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Risk sharing and asset commonality in the financial sector

Kenta Toyofuku
Nihon University

Abstract

This paper develops a model to explain why asset commonality was pervaded among financial intermediaries before the global financial crisis. We show that when there is a lack of opportunities for financial intermediaries to hedge idiosyncratic shocks, they shift their asset allocation to commonly accessible assets not only to diversify their own portfolios but also to share the risk among themselves. We also show that this asset commonality arises even if the commonly held assets are risky and unproductive. In this sense, asset commonality in the financial sector can mitigate the risk of each financial intermediary but may decrease aggregate output in the economy.

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Contact: Kenta Toyofuku - toyofuku.kenta@nihon-u.ac.jp.
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1 Introduction

Following the global financial crisis of 2007–09, policy makers and researchers began paying more attention to the contagion caused by asset commonality in the financial sector. Recent empirical studies have documented evidence that prior to the global financial crisis, the phenomenon of asset commonality or portfolio overlap had pervaded financial intermediaries and induced systemic risk in the financial sector (e.g., Billio et al. 2012, Blei and Ergashev 2014, Siedlarek and Fritsch 2019)[1]. As asset portfolios of financial intermediaries become similar and interconnected, they are directly exposed to common asset-side shocks. Thus, declines in asset prices or unexpected loan defaults can damage their balance sheets simultaneously and trigger contagion in the financial sector[2].

The theoretical literature emphasizes that asset commonality among financial intermediaries arises from portfolio diversification of each financial intermediary (e.g., Ibragimov et al. 2011, Beale et al. 2011, Liu 2018, Cai et al. 2018, Kopytov 2019)[3]. In contrast, we characterize the phenomenon of asset commonality as a consequence of not only the portfolio diversification of each financial intermediary but also risk sharing among the portfolios of the financial intermediaries.

We outline the model as follows. There are two periods and two types of players: agents and buyers. The agents have two investment opportunities to produce a commodity. One is an agent-specific investment (hereafter, a local investment) and the other is a general investment that is commonly accessible to all agents (hereafter, a general investment). In this sense, agents can be thought of as local banks issuing both local and universal loans or entrepreneurs undertaking both firm-specific investments and generic investments. In period 0, each agent allocates its endowment between local and general investments. In this paper, the proportion of the endowment allocated to a general investment represents the degree of asset commonality among agents. At the beginning of period 1, productivity shocks to both investments are realized. While a productivity shock to a general investment is common to all agents, the shock to a local investment is idiosyncratic so that the productivity of agents may differ *ex post*. Finally, agents sell the commodity to buyers at the end of period 1. We introduce two market frictions into the model. The first is an incomplete market in which each agent can access only one local investment so that agents cannot fully diversify their investment portfolio. The second is that the number of agents is finite so that we allow an oligopolistic commodity market.

[1]Cai et al. (2018) and Kopytov (2019) show that overlap in syndicated loan portfolios and securitization increased in the early and mid-2000s.

[2]This asset-based contagion is different from debt-based contagion in which the coordinated behavior of debt holders triggers the systemic risk (see Diamond and Dybvig 1983, Rochet and Vives 2004, and others).

[3]Allen et al. (2012) show that funding the maturity structure of financial intermediaries also affects the interaction between asset commonality and diversification of their portfolios.

First, we derive the competitive market equilibrium as a benchmark case and show that agents allocate more goods to a general investment as the productivity shocks become large. This result implies that diversification of each agent induces asset commonality among agents. Next, if the commodity market becomes oligopolistic so that each agent has pricing power, agents allocate more goods to a general investment to further share the risk among themselves even if it is less productive and riskier than a local investment. This result implies that the asset commonality among agents benefits the payoff of agents but decreases aggregate output in the economy.

The reminder of this paper is organized as follows. Section 2 develops the model. Section 3 describes the social optimal allocation. Section 4 shows the competitive market equilibrium. Section 5 shows the oligopolistic market equilibrium and shows that risk sharing among agents implies asset commonality. Section 6 concludes.

2 The model

There are two periods, 0 and 1, and there is no time discounting. There are two types of players: agents and buyers. We discuss agents, buyers, and the market, in that order.

