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CES technology and comovement problem

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Abstract

One of the stylized facts in open economies is countercyclical trade balance. In this paper, I develop and estimate a canonical small open-economy (SOE) model that features the constant elasticity of substitution (CES) production function. The estimation result reveals that the model can deliver the countercyclical trade balance. This paper also finds that the model can produce a negative response of hours to a productivity shock, which is in line with early studies. These results are corroborated by business-cycles statistics based on actual data.

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1. Introduction

Countercyclical behavior of trade balance and hours following a productivity shock is an important business-cycle characteristic corroborated by early studies.[1] It can be explained by a channel that a rise in overall income during booms makes people enjoy more consumption by reducing labor and leads countries to import more than they export. Despite the importance of the productivity as a driving force of business cycles, it has proven challenging to deliver the result in the context of standard small open-economy (SOE) models with the Cobb-Douglas (CD) technology since either trade balance or labor shows procyclical behavior.[2]

In this paper, I take a fresh approach to delivering this result. To this end, I develop a SOE model that features the constant elasticity of substitution (CES) technology and estimate it using the Bayesian technique.[3] To substantiate the empirical validity of the CES technology, I also estimate a standard SOE model with the CD technology to compare these two models.[4] Estimated parameters from these two models enable me to explore different dynamic responses of key macroeconomic aggregates and different business-cycle properties.

Main results reveal that actual data give an endorsement of the CES technology as the aggregate production function based on a statistic of model fit comparison. Furthermore, the estimated elasticity of substitution of 0.51 also validates this result, implying that it lends credence to the literature that the aggregate production function is better characterized by the CES technology.[5] The analysis on dynamic behavior of key macroeconomic variables shows that CES model can deliver the right comovement by producing the countercyclical movements of both trade balance and hours, whereas CD model fails to produce them. This result from CES model is also documented by implied business-cycle statistics computed by actual data. Furthermore, output in CES model remains lower for 5 periods and then higher after that period than CD model. This result comes out of dynamic movements in consumption and hours in CES model.

To the best of my knowledge, this paper is the first to show that a SOE model with the CES technology can resolve this comovement problem. In this regard, the use of the CES technology in various models is able to help explain and solve various macroeconomic problems.

The rest of this paper is organized as follows. Section 2 lays out the SOE model that features the CES technology. Section 3 discusses the results by highlighting differences between two models. Section 4 concludes.

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[1] See, for example, Backus et al. (1994), Aguiar and Gopinath (2007), and Martínez-García and Sondergaard (2009) for countercyclical trade balance, and Galí (1999), Francis and Ramey (2005), Tervala (2007), Rebei (2014) and Choi (2017) for the negative response of hours to a productivity shock.

[2] In particular, hours typically show procyclical behavior. For instance, though Schmitt-Grohé and Uribe (2003) present countercyclical trade balance using typical SOE models, the same models deliver procyclical hours. This is the typical result from standard SOE models.

[3] Martínez-García and Sondergaard (2009) argue that a two-country New Keynesian style-model with price stickiness does not seem to produce better performance than the flexible-price two-country model used in Backus et al. (1994). It implies that a prototypical flexible-price SOE model that features the CES technology merits an analysis to test whether it can deliver countercyclical trade balance and hours.

[4] For brevity, I will refer to the model with the CES (CD) technology as CES (CD) model henceforth.

[5] A mass of previous studies supports this result (e.g., Acemoglu 2002, 2003; Antrás 2004; Chirinko 2008). A merit of the CES function, compared to the CD one, is that it allows the elasticity of substitution to vary in an admissible range.

2. The model

The model economy consists of two private sectors, households and firms. The representative household derives utility from consumption, c_t, and disutility from labor, h_t, by maximizing

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{\mu}\left(1-h_t\right)^{1-\mu}\right]^{1-\gamma}}{1-\gamma}, \tag{1}$$

subject to a budget constraint

$$c_t + x_t + (1+r_{t-1})d_{t-1} + z_t = w_t h_t + u_t k_t + d_t, \tag{2}$$

where β determines the degree of patience, γ is the curvature parameter that represents relative risk aversion, and μ is the relative weight that the household assigns to consumption. The variables, x_t, k_t, w_t, u_t and d_t, denote investment, capital, real wage, real rental price of capital and foreign debt, respectively. The household can borrow from international financial markets in period *t* at an interest rate $r_t = r^* + \phi(e^{d_t - d} - 1)$, where r^* and ϕ denote the world real interest rate and the degree of risk premium.[6]

Government receives a lump-sum tax, z_t, from the household and spends it in the form of government spending, g_t. The government budget constraint is then given by $z_t = g_t$, and government spending obeys an exogenous stochastic process, $\ln g_t = \rho_g \ln g_{t-1} + \sigma_g \varepsilon_t^g$, where $\varepsilon_t^g \sim N(0,1)$.

