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Eliminate the normative worst, then choose

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Abstract

We model the behavior of a decision maker (DM) who is psychologically constrained from choosing according to her tastes by her normative preferences that capture her values and ideals. In any menu, choosing the worst alternative according to her normative preferences may produce overwhelming feelings of guilt. Hence, to mitigate such feelings, she eliminates this alternative and chooses the best amongst the remaining ones according to her tastes. We formally define this sequential choice procedure and behaviorally characterize it. We show that the parameters of the model—the DM's tastes and norms—can be (almost) uniquely identified from choices. We also highlight the model's implications for "non-standard" choices.

1. Introduction

Consider a decision maker (DM) with tastes captured by a strict preference ranking over a given set of alternatives as is standard. But, additionally, suppose this DM also has normative preferences that capture her values and ideals in terms of how she thinks she *should* behave. In any menu, the DM wants to choose the best alternative according to her tastes like a standard agent. But, unlike such an agent, suppose our DM experiences an overwhelming sense of guilt if the chosen alternative happens to be the worst one in the menu according to her normative preferences. Such guilt brought about by acting in complete dissonance with her values may be psychologically quite costly. Hence, it is not unreasonable to imagine that to avoid experiencing such feelings of guilt, she may simply eliminate this alternative from her consideration and choose the best one amongst the remaining alternatives according to her tastes.

Such a way of choosing may be behaviorally quite plausible in many choice settings and capture empirically relevant departures from the rational choice model. For instance, consider a DM on a diet whose normative goal is to minimize her calorie intake. Suppose she has to choose between foregoing dessert or having a 200 calorie apple pie. In this case, although according to her tastes, she may want to have the dessert, doing so may make her feel guilty given that her normative preference is to minimize her calorie intake. Therefore, she may forgo having the dessert. Now, suppose the menu is expanded and along with the apple pie a 1000 calorie chocolate chip cookie sundae is also available. In this case, eliminating the decadent sundae from her consideration may be enough to assuage her feelings of guilt and she may end up having the apple pie. This pattern of choices illustrates the compromise effect, which is a leading example of menu effects. Such menu effects, of course, violate the rational choice benchmark, characterized by the weak axiom of revealed preferences (WARP).

In this note, we formally define such a sequential choice procedure and provide a behavioral characterization for it. We show that amongst its desirable properties is the fact that the parameters of the model—the DM’s tastes and norms—can be (almost) uniquely identified from behavior. In the next section, we set up the primitives and formally define our choice procedure. In Section 3, we behaviorally characterize the model and establish the extent of its identification. Finally, in Section 4, we conclude with a few comments situating our model in the literature, especially in terms of its implications for non-standard choices. Proofs of results appear in the Appendix.

2. Primitives

Let X be a finite set of alternatives with typical elements denoted by x, y, z etc. \mathcal{X} denotes the set of non-empty, non-singleton subsets of X with typical elements denoted by S, T etc., which we refer to as menus. A choice function $c : \mathcal{X} \rightarrow X$ is a mapping that, for any menu $S \in \mathcal{X}$, specifies the alternative $c(S) \in S$ that the DM chooses in that menu. In our model, the DM makes these choices following a sequential procedure that is based on a pair of strict preference rankings (By a strict preference ranking, we mean

a binary relation that is total, asymmetric and transitive). The ranking $\succ \subseteq X \times X$ captures her tastes; and the ranking $\succ^* \subseteq X \times X$ captures her normative preferences. Faced with any menu $S \in \mathcal{X}$, first, she eliminates the worst alternative in S according to \succ^* . Then, amongst the remaining alternatives, she chooses the best one according to \succ . Since \succ^* and \succ are strict preference rankings, the \succ^* -worst and \succ -best elements of any $\emptyset \neq S \subseteq X$ are unique and the procedure results in a decisive choice in any menu. In the way of notation, denote the \succ^* -worst and \succ -best elements of any $\emptyset \neq S \subseteq X$ by $\min(S; \succ^*)$ and $\max(S; \succ)$, respectively. Before defining the procedure formally, note that in the subsequent analysis we abuse notation by often suppressing set delimiters; e.g., we write $S \setminus \min(S; \succ^*)$ instead of $S \setminus \{\min(S; \succ^*)\}$, $c(xy)$ instead of $c(\{x, y\})$, etc.

