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A real option to divest with two correlated sources of ambiguity

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Abstract

Real options models have recently been expanded to account for ambiguity in various economic and financial settings, proving they are precious, flexible, and efficient decision-making tools. Yet, these real option models under ambiguity usually only account for the existence of a unique source of ambiguity, while more complex real-life situations require the modeling of multiple uncertain factors. We expand the theoretical work of Roubaud et al. (2017) by applying their framework of correlated Choquet-Brownian motions to the case of a real option to divest, with two correlated sources of ambiguity over its project value and exit value. This is the first application with multiple Choquet-Brownian ambiguity sources to an irreversible investment decision. Our findings illustrate the explanatory power of the model as well as some of its limitations. It also deepens the analysis of the effect of ambiguity on the value of options and on the timing of investors' decisions, hence expanding the real option irreversible investment theory.

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1. Introduction

Despite their large application, traditional capital budgeting models, such as discounted cash flows or net present value criteria, still suffer from significant flaws when decision makers face *uncertainty* but also enjoy flexibility regarding the timing of their decisions. To address these limits, the real options approach to capital budgeting was proposed (Myers, 1977).

The very notion of *uncertainty* is vague at first and appears to be a multiform term, including (but not limited to) the familiar notion of *risk* (Knight, 1921). *Risk* is commonly defined as the existence of a unique, well-known probabilistic model on which decision makers can rely. But decision makers may find themselves in uncertain situations that are not characterized as just *risky*. The more elusive notion of *ambiguity,* as defined by Ellsberg (1961), has been useful in experimental research to explain behaviours when decision makers cannot rely on probabilistic expected utility models to describe the possible outcomes' likelihood. As it progressively became mainstream in decision theory, though often still seen as controversial and highly debated, *uncertainty* is the generic category, which includes *risk*, defined by the presence of known probability distributions and *ambiguity* in the absence of known probability distributions.

Ambiguity appears when the decision maker perceives the presence of ambiguity — not just risk — and cannot rely on known probability distribution. Ambiguity is often ignored in standard models, but ambiguity may lead to meaningful changes in behaviours and choices. However, these notions of risk and ambiguity require distinct sets of axioms, and various definitions of ambiguity have been proposed and evaluated through experiments. Risk and ambiguity may be objective in a situation but also may be purely subjective and only present in the mindset of a decision maker. Sometimes, hard-to-decipher decisions may result from subjective perceptions and preferences toward ambiguity. In contrast, ignoring ambiguity and relying only on the introduction of risk in an uncertain environment may often be misleading and prevent observers from fully understanding decisions taken by subjects when confronted with either risk or ambiguity, depending on the situation. Even though integrating ambiguity was shown to lead to meaningful changes in expected choices, it remains theoretically and practically challenging. Consequently, ambiguity remains ignored in standard real options models.

As underlined in Uppal and Wang (2003), most real-life situations are ambiguous rather than risky. It is remarkable that while primarily concerned with uncertainty, the real options theory was founded to address risk and preferences toward risk without integrating ambiguity for a long time. Applying the standard subjective expected utility framework to a real option model when the decision maker is confronted with ambiguity and not just risk may be misleading. Our study is based on the notion that ambiguity often matters, especially in the presence of options.

Several real options models under *ambiguity* have been proposed (such as Nishimura & Ozaki, 2007; Trojanowska & Kort, 2010; Miao & Wang, 2010), These models identify the contrasted effect of ambiguity on option valuation and their subsequent exercise timing, an interesting feature. They demonstrate how the introduction of ambiguity can drastically affect option value and their optimal exercise timing.[1] However, they all solved uncertainty by applying a *maximin* criterion over the potential outcomes of decision; these recursive multiple-priors real options models (Chen & Epstein, 2002) only consider the case of extreme aversion toward uncertainty, which reduces the behavioural bias to extreme pessimism.

[1] Please refer to Roubaud et al. (2017) for a detailed literature review.

To avoid taking such a limited normative stance on the nature of ambiguity preferences or their rationality, our study adopts the more recently axiomatized Choquet-Brownian motions to describe the dynamics of one (or more) ambiguous variable(s), as in Kast et al. (2014). This representation of ambiguity has been successfully applied to address ambiguous environments such as valuing an option (Driouchi et a., 2018; 2020), defining corporate finance decisions (Agliardi et al., 2016; Agliardi, 2017), and discussing the timing of ecologic policy adoption (Sereno, 2011).