Accumulating capital is costly, and an evolution of capital including this component is given by

$$k_{t+1} = x_t + (1-\delta)k_t - \frac{\kappa\left(k_{t+1} - k_t\right)^2}{2}, \tag{3}$$

where δ is the depreciation rate, and κ is the intensity of capital adjustment cost. The trade balance, tb_t, is given by

$$tb_t = y_t - c_t - x_t - g_t - \frac{\kappa\left(k_{t+1} - k_t\right)^2}{2}. \tag{4}$$

The representative profit-maximizing firm produces output using labor and capital under the CES technology with labor-augmenting technological change.[7] The production function is

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[6] Schmitt-Grohé and Uribe (2003) propose several alternative methods to close SOE models, and I use the external debt-elastic interest rate premium.

[7] Acemoglu (2002, 2003) stresses out the importance of this type of technological change for balanced growth path in growth models.

then given by

$$y_t = \left[\eta k_t^{\frac{\theta-1}{\theta}} + (1-\eta)\left(a_t^h h_t\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where y_t is output, η is the distribution parameter, θ is the elasticity of substitution between labor and capital, and a_t^h is a productivity shock that obeys a first-order process given by $\ln a_t^h = \rho_{a^h} \ln a_{t-1}^h + \sigma_{a^h} \varepsilon_t^{a^h}$, where $\varepsilon_t^{a^h} \sim N(0,1)$. It contains Leontief ($\theta \to 0$), CD ($\theta \to 1$) and linear ($\theta \to \infty$) production function as special cases. Using the household's budget constraint and the optimal choice of labor and capital from the firm's problem yields the aggregate resource constraint

$$c_t + x_t + g_t + (1+r_{t-1})d_{t-1} = y_t + d_t. \tag{6}$$

3. Quantitative analysis

Taking a likelihood-based approach to estimating a subset of important parameters is a better choice than the calibration method in this paper since it enables me to assess which model is preferred by actual data. Furthermore, another advantage of estimating CES model is to offer an estimated elasticity of substitution.[8] I, therefore, estimate CES model using the Bayesian technique.[9] Though the estimated elasticity of substitution that differs from unity does not have its peril to assess that the aggregate production function is characterized by the CES technology, presenting formal statistical evidence by estimating CD model can substantiate the result. Thus, I also estimate CD model using the same method and compare empirical fit using log marginal likelihood, Bayes factor and posterior model probability.[10]

The data that range from 1995:1 to 2019:4 are obtained from Bank of Korea.[11] To make a correct comparison, I use the same data set including two observables, HP-filtered per-capita real output and per-capita real government spending, to estimate two models.[12] The parameters that are not estimated are calibrated in a standard fashion as follows: R=1.01, β=0.99, δ=0.025, d/y=0.27 and g/y=0.13.[13]

Table 1 presents estimation results. The number without parenthesis denotes the result from CES model, while that inside parenthesis indicates the result from CD model. The top panel of Table 1 exhibits empirical performance based on log marginal likelihood, Bayes factor and

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[8] Calibration is another useful way to feed parameter values into the model. Since one of the linchpins of this paper is to compare which model is chosen by actual data, however, this paper uses the estimation technique.

[9] I use the random walk Metropolis-Hastings (HM) algorithm to sample from the posterior distribution. I create a sample of 500,000 draws and remove the first 100,000 draws.

[10] Rabanal and Rubio-Ramírez (2005) use log marginal likelihood and Bayes factor. I also report posterior model probability as complementary evidence.

[11] The fact that quarterly population data are available from 1995:1 leads me to set this data range.

[12] The two-sided HP-filter technique with the smoothing parameter of 1,600 is used to compute cyclical components of the variables.

[13] β is the inverse of the U.S. real gross interest rate as a proxy of the world real interest rate, which is computed by using the U.S. 3-month Treasury Bill rate and GDP deflator. The data source is the Federal Reserve Economic Data (FRED) of the Federal Reserve. The variables *d/y* and *g/y* are computed as a long-run average of Korean national debt over GDP from 1997 to 2019 and that of government spending over GDP from 1995 to 2019.

posterior model probability. All the statistics suggest that the aggregate production function for Korea is better characterized by the CES technology. Furthermore, the estimated elasticity of substitution that is decidedly below unity (complementarity between two inputs) also stands up for the CES technology as the aggregate production function.[14]

In addition to this main result of this paper, a noticeable feature of the estimation result deserves mention. The estimated parameters in both models show a relatively larger difference than those of the shock processes. For instance, the estimated parameters of curvature and capital adjustment cost function in CES model are 1.61 and 5.66, whereas those in CD model are 1.35 and 4.80. In contrast, the estimated parameters of persistence and standard deviation of productivity shock in CES model are 0.96 and 0.12, while those in CD model are 0.94 and 0.13. Despite the difference between these models, the overall estimation result from both models is quite close to perceived values in this literature.