Definition 2.1. *A choice function $c : \mathcal{X} \rightarrow X$ is a choice after eliminating the normative worst (CENT) if there exists an ordered pair of strict preference rankings (\succ^*, \succ) on X such that for any menu $S \in \mathcal{X}$,*

$$c(S) = \max(S \setminus \min(S; \succ^*); \succ)$$

3. Behavioral foundation and identification

We now draw on three well-understood behavioral conditions to characterize a CENT. The first, referred to as never chosen (NC), says that if an alternative in a menu is not chosen in pairwise choice comparisons with any other alternative from that menu, then it cannot be the choice in that menu.

Axiom (NC). *For all $S \in \mathcal{X}$ and $x \in S$:*

$$[c(xy) \neq x, \forall y \in S \setminus x] \implies c(S) \neq x$$

The second, referred to as no binary cycles (NBC), requires that there are no pairwise choice cycles.

Axiom (NBC). *For all $x_1, \dots, x_{n+1} \in X$:*

$$[c(x_i x_{i+1}) = x_i, i = 1, \dots, n] \implies c(x_1 x_{n+1}) = x_1$$

The standard WARP condition fails to hold in our set-up as seen with the compromise effect example in the Introduction, which our model can accommodate. Recall that WARP imposes the following consistency on a DM's choices: for all $S, T \in \mathcal{X}$ and $x, y \in S \cap T$, $x \neq y$, if $c(S) = x$ then $c(T) \neq y$. That is, if x is chosen in the presence of y , then y is not chosen in the presence of x . If choices satisfy WARP, then they can be rationalized by a single strict preference ranking that can be uniquely elicited from these choices. In the example from the Introduction, denoting the alternatives no dessert, apple pie and chocolate chip cookie sundae by x , y and z , respectively, we see that $c(xy) = x$ and $c(xyz) = y$. That is, introducing z to the menu $\{x, y\}$ changes the choice from x to y . In the context of our model, with the DM's norms and tastes specified by, say, the preference rankings $x \succ^* y \succ^* z$ and $z \succ y \succ x$, respectively, this is so because in the menu $\{x, y\}$, y is the worst alternative according to \succ^* and gets eliminated, resulting in the choice of x . On the other hand, in $\{x, y, z\}$, z being the \succ^* -worst alternative gets

eliminated, and of the two remaining alternatives x and y , $y \succ x$; hence y is chosen. From a revealed elicitation perspective, this means that when an outside observer sees x being chosen in the presence of y , this does not necessarily imply that $x \succ y$. Instead, it may be the case that y is the worst alternative in the menu according to \succ^* and gets eliminated, with possibly $y \succ x$. We now propose an axiom that weakens WARP to acknowledge this distinction.

Axiom (WARP-CENT). For all $S, T \in \mathcal{X}$ and $x, y \in S \cap T$, $x \neq y$:

$$[x = c(S), y = c(S'), S' \subseteq S, x = c(T'), T' \subseteq T] \implies c(T) \neq y.$$

Observe that if y is chosen in some subset S' of S , then it reveals that y is not the normatively worst alternative in S' so as to get eliminated and, accordingly, neither is it in S . Since y survives elimination in S and $x = c(S)$, it can be inferred that x is preferred to y according to the DM's tastes. Likewise, $x = c(T')$, $T' \subseteq T$, reveals that x is not eliminated in T . Hence, in the presence of x , choice from T is not y .

Theorem 3.1. A choice function is a CENT iff it satisfies NC, NBC and WARP-CENT.

Proof: Please refer to Section A.1.

As it turns out, a CENT can also be characterized in terms of the acyclicity of a binary relation defined from primitive choices. To establish this, first, note that we call a menu T to be a *normatively no worse expansion* of menu S if $S \subseteq T$ and $z \in T \setminus S \implies \exists T' \subseteq T$ s.t. $c(T') = z$. In other words, when the menu S is expanded to the menu T , since each of the “new” alternatives in $T \setminus S$ is chosen in some sub-menu of T , it reveals that none of them is normatively so unacceptable as to induce prohibitive guilt in T . Hence, no normatively inferior alternatives are introduced through this expansion and we can infer that the worst alternative in S according to the DM's norms continues to be so in T .¹ With this background, consider the following binary relation on X .

- $\forall x, y \in X$, $x \neq y$, $x P_c y$ if $\exists S, T \in \mathcal{X}$, with $x, y \in S$, $x = c(S)$, and T a normatively no worse expansion of S s.t. $y = c(\hat{T})$ for some $\hat{T} \subseteq T$.