To quote Trigeorgis et al. (2018): "Ambiguity is a dimension of uncertainty beyond probabilistic risk that can be estimated under a partial ignorance framework using Choquet expected utility (CEU) and Choquet-Brownian motions." This is precisely what our paper offers.

When based on such a framework, please note that real options models under ambiguity incorporates risk as a specific base case when ambiguity is absent; this is an improvement over some models of ambiguity where risk is purely absent from the representation. In the Kast et al. (2014; 2017) framework of ambiguity, risk and ambiguity cannot coexist as separate concepts. Rather, risk is the first level of uncertainty (i.e., in the absence of ambiguity we face risk with a full knowledge of probability distribution), and once ambiguity is present, we cannot refer to risk any longer (i.e., the presence of ambiguity leads to the absence of known probability distributions, and risk is absorbed by ambiguity as a higher level of uncertainty). As in most other ambiguity models, risk disappears when ambiguity appears,[2] so that there is an original attitude toward ambiguity modelled with a single parameter, an interesting feature.

Please note that in our model, we still do not ignore risk and preferences toward risk, as risk is integrated in our base model (i.e., the case without ambiguity). This only requires setting the parameters toward an absence of ambiguity; then, our base model is that of risk only (when ambiguity parameters are null). Once ambiguity is present, there is no risk detached from ambiguity that would cohabitate with ambiguity in a cognitively meaningful manner. Either we have known probability distributions, or we do not. But, thanks to our model, we can compare the results in the presence of risk and the results in the presence of ambiguity. The difference obtained is an interesting topic of analysis of the distinct effect of ambiguity vs. risk.

Ambiguity was first introduced inside real options models using dynamically consistent Choquet-Brownian motions in Roubaud et al. (2010), where ambiguous Brownian motions represent the trajectory of ambiguous expected cash flows in the context of a real option to invest. The attitude toward one single perceived source of ambiguity is captured when the stochastic cash flows of a project are perceived to be ambiguous. As with other early real options models under ambiguity, please note that this model is only concerned with a single source of ambiguity, affecting the dynamics of the cash flows generated by a project, in the context of a real option to invest.

In contrast, integrating a second source of ambiguity would further increase the real options under ambiguity models' ability to describe concrete real-life situations. Nevertheless, it is remarkable that even after Dixit and Pindyck (1994) first underlined the importance of dealing with multiple sources of risks, very few real options models were built with more than one random risk variable and none with multiple sources of ambiguity. The added complexity

[2] Very few models have explored the coexistence of risk and ambiguity simultaneously modelled with distinct parameters because this is certainly difficult to define and comprehend. It is unclear how this would be consistent with the adopted notions of risk (presence of a well-known probability distribution) and ambiguity (absence of known probability distributions). No doubt this remains an exciting research question for years to come.

remains very challenging to handle in real-world applications, as it is much more difficult to find an analytical solution.

Roubaud et al. (2017) first provided a theoretical framework to model two sources of ambiguity. They demonstrated how correlated random walks and their expansion in continuous time to correlated Choquet-Brownian motions can be used to model in front of ambiguity, and they applied them to optimal portfolio choices.

To expand on this work, our study's contribution is threefold. First, it offers an original model of a real option to divest (rather than to buy). Next, it models two sources of ambiguity (rather than one), as a frequent business situation when both project and exit values are stochastic and perceived ambiguous. Finally, it also models the impact of extreme events on the option to divest, expanding upon Clark et al. (2010) in the presence of ambiguity. Our study thereby enriches the real options literature by exploring the impact of contrasted ambiguity preferences (not limited to aversion) on the valuation and timing of exercise of a real option to divest in the presence of extreme events.

The remainder of the study is organized as follows. In Section 2, we clarify the set-up and the issues raised by the coexistence of two sources of ambiguity. Then we introduce our key assumption, the adoption of correlated sources of ambiguity, and show how it allows easier computation of results without significantly damaging the explanatory potential of our model. Finally, the optimization problem is solved, and the threshold option values are identified. Section 3 presents a sensitivity analysis of the optimal divestment rule and discusses main results. Section 4 concludes our study.

