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Quality differentiation in durable goods monopoly always yields strictly positive profits

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Abstract

A durable good monopolist can discriminate between buyers in each period by offering them a sequence of price-quality menus (second-degree price discrimination). We show that, contrary to the Coase conjecture for the homogeneous durable good monopoly, under vertical differentiation, when the consumers base their expectations on the size of the market at the end of each period, the profit of a monopolist that cannot commit to future price-quality menus is bounded below by a strictly positive value independent of the discount factor.

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1 Introduction

This paper shows that generally that the Coase Conjecture fails when durable goods may be vertical differentiated and second-degree price discrimination is available in each period. Indeed, at the non-commitment, Markov-Perfect, equilibrium (MPE) of such a dynamic model, the monopolist's profits are bounded below by a strictly positive value, independent of the value of the discount factor, provided that (i) it is socially efficient to serve all agents (the market is "strong") and (ii) consumers base their expectations of future offers on the size of the market at the *end* of each period so they can be influenced by the firm's current decisions (the "Stage-wise Stackelberg Leadership Assumption"). Our result holds even when the market is just strong[1], which corresponds to the No Gap Case in the literature. The reason for this result is that it is a feasible strategy for the monopolist to cover in the first period the whole market by offering *all* agents a modified static Mussa-Rosen price-schedule, which amounts to a commitment that the market will not be re-opened after the first period. Our result does not require immediate full market-coverage (IFMC) to be an equilibrium since, when it is not an MPE, it is anyway a possible deviation from the MPE.

This paper is related to two strands of literature: the second-degree static price discrimination model (Mussa and Rosen, 1978) and the dynamic durable good commitment model (Coase, 1972, Bulow, 1982, Gul et al., 1986). It goes farther than the two closest ones which dealt simultaneously with intratemporal and intertemporal discrimination: Inderst (2008) and Laussel et al. (2021)[2] who provided conditions for IFMC to be an MPE. Here, our strictly positive lower bound of profits is indeed shown to prevail even when IFMC is not an MPE. This is especially relevant in the case of a market which is just strong, corresponding to the traditional No Gap case, where we show that, as in the standard durable good monopoly literature, full market-coverage cannot occur in a finite number of periods.

2 The Model

A monopolist produces a durable good at different quality levels. The quality index q can take any value on $[0, +\infty)$. There is a continuum of mass one of consumer types, indexed by θ, where $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, $\overline{\theta} > \underline{\theta} \geq 0$. ($\theta$ is private information.) The cumulative distribution, $F(\theta)$, is

[1] Meaning that the net social surplus of serving the lowest type consumers is just equal to zero.

[2] Inderst deals with the two-types case, Laussel et al. with a general continuous distribution.

continuously differentiable, with the density function $f(\theta) \equiv F'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.The inverse hazard rate function $h(\theta)$ is positive and monotone decreasing over $[\underline{\theta}, \overline{\theta}]$. Consumers are infinitely lived and buy at most one unit of the good. A type $\theta-$consumer who makes use of a unit of durable at quality level q over $[t, +\infty)$ derives a utility flow of θq at each $\tau \in [t, \infty)$.

The unit cost of a durable good at quality level q is $c(q) \geq 0$, with $c'(q) > 0$ *and* $c''(q) > 0$ *for all* $q > 0$. There exists a unique $\widehat{q}$ such that $\widehat{q}c'(\widehat{q}) = c(\widehat{q})$.

Time is a continuous variable, $t \in [0, \infty)$, divided in discrete periods of length Δ numbered consecutively by $n \in \{0, 1, ..\}$. Let $\beta = e^{-r\Delta}$ denote the discount factor across periods, where $r > 0$ is the instantaneous rate of discount, common to all agents. The life time utility discounted back to time t of a type θ consumer buying a durable of quality q at price p is simply $\frac{\theta}{r}q - p$.

At the beginning of period n, all consumers whose types belong to $[\theta_n, \overline{\theta}]$ have already purchased the durable in previous periods. Consumer of type θ_n is defined as the marginal customer who is indifferent between buying the good in period $n-1$ and period n (i.e., θ_n is period $n-1$ cut-off type).

Define $\widehat{\theta} = rc'(\widehat{q})$. Under social welfare maximization, only consumers of type $\theta \geq \widehat{\theta}$, whose life time utility for a unit of durable is higher than its cost of production, are offered a quality $q^{se}(\theta) = c'^{-1}(\frac{\theta}{r})$ while other consumers are not served (see Laussel et al., 2021). We assume that the market is "strong", i.e., that $\underline{\theta} \geq \widehat{\theta}$, a strong market being equivalently a wholly economically viable one.[3] It is "super strong" if $\underline{\theta} > \widehat{\theta}$. The super strong case is an extension of the Gap case of the durable good literature while the just strong (or weak) market, when $\underline{\theta} = \widehat{\theta}$ corresponds to the No Gap case.