To understand the definition of P_c , first, note that since $y = c(\hat{T})$, $\hat{T} \subseteq T$, we can infer that y is not the normatively worst alternative in \hat{T} so as to get eliminated and, accordingly, neither is it in T . We can further infer that, since T is a normatively no worse expansion of S , y is not the normatively worst alternative in S as well and, hence, receives consideration in this menu. Given that $x = c(S)$, therefore, $x P_c y$ has the interpretation of a revealed preference.

Theorem 3.2. A choice function c is a CENT if and only if P_c is acyclic.

Proof: Please refer to Section A.2.

Finally, we address the issue of how uniquely the parameters underlying a CENT can be identified. In the way of notation, note that if B is a binary relation on X , then we denote the restriction of B to $S \subseteq X$ by B_S .

¹It is worth pointing out that, trivially, any menu S is a normatively no worse expansion of itself.

Proposition 3.1. *If (\succ^*, \succ) and $(\tilde{\succ}^*, \tilde{\succ})$ are both CENT representations of a choice function, then $\succ^* = \tilde{\succ}^*$ and $\succ_{X \setminus \min(X; \succ^*)} = \tilde{\succ}_{X \setminus \min(X; \tilde{\succ}^*)}$.*

Proof: Please refer to Section A.3.

In other words, the DM’s normative preferences are uniquely identified; and her tastes are identified almost uniquely with only the position of the worst alternative in X under the normative preferences being indeterminate.

4. Comments on the literature

We now compare our model with few prominent behavioral choice models in the literature, especially in the context of its implications for non-standard choices. When it comes to observed violations of rational choice theory seen in experiments and field studies, three prominent classes of violations have often been highlighted—those of no binary cycles (NBC), never chosen (NC) and always chosen (AC).² As we have seen, a CENT satisfies NBC and NC. Of the three, the only type of violation it can accommodate is that of AC. This observation immediately establishes that our model is distinct from the influential Rational Shortlist Method (RSM) or, more generally, the sequentially rationalizable model that Manzini and Mariotti (2007) introduce. An RSM satisfies AC and NC but can accommodate violations of NBC. An RSM is characterized by two axioms: weak WARP and expansion. Weak WARP is a key axiom in the behavioral choice theory literature.³ Several important models in the literature are characterized by it, e.g., the Rationalization model of Cherepanov et al. (2013), the Categorize then Choose model of Manzini and Mariotti (2012) and the Overwhelming Choice model of Lleras et al. (2017). In this context, it is worth noting that a CENT satisfies weak WARP (Refer to Section A.4 for a proof). Therefore, a CENT is a special case of these three aforementioned models. In terms of accommodating non-standard choices, all three models can accommodate violations of NC, AC and NBC. This last observation establishes that there exist choice functions that satisfy weak WARP but are not a CENT. Another prominent model in the literature involving choice from consideration sets is the Choice with Limited Attention (CLA) model of Masatlioglu et al. (2012). This model is characterized by the WARP(LA) axiom.⁴ A CENT satisfies WARP(LA) and, hence, is a special case of a CLA (Refer to Section A.4 for a proof). Finally, we would like to point out that another model that accommodates violations of AC but not those of NBC and NC is the Two-Stage Chooser model of Bajraj and Ülkü (2015). It can be shown that neither of these models are special cases of one another.

²A choice function $c : \mathcal{X} \rightarrow X$ satisfies AC if for all S and $x \in S$, $c(xy) = x, \forall y \in S \setminus \{x\} \implies c(S) = x$. Manzini and Mariotti (2007) show that all violations of WARP can be categorized as either violations of AC or violations of NBC (or both).

³A choice function $c : \mathcal{X} \rightarrow X$ satisfies weak WARP if for all $S, T \in \mathcal{X}$ and $x, y \in X$, $\{x, y\} \subseteq S \subseteq T$, $x = c(xy) = c(T) \implies y \neq c(S)$.

⁴A choice function $c : \mathcal{X} \rightarrow X$ satisfies WARP(LA) if for any S , there exists $x^* \in S$ such that, for any T including x^* , if $c(T) \in S$ and $c(T) \neq c(T \setminus x^*)$, then $c(T) = x^*$.