2. Model

2.1 Set up

We consider a decision maker enjoying the possibility of selling a generating cash flows asset at any moment over a period. As well established in the finance literature (Margrave, 1978; Stulz, 1982; Brennan & Schwartz, 1985; McDonald & Siegel, 1986; or Dixit & Pindyck, 1994), this situation may be modelled as a real option to divest (or abandon or sell). It directly relates to the American put option model in finance theory. Indeed, the asset value (or project value) may be interpreted as corresponding to the underlying value of an option, while the selling price (or exit value) would be its strike price.

What is the optimal moment for selling the asset? This requires solving a typical optimal stopping problem. But what if the decision maker is ambiguous toward the evolution over time of both project value and exit value? How does this affect the valuation of the option and the decision whether to keep the asset or to sell it? If we add such ambiguity in the model, how do we represent ambiguous stochastic processes?

Choquet-Brownian motions are non-probabilistic ambiguous stochastic processes; in contrast to the commonly used standard geometric Brownian processes, they account for ambiguity, and not just risk, including ambiguity loving and ambiguity aversion preferences. Risk is a special base case in the model.

A Choquet-Brownian motion (CBM) is an ambiguous distorted Wiener process where a constant conditional capacity c summarizes the decision makers' attitude towards ambiguity (aversion or love). The model is non probabilistic because c can take any value between 0 and 1 (in contrast with the probabilistic case of a binomial lattice with equal probability for up and down movements, which is the special case when $c = 1/2$).

As explained earlier, few real option models deal specifically with more than one random variable impacting a single real option, and so far, none exist in the presence of ambiguity. In our study, we simultaneously consider two correlated sources of ambiguity, stochastic *project value* and *exit value*, whose dynamics are modelled by Choquet-Brownian motions.

We use properties from correlated sources of ambiguity to compute the real option value, in the spirit of the construction proposed by Myers and Majd (1990) and sophisticated in Clark et al. (2010), preserving the explanatory potential of our model. We simplify the option value as a function of homogeneity of degree one using the ratio of two variables.

Obviously, and as discussed notably by Adkins and Paxson (2011), this correlation is a strong assumption, but an acceptable one, because many real options models are based on correlated sources of uncertainty. Correlated ambiguity factors are even more realistic in many capital budgeting situations than in situations where it is necessary to artificially distinguish non correlated sources. The adoption of this correlation factor is economically sound because sources of ambiguity are commonly subjected to the same contextual factors, such as a change in economic outlook or innovations that affect both expected cash flows of an asset and its potential resale price.

2.2 Preliminary Notations

Suppose a decision maker enjoys an option to sell an asset (project, subsidiary, etc.) at any time. This project presents the essential characteristics of a real option: it is irreversible (once decided, the selling is instantaneous); it is only affected by time decay; its exercise can be delayed; and the choice of timing belongs exclusively to the decision maker. The decision will be made based exclusively on observed information about stochastic cash flows (project value) and exit value. The project value at time *t* is equal to the discounted expected cash flows, noted V_t, and follows a Brownian motion. The exit value is also stochastic, and the price that a buyer is willing to pay at time *t* is noted S_t. When the decision maker enjoys an option to sell, this option has value (at least time value) and should be included in the valuation, hence the distinction between the "true" asset value (including option value) and the "naked" asset value (absence of option). The former is noted $F = F(V_t, S_t)$.

We must solve an optimal stopping timing problem to define the optimal moment when the exercise of the option to sell becomes beneficial. Time is continuous. The option is supposed to be perpetual. Uncertainty is represented via a filtration on the probability space $(\Omega, (\pi_t)_{t\geq 0}, Q)$, summarizing all information available on t. Cash flows are discounted at rate $r > 0$.

2.3 The Model

2.3.1 No ambiguity with geometric Brownian motions (standard risk model)

Suppose both the "naked" project value (i.e., a value which does not include the option to sell the project) and the exit value follow a geometric Brownian motion (risk, no ambiguity). If these values are correlated and we know the correlation coefficient, then we can identify the "true"[3] project value as follows:

[3] "True" in that it accounts for the existence of an option to sell the project.