Agents: There are N agents, where $N > 1$ and N is a finite integer. Agents are indexed by $i(i = 1, \cdots N)$. Each agent is endowed with one unit of nonstorable good in period 0. Agents can generate a commodity in period 1 by investing the good in period 0. Agents are identical *ex ante* except for their investment opportunities as shown below.

A representative agent i has access to two risky investment opportunities.

- Local investment: A local investment i is one that can be accessed only by an agent i. If a representative agent i invests L_i units of the good in period 0, where $L_i \in [0, 1]$, it generates $\gamma R_i L_i$ units of the commodity in period 1, where γ denotes the productive efficiency of the investment and $\gamma \geq 1$, and R_i denotes an idiosyncratic productivity shock of investment i such that:

$$R_i = \begin{cases} R & \text{with probability } 1/2, \\ 2 - R & \text{with probability } 1/2, \end{cases}$$

  where $0 < R < 1$. R_i is i.i.d. across local investments. Let $\bar{R} \equiv (R_1, \cdots R_N)$ denote the vector of productivity shocks.

- General investment: A general investment is one that can be accessed by all agents. If a representative agent i invests G_i units of the good in period 0, where $G_i \in [0, 1]$, it generates $R_\omega G_i$ units of the commodity in period 1 where R_ω denotes a productivity shock of the investment such that:

$$R_\omega = \begin{cases} R & \text{with probability } 1-\pi, \\ 2-R & \text{with probability } \pi, \end{cases}$$

where $0 < \pi \leq 1/2$[4].

If agents are local banks, a local investment represents a loan to a local firm and a general investment represents a loan to a firm operating extensively in a country or a syndicated loan. From these settings, it is assumed that a general investment is less productive (because $\gamma \geq 1$) and riskier (because $\pi \leq 1/2$) than a local investment.

The resource constraint for a representative agent i can be written as:

$$L_i + G_i = 1. \tag{1}$$

In period 1, productivity shocks to both investments are realized and the output of the investments, a commodity, is produced. Then, the amount of the commodity that a representative agent i produces, $Y_i(R_i, R_\omega)$, is denoted by

$$Y_i(R_i, R_\omega) = \gamma R_i L_i + R_\omega G_i. \tag{2}$$

Agents sell the commodity to buyers in return for receiving the good. Agents are risk averse and derive utility only from consuming the good. The value of utility from consumption for a representative agent i in period 1, U_i, is denoted by

$$U_i = \ln C_i(\bar{R}, R_\omega),$$

where C_i is consumption of the good by a representative agent i.

Buyers: There is a measure one of identical buyers who are indexed by $j \in [0,1]$. Buyers enter the economy in period 1. Each buyer is endowed with M units of the good. Buyers are risk neutral and derive utility only from consuming the commodity. The value of utility from consumption for a representative buyer j in period 1, V_j, is simply denoted by

$$V_j = A_j(\bar{R}, R_\omega),$$

where A_j is the consumption of the commodity by a representative buyer j[5].

Market: In period 1, the market for the commodity opens. Agents sell the commodity in return for receiving the good from buyers. In this sense, the good is not only used as *a medium of exchange*, but also serves as *a unit of account* in the transaction. Let $P(\bar{R}, R_\omega)$ be the price of the commodity in terms of the good. The transactions in period 1 are shown in Figure 1.

[4]Our results generalize to a more general distribution of R_i and R_ω. This particular distribution is adopted to simplify the exposition.

[5]This assumption is intended for simplicity of analysis. However, even if we consider the different utility functions of buyers, our main results still hold.