A Appendix

A.1 Proof of Theorem 3.1

Necessity: Let $c : \mathcal{X} \rightarrow X$ be a CENT with (\succ^*, \succ) as the ordered pair of strict preference rankings. We show below that c satisfies NC, NBC and WARP-CENT.

NC: Consider $x \in S$, s.t., $x \neq c(xy)$, for all $y \in S \setminus x$. This implies that $y \succ^* x$, for all $y \in S \setminus x$ and, accordingly, $x = \min(S; \succ^*)$. Therefore, $x \neq c(S)$.

NBC: Consider $x_1, \dots, x_{n+1} \in X$ with $c(x_i x_{i+1}) = x_i, i = 1, \dots, n$. This implies that $x_i \succ^* x_{i+1}$, for all $i = 1, \dots, n$. Since, \succ^* is transitive, it follows that $x_1 \succ^* x_{n+1}$. Accordingly, $c(x_1 x_{n+1}) = x_1$.

WARP-CENT: Let $x, y \in S \cap T, x \neq y$, be s.t. $x = c(S), y = c(S'), S' \subseteq S$, and $x = c(T'), T' \subseteq T$. Since $y = c(S')$, it follows that there exists $z \in S'$ such that $y \succ^* z$ and, accordingly, $y \neq \min(S; \succ^*)$. Hence, $x \succ y$. Similarly, since $x = c(T')$, a similar argument establishes that $x \neq \min(T; \succ^*)$ and, hence, $x \in T \setminus \min(T; \succ^*)$. Now, if $y \in T \setminus \min(T; \succ^*)$, since $x \succ y$, we can conclude that $y \neq c(T)$. Of course, if $y \notin T \setminus \min(T; \succ^*)$, the conclusion is obvious.

Sufficiency: Let $c : \mathcal{X} \rightarrow X$ satisfy NC, NBC and WARP-CENT. We show below that we can identify strict preference rankings \succ^* and \succ on X such that with respect to the ordered pair (\succ^*, \succ) , c is a CENT.

Define $\succ^* \subseteq X \times X$ as follows: for any $x, y \in X, x \neq y, x \succ^* y$ if $x = c(xy)$. We establish that \succ^* is a strict preference ranking, i.e., \succ^* is:

Total: $c(xy) \neq \emptyset$, for all $x, y \in X, x \neq y$. Thus, either $x \succ^* y$ or $y \succ^* x$.

Asymmetric: Suppose, towards a contradiction, $x \succ^* y$ and $y \succ^* x$. Then by definition, $x = c(xy)$ and $y = c(xy)$!

Transitive: Let $x \succ^* y$ and $y \succ^* z$. This implies $x = c(xy)$ and $y = c(yz)$. Since c satisfies NBC, it follows that $x = c(xz)$. Hence $x \succ^* z$.

Define $\succ \subseteq X \times X$ as follows: for any $x, y \in X, x \neq y, x \succ y$ if either (i) there exists $S \in \mathcal{X}$ with $x, y \in S$ such that $x = c(S)$ and $y = c(S')$, for some $S' \in \mathcal{X}, S' \subseteq S$; or (ii) $y \neq c(S)$ for any $S \in \mathcal{X}$. We establish that \succ is a strict preference ranking, i.e., \succ is:

Total: First note that, since X is a finite set and c satisfies NBC, there exists a unique alternative, call it \underline{z} , such that $c(\underline{z}z) \neq \underline{z}$ for all $z \in X \setminus \underline{z}$. Now take any $x, y \in X, x \neq y$. First, consider the case $x, y \neq \underline{z}$ and the menu $\{x, y, \underline{z}\}$. Since, $x = c(x\underline{z})$ and $y = c(y\underline{z})$, by NC, we know that $c(xy\underline{z}) \neq \underline{z}$. If $c(xy\underline{z}) = x$, then $x \succ y$, otherwise $y \succ x$. Next, consider the case that one of x or y , wlog say y , is \underline{z} . Accordingly, since $c(yz) \neq y$, for any $z \in X \setminus y$, by NC it follows that there exists no $S \in \mathcal{X}$ such that $c(S) = y$. Hence, $x \succ y$. This establishes that \succ is total.