3 Markov-Perfect Equilibria

We deal here with non-commitment equilibria of a dynamic game between the monopolist and the consumers. The state variable of our model in period n is denoted by $\Theta(n) = \theta_n$ and is simply the cut-off type of the previous period $n-1$ whose initial value $\Theta(0) = \overline{\theta}$. $X(n) = 1 - F(\Theta(n))$ denotes the fraction of the total population that has purchased the good prior to period n, and is linked to $\Theta(n)$ in a simple way. The real interval $[\underline{\theta}, \Theta(n))$ represents the set of customers who have not purchased the good prior to period n.

[3] This is not really restrictive. A "weak" market is easily truncated by considering only the "strong", economically viable, segment $[\widehat{\theta}, \overline{\theta}]$.

At the beginning of each period n, the monopolist offers to all potential new customers who have not bought the durable a menu of (quality, price) pairs $\left(q_n(\widetilde{\theta}), p_n(\widetilde{\theta})\right)$, depending on their reported type $\widetilde{\theta}$ when buying in period n. A type θ-customer who purchases the durable in period n is willing to report her true type if and only if it does not pay to pretend to be a different type, i.e., iff $\theta = \arg\max_{\widetilde{\theta}} \frac{1}{r}\theta q_n(\widetilde{\theta}) - p_n(\widetilde{\theta})$. Accordingly, denoting by $U_n(\theta)$ the net utility (over the entire life-time) of a type θ customer who buys the durable good in period n, the price-quality schedule$(q_n(\theta), p_n(\theta))$ is incentive compatible iff $U_n'(\theta) = \frac{1}{r}q_n(\theta)$. Integrating we obtain that

$$U_n(\theta) = U_n(\theta_{n+1}) + \int_{\theta_{n+1}}^{\theta} \left(\frac{1}{r}q_n(s)\right) ds. \tag{1}$$

Notice that incentive compatibility also implies that $q_n(\theta)$ is non-decreasing.[4]

The participation constraints imply that the type θ_{n+1} must be indifferent between buying in period n or waiting until period $n+1$. In the latter case, it is optimal to report the highest possible type $\widehat{\theta} = \theta_{n+1}$ among all thoise who buy in $n+1$.[5] It follows that

$$U_n(\theta_{n+1}) = \beta U_{n+1}(\theta_{n+1}) \tag{2}$$

and that infra-marginal types of vintage n should also prefer to buy in period n, i.e., $q_n(\theta_{n+1}) - \beta q_{n+1}(\theta_{n+1}) \geq 0$. From (1) and (2),[6]

$$U_n(\theta_{n+1}) = \sum_{j=1}^{\infty} \beta^j \left(\int_{\theta_{n+j+1}}^{\theta_{n+j}} \left[\frac{1}{r}q_{n+j}(\theta)\right] d\theta\right) \tag{3}$$

The consumers decide whether to buy in a given period based on their forecast future rent, given by (3). We assume that they have a Markovian expectations' rule, denoted by $\Phi(.)$, that is a function of $\Theta(n+1)$, the cut-off value at the *end* of period n selected by the monopolist. This implies that *the monopolist can in period n influence the expected future rents of customers.*

Stage-wise Stackelberg Leadership (SL) Assumption: *At each stage n, given $\Theta(n)$, the monopolist moves first and announces a value*

[4]The convexity of $U_n(\theta)$ follows from a standard revealed preference argument, so that $U_n'(\theta)$ is monotone increasing: for any two values θ' and θ'' in $[\theta_{n+1}, \theta_n]$, with $\theta'' > \theta'$, it holds that $q_n(\theta'') \geq q_n(\theta')$.

[5]This Claim is proved in the Appendix. This is mutatis mutandis Claim 1 of Laussel et al. (2020).

[6]For more details see Laussel et al. (2021).

$\Theta(n+1) \leq \Theta(n)$, and then consumers' expectations of the period n marginal customer's life-time net surplus are given by $U_n(\theta_{n+1}) = \Phi(\Theta(n+1))$.

From the SL assumption and rational expectations, $\Phi(\underline{\theta}) = 0$: if the consumers observe that in period n the firm covers the whole remaining market, they expect zero future rents.