Table 1: Estimation result

Model fit comparison	CES model	CD model
Log marginal likelihood	271.26	229.61
Bayes factor	1.00	exp (41.65)
Posterior model probability	1.00	0.00

Bayesian estimation	Prior distribution			Posterior distribution		
	Density	Mean	SD	Mean	5%	95%
γ	Gamma	2.00	0.50	1.61 (1.82)	1.35 (0.96)	1.83 (2.68)
μ	Beta	0.30	0.05	0.27 (0.25)	0.22 (0.17)	0.32 (0.33)
κ	Gamma	5.00	0.50	5.66 (5.09)	4.80 (4.08)	6.47 (6.11)
ϕ	Gamma	0.10	0.05	0.13 (0.11)	0.09 (0.03)	0.16 (0.20)
η	Beta	0.30	0.05	0.46 (0.31)	0.43 (0.22)	0.49 (0.40)
θ	Gamma	1.00	0.50	0.51	0.29	0.78
ρ_g	Beta	0.80	0.10	0.83 (0.78)	0.67 (0.61)	0.95 (0.92)
ρ_{a^h}	Beta	0.80	0.10	0.96 (0.94)	0.89 (0.89)	0.99 (0.98)
σ_g	Igamma	1.00	3.00	0.13 (0.13)	0.12 (0.12)	0.13 (0.15)
σ_{a^h}	Igamma	1.00	3.00	0.12 (0.13)	0.12 (0.12)	0.13 (0.13)

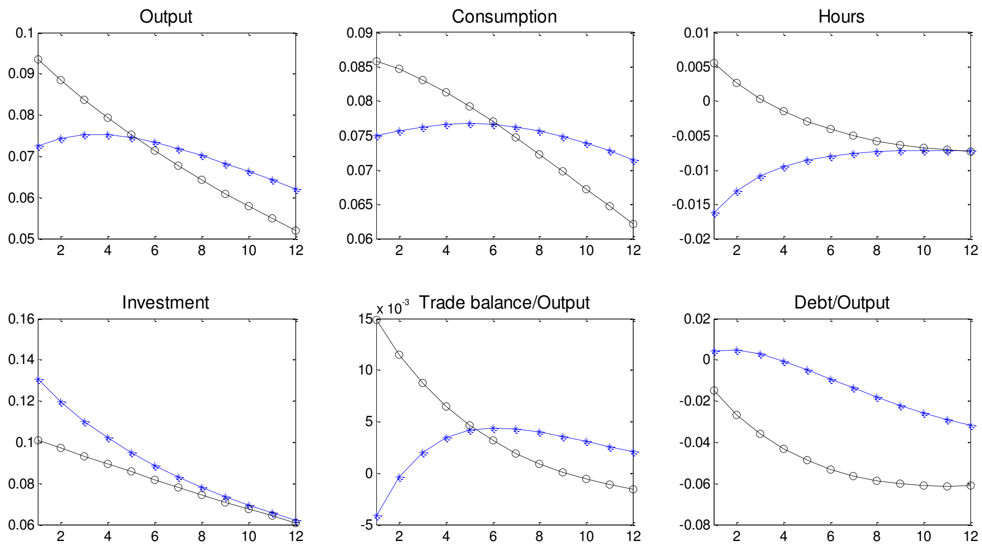
Note: Igamma and SD denote the inverse gamma and standard deviation. The value without parenthesis indicates the result from CES model, whereas that inside parenthesis denotes the result from CD model.