“Naked” Project Value:

π at time t represents the present value of expected future cash flows, so that:

$$dV_t = \alpha V_t dt + \sigma V_t dB_t \qquad (1)$$

with: B represents a Wiener process with respect to the original probability measure; $\alpha, \sigma \in R$ **;** $\sigma > 0$ **;** $\alpha > 0$ is the growth rate of Project Value V_t; σ^2 is the variance of Project Value V_t

Exit value S at time t (corresponding to the selling price of project, also stochastic):

$$dS_t = \pi S_t dt + \omega S_t dB'_t \qquad (2)$$

with: B' represents a Wiener process,

π is the trend parameter for the Exit Value S_t; ω^2 is the variance of the Exit Value S_t

Relation between Project and Exit Values:

$$dB_t dB'_t = \rho dt \qquad (3)$$

where ρ is the instantaneous correlation coefficient between V and S

« True » Project Value (including value of option to abandon/sell project):

$$F = F(V_t, S_t) \qquad (4)$$

The impact of risk preferences on real options models is well known and holds for our real option to sell as for other real option models under ambiguity.

2.3.2 Ambiguity through Choquet-Brownian motions (ambiguous model)

Suppose now that the decision maker is ambiguous toward the evolution of both “naked” and exit values, hence not perfectly confident about the extent to which the standard geometric Brownian motions properly model the expected profit and exit values dynamics. The decision maker expresses preferences relative to the ambiguous payoffs generated by a real option project at various dates. Then, using the results from Roubaud et al. (2017), we obtain the key formulas necessary to describe and solve the real option optimal stopping problem, as such:

Project Value V at time t (present value of expected future cash flows)

$$dV_t = (\alpha + m\sigma)V_t dt + s\sigma V_t dB_t \qquad (5)$$

B represents a standard Wiener process, with mean m and variance s^2, such that $\begin{matrix} m = 2c - 1 \\ s^2 = 4c(1-c) \end{matrix}$

This relation is like the « no ambiguity » case, with the difference that some parameters m and s are now to be considered; they directly derive from parameter c, which is the index of preference toward ambiguity.

Exit value

$$dS_t = (\pi + m\omega)S_t dt + s\omega S_t dB'_t \qquad (6)$$

B' represents a general Wiener process, with mean m and variance s^2, such that $\begin{matrix} m = 2c - 1 \\ s^2 = 4c(1-c) \end{matrix}$

Relation between Project and Exit Values:

$dB_t dB'_t = \rho dt$ where ρ is the instantaneous correlation coefficient (7)

With correlated CBM, $\rho = \dfrac{-c(1-c) + cb + (1-2c)a}{c(1-c)}$ see Roubaud et al. (2017).

« True » Project Value (including full value of option to abandon/sell project):

$$F = F(V_t, S_t) \qquad (8)$$

To solve the optimal stopping optimization problem, we must identify the crucial limit value (or reservation value) for which, at a critical moment in time, it becomes more advantageous to exercise the option than to keep it. Consequently, the decision to divest the project is taken only once the critical stopping time is reached; before that, the project is kept in the continuation region.

Let us introduce the project value per dollar of exit value at each moment in time, noted $g = f(V_t, S_t)$. We must determine the critical value g^*, which will lead to the option exercise. We rely on dynamic programming[4] to identify the optimal sequential decision under uncertainty.[5]

We wish to solve (4) and (8) using Itô's Lemma to identify continuation region and critical stopping time; from (1) and (2) in the absence of ambiguity, or (5) and (6) if we introduce ambiguity, we get:

$$dg = \mu g dt + \delta g ds \qquad (9)$$

That leads to the following parameters:

a. *In the absence of ambiguity (risk standard case):*

$$\begin{cases} \mu = \alpha - \pi - \sigma\omega\rho + \omega^2 \\ \delta^2 = \sigma^2 - 2\sigma\omega\rho + \omega^2 \\ ds = \dfrac{\sigma dB - \omega dB^{'}}{\delta} \end{cases}$$

b. *Under Choquet-Brownian Ambiguity (ambiguous case):*

$$\begin{cases} \mu = (\alpha + m\sigma) - (\pi + m\omega) - (s\sigma)(s\omega)\rho + (s\omega)^2 \\ \delta^2 = (s\sigma)^2 - 2(s\sigma)(s\omega)\rho + (s\omega)^2 \\ ds = \dfrac{(s\sigma)dB - (s\omega)dB^{'}}{\delta} \end{cases}$$