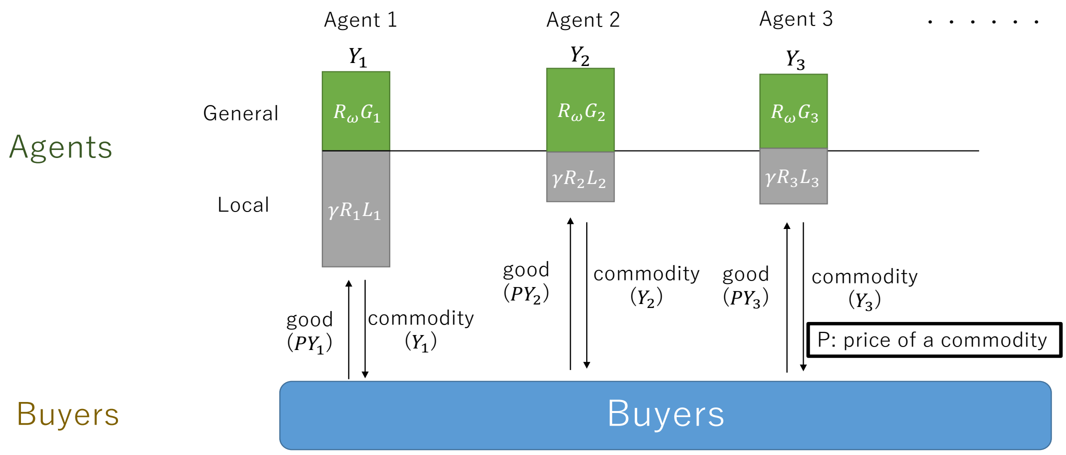

Figure 1: Transactions in period 1

3 Socially optimal allocation

First, we consider the socially optimal allocation. In the socially optimal allocation, the social planner solves the following maximization problem.

$$\begin{aligned} \max_{C_i, L_i, G_i, A_j} \quad & \sum_i E\big[U_i(\bar{R}, R_\omega)\big] + \sum_j E[V_j(\bar{R}, R_\omega)] \\ \text{s.t.} \quad & (1), \ (2) \\ & \sum_i C_i(\bar{R}, R_\omega) = M \qquad (3) \\ & \sum_j A_j(\bar{R}, R_\omega) = \sum_i Y_i(R_i, R_\omega). \qquad (4) \end{aligned}$$

Here, (3) and (4) denote the resource constraint for the good and the commodity, respectively. Solving this maximization problem, we can derive the following proposition.

Proposition 1 (Social optimal allocation)**.** The social optimal allocation is achieved when $L_i = 1, C_i = M/N$, and $A_j = \gamma \sum_i R_i \ \ \forall i \in \{1, \cdots N\}$ and $j \in [0, 1]$.

Proposition 1 states that it is socially optimal for risk-averse agents to allocate the same amount of the good, M/N, irrespective of the state that occurs in period 1. In addition, because a local investment is more productive than a general investment, $L_i = 1$ is optimal and this implies that each buyer consumes $\gamma \sum_i R_i$ units of the commodity.

4 Competitive market equilibrium

Next, we consider the case where the number of agents, N, is so large that the market for the commodity is competitive. In this case, a representative agent i sells its commodity at the competitive price $P(\bar{R}, R_\omega)$.

First, the maximization problem of a representative agent i is defined as:

$$\begin{aligned} \max_{C_i, L_i, G_i} \quad & E\big[U_i(R_i, R_\omega)\big] \\ \text{s.t.} \quad & (1), (2) \\ & C_i(R_i, R_\omega) = P(\bar{R}, R_\omega) Y_i(R_i, R_\omega). \end{aligned} \tag{5}$$

Here, (5) shows that the consumption level of a representative agent i equals the amount of the good received from buyers as the price of the commodity.

Next, the maximization problem of a representative buyer j is defined as:

$$\begin{aligned} \max_{A_j} \quad & E\big[V_j(\bar{R}, R_\omega)\big] \\ \text{s.t.} \quad & P(\bar{R}, R_\omega) A_j(\bar{R}, R_\omega) = M. \end{aligned} \tag{6}$$

Here, (6) represents the budget constraint for a representative buyer j.

The market clearing conditions for the good and the commodity are given by (3) and (4), respectively, and the competitive price of the commodity is determined by

$$P(\bar{R}, R_\omega) = \frac{M}{\bar{Y}(\bar{R}, R_\omega)}, \tag{7}$$

where $\bar{Y}(\bar{R}, R_\omega)$ denotes the aggregate amount of the commodity, $\sum_i Y_i(R_i, R_\omega)$.

Then, the competitive market equilibrium can be characterized as follows.

Definition 1. Given the set of parameters, (γ, R, π, N), a competitive equilibrium in this economy consists of a vector of parameters (C_i, L_i, G_i, A_j, P), such that: 1) a representative agent i solves its maximization problem given $P(\bar{R}, R_\omega)$; 2) a representative buyer j solves its maximization problem given $P(\bar{R}, R_\omega)$; 3) the competitive price of the commodity is determined by (7); and 4) markets for the good and the commodity clear.