The profit that the monopolist makes in period n from selling the durable good to customers in period n is

$$\begin{aligned} \pi_n &= \int_{\theta_{n+1}}^{\theta_n} \left[(p_n(\theta) - c(q_n(\theta)) \right] f(\theta) d\theta \\ &= \int_{\theta_{n+1}}^{\theta_n} \left[\frac{1}{r} \theta q_n(\theta) - c(q_n(\theta)) - U_n(\theta) \right] f(\theta) d\theta, \end{aligned} \tag{4}$$

and, after integration by parts

$$\pi_n = \int_{\theta_{n+1}}^{\theta_n} \left\{ \begin{array}{c} \frac{1}{r} \left[\theta - h(\theta|\theta_n) \right] q_n(\theta) \\ -c(q_n(\theta)) - U_n(\theta_{n+1}) \end{array} \right\} f(\theta) d\theta, \tag{5}$$

where $U_n(\theta_{n+1})$ follows from (3), and

$$h(\theta|\theta_n) \equiv \frac{F(\theta_n) - F(\theta)}{f(\theta)} \text{ for } \theta \in \left[\theta_n, \overline{\theta} \right]. \tag{6}$$

The firm's Markovian strategy is a pair (ψ, η), consisting of two components: (a) a *Markovian cut-off rule* ψ, which, at the beginning of each period n, given $\Theta(n)$, specifies the next $\Theta(n+1)$, thus determining the fraction of the currently unserved customer base that will be served in period n; and (b) a *Markovian quality-schedule rule* η, defining the monopolist's type-dependent quality offers to consumers who buy the durable good in period n.

The monopolist's Bellman equation is

$$V(\Theta(n)) = \max_{q_n(.), \theta_{n+1}} \left\{ \begin{array}{c} \int_{\theta_{n+1}}^{\theta_n} \left[\begin{array}{c} \frac{\theta - h(\theta|\Theta(n))}{r} q_n(\theta|\Theta(n)) \\ -c(q_n(\theta|\Theta(n))) - \Phi(\Theta(n+1)) \end{array} \right] f(\theta) d\theta \\ +\beta V(\Theta(n+1)) \end{array} \right\}, \tag{7}$$

where the RHS is to be maximized with respect to $q_n(\theta|\Theta(n))$ and $\Theta(n+1)$, subject to the constraint

$$q_n(\Theta(n+1)|\Theta(n)) \geq \beta q_{n+1}(\Theta(n+1)|\Theta(n+1)). \tag{8}$$

It follows from pointwise maximization of the RHS of the Bellman equation that, given $\Theta(n)$,

$$q^m(\theta|\Theta(n)) = \max\{q^{**}(\theta|\Theta(n), \beta q^{se}(\Theta(n+1))\}, \tag{9}$$

where

$$q^{**}(\theta|\Theta(n)) \equiv c'^{-1}\left[\frac{\theta - h(\theta|\Theta(n))}{r}\right]. \tag{10}$$

Definition 1 A Markov-Perfect Equilibrium (MPE) is a Markovian expectations rule Φ, a Markovian cut-off rule ψ and a Markovian quality-schedule rule η, such that (i) consumers" expectations are rational ($U_n(\theta_{n+1}) = \Phi(\Theta(n+1))$) given the firm's strategy (ψ, η) and (ii) the firm's strategy maximizes its expected profits given the consumers' expectations rule Φ.

Example 1 *Assume the uniform distribution of types and the cost function* $c(q) = B + \frac{1}{2}q^2$. *From Laussel et al. (2021, Proposition 2), IFMC is an MPE when the market is super strong and additional conditions are met. When* $\underline{\theta} = \widehat{\theta}$ and $\beta < 1$, *IFMC is not an MPE. Moreover, starting from any* $\Theta(n) \in (\underline{\theta}, \overline{\theta}]$, *immediate covering of the remaining part of the market is not an equilibrium strategy.*[7] *It follows that the market, when just strong, is never covered in a finite number of periods. Intermediate cases where the market is fully covered in a finite number of periods* $N > 1$ *obtain in the super-strong market case when* β *is low and the market is wide (*$\frac{\overline{\theta}}{\underline{\theta}}$ *is great).*

4 Analysis

We show below that the monopolist can always ensure for itself a strictly positive profit by covering immediately the whole market and offering to the customers a price-quality schedule which is the optimal static Mussa-Rosen one under a full market coverage constraint.