Asymmetric: Suppose $x \succ y$. Clearly, $x \neq \underline{z}$ since by NC there exists no $S \in \mathcal{X}$ such that $c(S) = \underline{z}$, and \underline{z} is the only alternative for which this is true. On the other hand, if $y = \underline{z}$, then for the same reason, $\neg[y \succ x]$. Now consider the case $y \neq \underline{z}$. Then, $x \succ y$ implies that there exists $S \in \mathcal{X}$ with $x, y \in S$ such that $x = c(S)$ and $y = c(S')$, for some $S' \in \mathcal{X}$,

$S' \subseteq S$. WARP-CENT then implies that there does not exist $T \in \mathcal{X}$ with $x, y \in T$, such that $y = c(T), x = c(T')$, for some $T' \in \mathcal{X}, T' \subseteq T$. Hence, $\neg[y \succ x]$.

Transitive: Let $x \succ y$ and $y \succ z$, for some $x, y, z \in X$. Clearly, by the argument made above, $x, y \neq \underline{z}$. Further, if $z = \underline{z}$, then clearly $x \succ z$ and our desired conclusion is immediate. So, assume $z \neq \underline{z}$. Now, consider the menu $\{x, y, z, \underline{z}\}$. We know that $a = c(a\underline{z})$, for $a = x, y, z$. Therefore, by NC, $\underline{z} \neq c(xyz\underline{z})$. Further, since $x = c(x\underline{z})$ and $y = c(y\underline{z})$, by WARP-CENT it follows that $y, z \neq c(xyz\underline{z})$. Hence, $x = c(xyz\underline{z})$ and it follows that $x \succ z$.

To show: (\succ^*, \succ) is a CENT representation of c .

Pick any menu $S \in \mathcal{X}$ and let $x = c(S)$. Since \succ^* , as shown above, is a preference ranking, there exists a unique \succ^* -worst alternative in S , denote it by $\min(S; \succ^*)$. First, consider the case when $|S| = 2$, i.e., $S = \{x, y\}$ for some $y \neq x$. By the definition of \succ^* , it follows that $x \succ^* y$; therefore, $\{x\} = S \setminus \min(S; \succ^*)$ and the desired conclusion follows. Next, consider the case $|S| > 2$. Since c satisfies NC, there exists $z \in S$, such that, $c(xz) = x$. This implies $x \succ^* z$. Thus, $x \in S \setminus \min(S; \succ^*)$. Now consider any $y \in S \setminus \min(S; \succ^*), y \neq x$; i.e., $y \succ^* z$ or, equivalently, $y = c(yz)$, for some $z \in S$. In other words, there exists $S' \subseteq S$ such that $c(S') = y$. Hence, $x \succ y$ and we have,

$$c(S) = x = \max(S \setminus \min(S; \succ^*); \succ)$$

A.2 Proof of Theorem 3.2

It is straightforward to establish that if c is a CENT, then P_c is acyclic. To establish the converse, we show that P_c acyclic implies that c satisfies NBC, NC and WARP-CENT.

To show that c satisfies NBC, we prove the contrapositive, i.e., if c violates NBC then P_c is not acyclic. The proof is by induction on the number of alternatives involved in the NBC violation, denote this number by k . First, consider the case of $k = 3$. Let $c(x_1x_2) = x_1, c(x_2x_3) = x_2, c(x_1x_3) = x_3$ and wlog suppose $c(x_1x_2x_3) = x_1$. The menu $\{x_1, x_2, x_3\}$ is a normatively no worse expansion (NNWE) of itself. Hence, $c(x_1x_2x_3) = x_1$ and $c(x_1x_3) = x_3$ implies that $x_1 P_c x_3$. Now w.r.t. the menu $\{x_1, x_3\}$, note that $\{x_1, x_2, x_3\}$ is a NNWE since $c(x_2x_3) = x_2$. Further, $c(x_1x_3) = x_3$ and $c(x_1x_2x_3) = x_1$ together imply that $x_3 P_c x_1$. Hence, P_c is not acyclic and the desired conclusion follows for $k = 3$. Now suppose the result has been proven for $k = n - 1$. We wish to prove it for $k = n$. To that end, suppose $c(x_1x_2) = x_1, c(x_2x_3) = x_2, \dots, c(x_{n-1}x_n) = x_{n-1}$ and $c(x_1x_n) = x_n$. Now, either (a) $c(x_1x_3) = x_3$ or (b) $c(x_1x_3) = x_1$. If (a), then $c(x_1x_2) = x_1, c(x_2x_3) = x_2, c(x_1x_3) = x_3$ and the conclusion that P_c is not acyclic follows from the case of $k = 3$. If (b), then $c(x_1x_3) = x_1, c(x_3x_4) = x_3, \dots, c(x_{n-1}x_n) = x_{n-1}$ and $c(x_1x_n) = x_n$. This is a violation of NBC with $n - 1$ alternatives and the conclusion that P_c is not acyclic follows from the case of $k = n - 1$.