As the equilibrium cannot be solved analytically, we investigate numerical computations about the equilibrium level of a local investment, L_i^{c*}, where superscript c denotes that the market is competitive. Proposition 2 shows the results.

Proposition 2 (Competitive equilibrium)**.** In a competitive equilibrium, L_i^{c*} decreases as γ becomes smaller, R becomes smaller and π becomes larger.

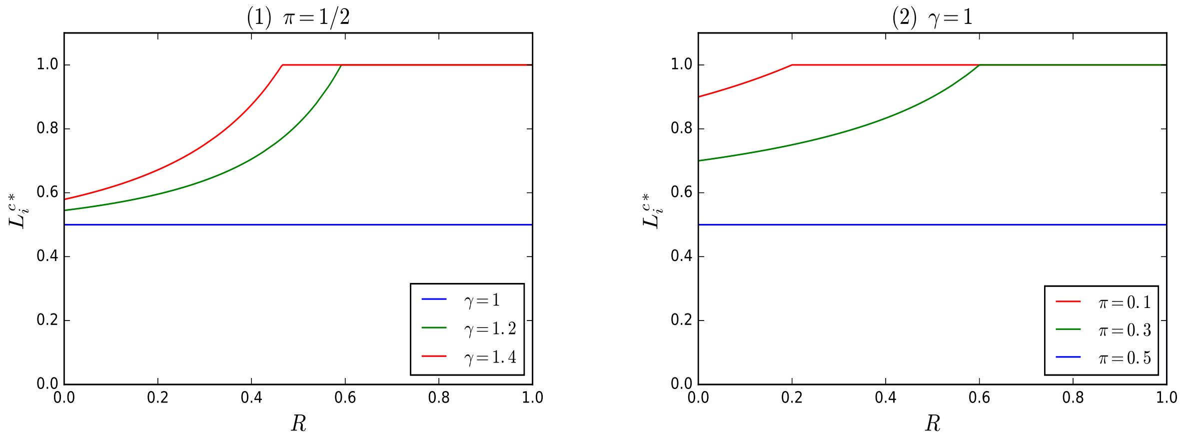

Figure 2: (Competitive market) The optimal level of L_i^{c*}

(Proof) See Appendix A.

In Figure 2(1), we plot the equilibrium level of a local investment with respect to R, $L_i^{c*}(R)$, when $\gamma \in \{1, 1.2, 1.4\}$ and $\pi = 1/2$. First, as a benchmark equilibrium, we consider the case where $\gamma = 1$. As Figure 2(1) shows, $L_i^{c*}(R) = 0.5 \quad \forall R \in (0, 1)$ because, in this case, there is no difference in productivity and risk between local and general investments; therefore, risk-averse agents split their good equally to both investments. This diversification enables agents to reduce their consumption risks.

Next, consider the case where $\gamma > 1$. Compared with the benchmark case, L_i^{c*} increases as γ becomes larger. This is because a local investment becomes more productive as γ becomes larger. In addition, L_i^{c*} decreases as R becomes smaller. This is because, as R becomes smaller, the fluctuations in output increase so that agents have more incentive to diversify their investment portfolios.

Figure 2(2) illustrates the case where $\gamma = 1$ and $\pi \in \{0.1, 0.3, 0.5\}$. It shows that $L_i^{c*}(R)$ decreases as π becomes larger. This shift shows that agents invest more goods in a general investment as it becomes safer[6].

These results show that in a competitive equilibrium, asset commonality is advanced when each agent seeks to diversify their investment portfolio, which is consistent with previous studies that relate asset commonality to diversification of investment portfolios.

Note that the allocation in the competitive market equilibrium cannot achieve the first best allocation[7]. This is because agents cannot hedge a productivity shock because of the imperfect financial market. Moreover, because a general investment is less productive, buyers consume less of the commodity as agents increase general investment. In this sense,

[6]If π were greater than 1/2, $L_i^{c*}(R)$ might be less than 0.5.

[7]The first best allocation can be achieved only when $\pi = 1/2$ and $\gamma = 1$.

while asset commonality benefits the agents' payoff, it decreases the aggregate output of the commodity, which worsens the buyers' payoff.

5 Risk sharing and asset commonality

Next, we consider an oligopolistic market where the number of agents, N, is limited. Agents compete in terms of quantity in the commodity market, so that the decision of each agent in period 0 affects the price level of the commodity in period 1.