Lemma 1 *Given the strong market and the SL assumptions, an IFMC strategy, such that* $\psi(\overline{\theta}) = \underline{\theta}$ *and* $\eta(\overline{\theta}) \equiv q^{**}(.|\overline{\theta})$, *where* $q^{**}(\theta|\overline{\theta}) \equiv c'^{-1}\left[\frac{\theta - h(\theta)}{r}\right]$, *yields strictly positive profits to the monopolist.*

Proof. See Appendix. ■

Example 1 (continued) *In the linear-quadratic case considered, straightforward computations show that the profits from IFMC are:*

$$\frac{1}{r^2}(\overline{\theta} - \underline{\theta})\left[(\overline{\theta} - \underline{\theta})^2 + 3\left(\underline{\theta}^2 - \left(\widehat{\theta}\right)^2\right)\right] > 0,$$

[7] The necessary conditions provided in Proposition 2 of Laussel et al. (2021) are necessary conditions for covering the remaining market when one substitutes $\Theta(n)$ for $\overline{\theta}$.

It reduces to $\frac{1}{r^2}(\overline{\theta} - \underline{\theta})^3 > 0$ *when the market is just strong (No Gap).*

Proposition 1 *The monopolist's profits at any MPE of the game are bounded below by a strictly positive value, independent of the discount factor.*

The proof of Proposition 1 is obvious since either IFMC is an MPE, or it is not but is then a profitable deviation from the MPE. Example 1 in the previous Section shows that Proposition 1 is not trivial since, depending on parameter values, there exist other MPEs than the IFMC one, among which, in the equivalent of the No Gap case, MPE where the number of periods it takes to clear the market is infinite as in the standard durable good literature.

Appendix

Claim: *A type* $\theta \in (\theta_{n+1}, \theta_n]$ *when considering to delay her purchase to period* $n+1$ *instead of period* n *would find optimal in that case to report the highest possible type* $\widehat{\theta} = \theta_{n+1}$ *among all those who buy in* $n+1$.

Proof:

Suppose that on the contrary that there exists a type $\theta > \theta_{n+1}$ and a type $\theta' < \theta_{n+1}$ such that

$$\frac{1}{r}(\theta q_{n+1}(\theta') - p_{n+1}(\theta') > \frac{1}{r}(\theta q_{n+1}(\theta_{n+1}) - p_{n+1}(\theta_{n+1})). \tag{11}$$

On the other hand incentive compatibility at $n+1$ implies that

$$\frac{1}{r}(\theta_{n+1} q_{n+1}(\theta_{n+1}) - p_{n+1}(\theta_{n+1})) \geq \frac{1}{r}(\theta_{n+1} q_{n+1}(\theta') - p_{n+1}(\theta')). \tag{12}$$

Subtracting the RHS of (12) from the LHS of (11) and symplifying we obtain

$$(\theta - \theta_{n+1})(q_{n+1}(\theta') - q_{n+1}(\theta_{n+1})) > 0,$$

Since $\theta > \theta_{n+1}$, we then should have $q_{n+1}(\theta') > q_{n+1}(\theta_{n+1})$. This is however impossible since by incentive compatibility, $q_{n+1}(\theta)$ must be non-decreasing in θ.■

Proof of Lemma 1:

Whenever $\psi(\Theta(n)) = \underline{\theta}$, from consumers' rational expectations and (3), it must be that $\Phi(\Theta(n+1)) = \Phi(\underline{\theta}) = 0$. Given the value of θ_n observed at the beginning of period n, full market-coverage in period n yields profits

$$\Pi(\theta_n, \underline{\theta}) \equiv \int_{\underline{\theta}}^{\theta_n} \left[\frac{\theta - h(\theta|\Theta(n))}{r} q^m(\theta|\Theta(n)) - c(q^m(\theta|\Theta(n)))\right] f(\theta) d\theta, \tag{13}$$

Taking the derivative with respect to θ_n, making use of the Envelope Theorem, and noting that there is no distortion at the top, i.e., for type θ_n, one obtains

$$\frac{\partial Z(\theta_n, \underline{\theta})}{\partial \theta_n} = f(\theta_n) \left[\left(\frac{\theta_n}{r} q^{se}(\theta_n) - c(q^{se}(\theta_n))\right) - \int_{\underline{\theta}}^{\theta_n} \frac{1}{r} q^m(\theta|\Theta(n)) d\theta\right].$$

From the strong market assumption, this derivative is positive if θ_n is evaluated at $\underline{\theta}$. To show that it is positive for any $\theta_n > \underline{\theta}$, it suffices to show that the bracketed term is increasing in θ_n. Differentiating it wrt θ_n and using again the Envelope Theorem[8], one obtains $-\int_{\underline{\theta}}^{\theta_n} \frac{1}{r} \frac{\partial q^m(\theta|\Theta(n))}{\partial \Theta(n)} d\theta$ which is > 0 since $\frac{\partial q^m(\theta|\Theta(n))}{\partial \Theta(n)} < 0$.■

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[8]Notice that $q^{se}(\theta) = \arg\max_q \frac{\theta}{r} q - c(q)$.