W^*))\right] = 0$$

where the last equality is implied by the first order condition (2.10).

B Proof of Proposition 3.2

The proof proceeds in a similar manner as the one in Appendix A.
Define $\tilde{J}(x;\delta(x_W^*),x_W^*) = (1-\delta(x_W^*))Y(x) + \phi(x-x_W^*) + \beta V(\delta(x),x)$. We then get the following:

$$\frac{\partial \tilde{J}}{\partial x} = (1-\delta(x_W^*))Y'(x) + \phi'(x-x_W^*) + \beta\delta'(x)V_\delta(\delta(x),x) - \beta\phi'(1-x)$$

Suppose $x_J^* = x_W^*$. We must show that $\left.\frac{\partial \tilde{J}}{\partial x}\right|_{x=x_W^*} \geq 0$ necessarily implies that $\phi'' < 0$. Since $Y(\cdot)$ is strictly concave, $\delta(\cdot)$ is convex, $V_\delta \equiv -1$, and $J_\delta, W_\delta > -1$, under the assumption that $x_J^* \leq x_W^*$ we get:

$$\begin{aligned}
\left.\frac{\partial \tilde{J}}{\partial x}\right|_{x=x_W^*} &= (1-\delta(x_W^*))Y'(x_W^*) + \phi'(x_W^* - x_W^*) + \beta\delta'(x_W^*)V_\delta(\delta(x_W^*),x_W^*) - \beta\phi'(1-x_W^*) \\
&= (1-\delta(x_W^*))Y'(x_W^*) + \phi'(x_W^* - x_W^*) + \beta\delta'(x_W^*)[\nu J_\delta + (1-\nu)W_\delta] - \beta\phi'(0) \\
&\quad -\beta\phi'(1-x_W^*) + \beta\phi'(0) \\
&\quad +\beta\delta'(x_W^*)V_\delta(\delta(x_W^*),x_W^*) - \beta\delta'(x_W^*)[\nu J_\delta + (1-\nu)W_\delta]
\end{aligned}$$

The First Order Condition for x_W^* then implies that the first line after the strict inequality is zero, so we get:

$$\left.\frac{\partial \tilde{J}}{\partial x}\right|_{x=x_W^*} = -\beta\phi'(1-x_W^*) + \beta\phi'(0) - \beta\delta'(x_W^*) + \beta\delta'(x_W^*)Y(x_W^*)$$

For $\left.\frac{\partial \tilde{J}}{\partial x}\right|_{x=x_W^*} \geq 0$ we therefore need:

$$-\beta\phi'(1-x_W^*) + \beta\phi'(0) - \beta\delta'(x_W^*) + \beta\delta'(x_W^*)Y(x_W^*) \geq 0$$

which implies:

$$\phi'(0) \geq \phi'(1-x_W^*) + \delta'(x_W^*)\left[1-Y(x_W^*)\right] > \phi'(1-x_W^*)$$

which requires that $\phi'' < 0$.