Under this modification, the maximization problem of a representative agent i is defined as:

$$\max_{C_i,L_i,G_i} \quad E\big[U_i(R_i, R_\omega)\big]$$
$$\text{s.t.} \quad (1), (2), (3), (7).$$

The difference from the previous case is that (7) is included in the constraint. That is, a representative agent i considers not only the quantity of the commodity it produces but also the price level of the commodity in the market.

The maximization problem of a representative buyer j is the same as in the previous case. Then, the oligopolistic market equilibrium is characterized as follows.

Definition 2. An oligopolistic market equilibrium, given the set of parameters (γ, R, π, N), consists of parameters, (C_i, L_i, G_i, A_j, P), such that: 1) a representative agent i solves its maximization problem given other agents' investment decisions; 2) a representative buyer j solves its maximization problem given $P(\bar{R}, R_\omega)$; 3) the price of the commodity is determined by (7); and 4) markets for the good and the commodity clear.

Proposition 3 shows the results.

Proposition 3. In an oligopolistic market equilibrium, agents increase their general investment compared with the case of a competitive market equilibrium. Furthermore, they increase their general investment as N becomes smaller.

(Proof) See Appendix B.

Figure 3(1) shows the optimal level of a local investment with respect to R for a representative agent i, $L_i^*(R)$, when $\pi = 1/2, N = 2$ and $\gamma \in \{1, 1.2, 1.4\}$. By comparing Figure 2(1) with Figure 3(1), we find that L_i^* locates lower than L_i^{c*}. Interestingly, as the figure shows, when $\gamma = 1$, $L_i^* < 0.5 \ \ \forall R$. This means that even if local and general investments are *ex ante* identical, agents allocate more goods to a general investment. This implies that a representative agent i invests more goods to a general investment in an oligopolistic market.

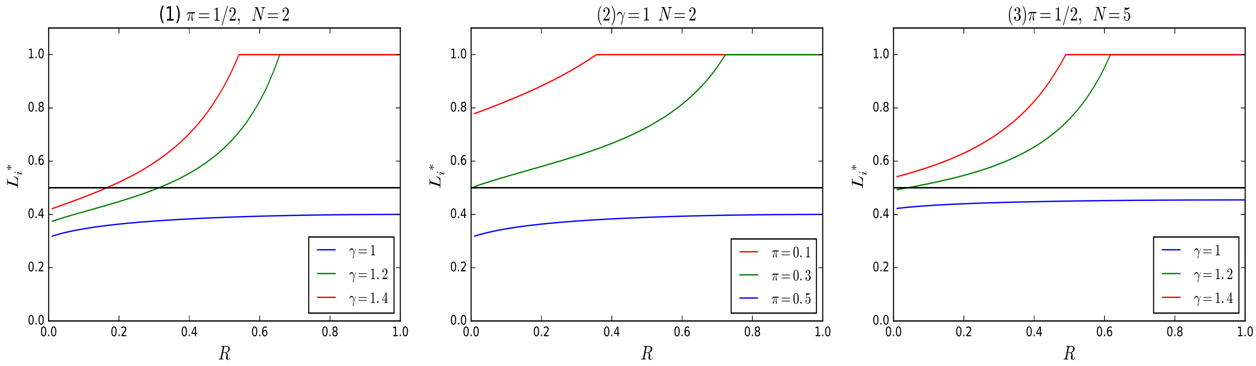

Figure 3: (Oligopolistic market) Equilibrium level of L_i^*

This is because in the oligopolistic market, agents can implicitly coordinate with each other to prevent an *ex post* fluctuation of purchasing power among agents.

Next, Figure 3(2) shows the case when $\gamma = 1, N = 2$ and $\pi \in \{0.1, 0.3, 0.5\}$. The figure shows that even if a general investment is less productive and riskier than a local investment, a representative agent i increases their general investment as R becomes smaller. This result shows that as the fluctuations in output increase, investing in a general investment is useful for sharing the consumption risk among agents.

Next, by comparing Figure 3(1) with Figure 3(3), the level of L_i^* increases as N becomes larger. This is because the pricing power of each agent decreases as N becomes larger. Moreover, as N becomes larger, $L_i^*(R)$ gradually approaches the case of a competitive market. Therefore, when the number of agents is small, agents have more incentive to increase their general investment so that the degree of asset commonality increases.