To show that c satisfies NC, we again prove the contrapositive, i.e., if c violates NC then P_c is not acyclic. So assume that for some menu S and $x \in S, x \neq c(xy)$ for all $y \in S \setminus x$ and $c(S) = x$. Consider the menu $\{x, \hat{y}\}$ with $c(x\hat{y}) = \hat{y}$, for some $\hat{y} \in S \setminus x$. It is straightforward to verify that S is a NNWE of $\{x, \hat{y}\}$, since $c(xy) = y$ for any $y \in S \setminus \{x, \hat{y}\}$. Since $c(x\hat{y}) = \hat{y}$ and $c(S) = x$, it follows that $\hat{y} P_c x$. On the other hand,

S is a NNWE of itself. Hence, $c(S) = x$ and $\hat{y} = c(x\hat{y})$ together imply that $xP_c\hat{y}$ and brings us to the conclusion that P_c is not acyclic.

Finally, suppose c violates WARP-CENT, i.e., $\exists S, T \in \mathcal{X}$ and $x, y \in S \cap T$, $x \neq y$ s.t., $x = c(S), y = c(S')$, for some $S' \subseteq S$, and $y = c(T), x = c(T')$, for some $T' \subseteq T$. Since, S and T are NNWE-s of themselves, we have xP_cy and yP_cx , violating acyclicity of P_c .

A.3 Proof of Proposition 3.1

Let (\succ^*, \succ) and $(\tilde{\succ}^*, \tilde{\succ})$ be two CENT representations of a choice function c . Then, for any $x, y \in X$, $x \neq y$,

$$x \succ^* y \iff x = c(xy) \iff x \tilde{\succ}^* y$$

Further, for any $x, y \in X$, $x \neq y$, $x, y \neq z := \min(X; \succ^*) = \min(X; \tilde{\succ}^*)$,

$$x \succ y \iff x = c(xyz) \iff x \tilde{\succ} y$$

A.4 Claims in Section 4

Proposition A1. *A CENT satisfies weak WARP.*

Proof. Let c be a CENT with (\succ^*, \succ) as the ordered pair of strict preference rankings. Further, let $\{x, y\} \subseteq S \subseteq T$ and $x = c(xy) = c(T)$. $x = c(xy)$ implies $x \succ^* y$. If $y = \min(S; \succ^*)$, then clearly $y \neq c(S)$. Alternatively, if $y \neq \min(S; \succ^*)$, then $y \neq \min(T; \succ^*)$; and $x = c(T)$ implies $x \succ y$. Further, since $x \succ^* y$, $x \neq \min(S; \succ^*)$. Hence, $y \neq c(S)$. \square

Proposition A2. *A CENT satisfies WARP(LA).*

Proof. To show this, we draw on Lemma 1 in Masatlioglu et al. (2012) that establishes that a choice function c satisfies WARP(LA) iff the binary relation \tilde{P} on X defined next is acyclic: $x\tilde{P}y$ if there exists $S \in \mathcal{X}$ s.t. $x = c(S) \neq c(S \setminus y)$. Let c be a CENT with (\succ^*, \succ) as the ordered pair of strict preference rankings and consider $x_1, \dots, x_n \in X$ s.t. $x_i\tilde{P}x_{i+1}$, for $i = 1, \dots, n-1$. $x_i\tilde{P}x_{i+1}$ implies that there exists S_i s.t. $x_i = c(S_i) \neq c(S_i \setminus x_{i+1})$. This implies that $x_{i+1} = \min(S_i; \succ^*)$ and, accordingly, $x_i \succ^* x_{i+1}$, for all $i = 1, \dots, n-1$. Since \succ^* is transitive, we have $x_1 \succ^* x_n$. This means there does not exist S s.t., $x_n = c(S) \neq c(S \setminus x_1)$ for this would imply that $x_n \succ^* x_1$. Thus, $\neg[x_n\tilde{P}x_1]$ and, hence, \tilde{P} is acyclic. \square

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