Figure 1 plots dynamic impacts conditional on one standard deviation of productivity shock on the main macroeconomic variables, which are expressed in percent deviation from the steady-state value of each model. The line with asterisk (circle) denotes the result from CES (CD) model. A glance at Figure 1 implies markedly different results across two models. In particular, transitional path of trade balance and hours in CES model stands in contrast to that in CD model: The dynamic response of these two variables in CES model shows an immediate decline following the shock, whereas the same variables jump on impact in CD model. In CES model with complementarity, a rise in the productivity of labor reduces the relative marginal product of labor, compared to that of capital. It increases the demand for capital by more than the demand for labor, resulting in excess demand for capital. It, therefore, gives an incentive

[14] The absence of the estimated CES parameter with its confidence interval in CD model stems from the fact that CD model does not contain the CES parameter because it is unity.

for firms to purchase capital via more investment. This result is illustrated by dynamic behavior of investment in Figure 1: On impact, investment in CES model shows a higher rise than that in CD model during the transition period. Another important result from Figure 1 is that output in CES model intersects that in CD model in period 5. More specifically, output in CES model remains lower for 5 periods and then higher after that period than CD model. This result comes out of dynamic movements in consumption and hours in CES model. Consumption (the largest fraction of GDP) in CES model shows a smaller rise than that in CD model during the first 5 periods and then it stays higher in CES model. Moreover, hours (an essential input for production) in CES model show an immediate drop but gradually rise as time goes on, which is an opposite movement to those in CD model.

Figure 1: Dynamic response of key macroeconomic aggregates



Note: The result is the impulse response functions conditional on one standard deviation of the productivity shock. The line with asterisk and circle denotes the result from CES and CD model, respectively.

Though the graphical analysis in Figure 1 may be enough to account for countercyclical behavior of both trade balance and hours in CES model, formal statistical evidence can reinforce the result. Table 2 offers correlation coefficients predicted by these models and those estimated by actual data.[15] Since the presence of two shocks in two models is likely to yield mixed results, I also estimate and simulate the models by isolating the productivity shock (i.e., models with only the productivity shock without the government spending shock). In Table 2, the value without parenthesis denotes the result from models with two shocks, whereas that inside parenthesis indicates the result from models with the isolated productivity shock only.

Casual inspection of Table 2 suggests that irrespective of whether the government spending shock is included, CES model does a reasonably good job of matching predicted correlations with actual ones based on negative correlation of output with trade balance and hours. In the case of models with two shocks (one shock), for example, trade balance over GDP and hours in CES model show -0.32 and -0.93 (-0.14 and -0.29), whereas those in CD model are 0.96 and

[15] Simulated data are created by 10,000 simulations with 100 time-series observations each, and these are converted to HP-filtered variables to compute mean correlation coefficients.

0.78 (0.78 and 0.99). Moreover, the relationship between productivity and other variables generated by CES model shows negative correlation of productivity with trade balance and hours, which is in line with actual data. For instance, correlation of these two variables with productivity in CES model with two shocks (one shock) is -0.42 and -0.97 (-0.57 and -0.69), respectively. In contrast, CD model with two shocks (one shock) yields the result of 0.96 and 0.80 (0.74 and 0.98), which is decidedly different from the actual correlation coefficients.

Table 2: Observed and implied business-cycle statistics

	$Corr\left(\frac{tb_t}{y_t}, y_t\right)$	$Corr(h_t, y_t)$	$Corr(c_t, y_t)$	$Corr(x_t, y_t)$	$Corr(y_t, y_t)$
Data	-0.66	-0.37	0.89	0.81	1.00
CES model	-0.32 (-0.14)	-0.93 (-0.29)	0.99 (0.96)	0.98 (0.65)	1.00 (1.00)
CD model	0.96 (0.78)	0.78 (0.99)	0.99 (0.99)	0.99 (0.99)	1.00 (1.00)
	$Corr\left(\frac{tb_t}{y_t}, a_t^h\right)$	$Corr(h_t, a_t^h)$	$Corr(c_t, a_t^h)$	$Corr(x_t, a_t^h)$	$Corr(y_t, a_t^h)$
Data	-0.41	-0.57	0.63	0.50	0.76
CES model	-0.42 (-0.57)	-0.97 (-0.69)	0.99 (0.97)	0.98 (0.92)	0.99 (0.89)
CD model	0.96 (0.74)	0.80 (0.98)	0.99 (0.99)	0.99 (0.99)	0.99 (0.99)

Note: The value without parenthesis is the result from the model with two shocks (i.e., productivity and government spending shocks), whereas that inside parenthesis is the result from the model with the isolated productivity shock (i.e., productivity shock only).

4. Conclusion

Actual data suggest that trade balance and hours are negatively correlated with output and productivity. Despite their widespread use in macroeconomics, however, standard SOE models with the CD technology have difficulty producing the comovement observed in the actual data. In this paper, I formulate a SOE model with the CES technology and estimate the model on Korean data to document that it can successfully deliver countercyclical behavior of both trade balance and hours that is in line with the actual data.

The result that the use of the CES technology can generate the right comovement implies that models with this type of technology can pave the way for more interesting research that can help explain a wide swathe of macroeconomic phenomena.

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