However, as in the previous section, while increasing general investment is beneficial for agents, it decreases the total output of the commodity, which lowers the buyers' consumption levels.

6 Conclusion

We have presented a model in which asset commonality among agents arises not only from diversification of the investment portfolio of each agent, but also from risk sharing among agents. In addition, as the productivity shock becomes larger, agents increase their general investment even if the investment is risky and unproductive. These results show that asset commonality in the financial sector can mitigate the risk of each financial intermediary but may decrease aggregate output in the economy. This insight will provide a new perspective about the relationship between stability in the financial system and efficiency in economic activity.

Appendix A: Proof of Proposition 2

First, the first-order condition with respect to L_i in the representative agent i's utility maximization problem implies that:

$$\frac{\partial E[U_i]}{\partial L_i} = E\left[\frac{P(\bar{R}, R_\omega)}{C_i(R_i, R_\omega)}(\gamma R_i - R_\omega)\right] \geq 0. \tag{A.1}$$

If (A.1) holds with inequality, $L_i^{c*} = 1$ is optimal. As agents are symmetric in period 0, all agents adopt the same investment decision, $L_i^{c*} = L^{c*}$ and $G_i^{c*} = G^{c*}$ $\forall i$.

Next, in a competitive market equilibrium where N is large enough, by the law of large numbers a fraction $N/2$ of local investments indicates high productivity (i.e., $R_i = 2 - R$) while the rest indicates low productivity (i.e., $R_i = R$). Then, the total amount of the commodity, $\bar{Y}^c(\bar{R}, R_\omega)$, can be written as:

$$\begin{aligned}
\bar{Y}^c(\bar{R}, R_\omega) =& \ \gamma(2 - R)L^{c*}(N/2) + \gamma R L^{c*}(N/2) + R_\omega G^{c*} N \\
=& \ \gamma L^{c*} N + R_\omega G^{c*} N \\
=& \ \{\gamma L^{c*} + R_\omega(1 - L^{c*})\}N..
\end{aligned} \tag{A.2}$$

Then, by inserting (A.2) into (7), we have

$$P^c(\bar{R}, R_\omega) = \frac{M}{\{\gamma L^{c*} + R_\omega(1 - L^{c*})\}N}. \tag{A.3}$$

Therefore, $P^c(\bar{R}, R_\omega)$ varies depending on the realized value of R_ω. Then, by inserting (A.1) into (A.3), L^{c*} is determined. (q.e.d.)

Appendix B: Proof of Proposition 3

First, the first-order condition with respect to L_i in the representative agent i's utility maximization problem implies that:

$$\begin{aligned}
\frac{\partial E[U_i]}{\partial L_i} =& E\left[\frac{1}{C_i(R_i, R_\omega)}\Big(P(\bar{R}, R_\omega)(\gamma R_i - R_\omega) + \frac{\partial P(\bar{R}, R_\omega)}{\partial L_i} Y_i(R_i, R_\omega)\Big)\right] \\
=& E\left[\Big(\frac{1}{Y_i(R_i, R_\omega)} - \frac{1}{\bar{Y}(\bar{R}, R_\omega)}\Big)(\gamma R_i - R_\omega)\right] \geq 0.
\end{aligned} \tag{B.1}$$

The second equation is derived by inserting (5) and (7) into the right-hand side of the first equation. The difference from (A.1) is that the second term in the first equation is added.

In a symmetric Nash equilibrium, $L_i^* = L^*$ and $G_i^* = G^*$ $\forall i$ are satisfied. Then, the total amount of the commodity, $\bar{Y}(\bar{R}, R_\omega)$, can be written as:

$$\bar{Y}(\bar{R}, R_\omega) = \gamma L^* \sum_{i=1}^{N} R_i + N(1 - L^*)R_\omega. \tag{B.2}$$

Then, by inserting (B.2) into (7), we have

$$P(\bar{R}, R_\omega) = \frac{M}{\gamma L^* \sum_{i=1}^{N} R_i + N(1 - L^*)R_\omega}.. \tag{B.3}$$

Different from the competitive price, $P(\bar{R}, R_\omega)$ varies depending not only on the value of R_ω, but also on that of R_i. Then, by inserting (B.1) into (B.3), L^* is determined. (q.e.d.)

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