Proof: Please note that it is not possible to apply ordinary rules of derivation to Itô processes, but the use of Itô's lemma allows differentiation and integration of functions of stochastic processes. A *multidimensional* Itô's lemma (see Yuh-Dauh Lyuu, 2011, p. 501), applied to the case of the quotient of two correlated Choquet-Brownian motions, is employed to show that:

$$\begin{aligned} dg &= (1/S)dV - (V/S^2)dS - (1/S^2)dVdS + (V/S^3)(dS)^2 \\ &= (1/S)\left[(\alpha + m\sigma)Vdt + (s\sigma)VdB_t\right] - (V/S^2)\left[(\pi + m\omega)Sdt + (s\omega)ScB_t^{'}\right] \\ &\quad -(1/S^2)\left[(s\sigma)(s\omega)VS\rho dt\right] + (V/S^3)\left[(s\omega)^2 S^2 dt\right] \\ &= g\left[(\alpha + m\sigma)dt + (s\sigma)dB_t\right] \text{-g}\left[(\pi + m\omega)dt + (s\omega)dB_t^{'}\right] \\ &\quad -g\left[(s\sigma)(s\omega)\rho dt\right] + g\left[(s\omega)^2 dt\right] \end{aligned}$$

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4 See Stokey et al.(1989) for deep treatment of dynamic programming in economic settings.

5 See Markov stopping rule problems in Chow et al. (1971).

$= g\left[(\alpha + m\sigma) - (\pi + m\omega) + (s\omega)^2 - (s\sigma)(s\omega)\rho\right]dt + g(s\sigma)dB_t - g(s\omega)dB_t^{'}$ QED

By introducing and rearranging δ and ds, we obtain equation (9) and the expressions for the parameters μ, δ^2 and ds as in b).

Then, the optimal stopping time is the one which maximises the value of the project at $t = 0$, over the whole period considered (principle of optimality), considering the discounted cost of investing, at discount rate r.

If the project is *perpetual*, computation is eased, and it is common to adopt an infinite planning horizon and a never expiring project (cf. Dixit & Pindyck, 1994, or Trigeorgis, 1996).

Suppose now that the project's external events (such as a change in ownership of the company, the adoption of new regulations, an economic crisis, etc.) may force the abandonment of the asset. These infrequent but unfortunate events may be modelled through the discrete-time dynamics of a random variable q following a Poisson process.

We adopt such a process to give our representation a more realistic touch, as the possible impact of unexpected brutal external forces should not be ignored in many real options models. Moreover, adopting this assumption will ease direct comparison of our results with those presented in Clark et al. (2010). We note the instantaneous probability of abandonment γ as:

$dq(t) = 1$ with probability γ; $dq(t) = 0$ with probability $1 - \gamma dt$

An earlier exit causes a loss in value, as S is lower than F. Indeed, when the earlier exit is triggered by external forces, the expected cash flows from exercising the option are: $\gamma(S - F)dt$

If we note g as the project value and f as the option value, we change variables so that:

$$f(g,1) = F(V,S)/S \tag{10}$$

Let k represent the instantaneous dividend (or convenience yield) generated by the project (until sold. Then, with the instantaneous pay-out equal to $kgdt$ and cash flows from a premature exit equal to $\gamma(1-f)dt$, setting up a riskless hedge consisting of one unit of the option and $-f'(g)$ units of g and applying Itô's lemma leads to:

$$\frac{\delta^2}{2} g^2 f^{''}(g) + (r-k)gf^{'}(g) - (r+\gamma)f + kg + \gamma = 0 \tag{11}$$

We identify characteristics of the optimal investment strategy: existence of a (unique) critical value g^* such that the option is exercised if and only if $g_t \geq g^*$; if not, the option is kept moving forward, defining a continuation region where $g_t < g^*$. Further assumptions are necessary to solve equation (11), holding in the continuation region. We adopt the following standard boundary conditions: value matching and smooth pasting.

The intuition behind the boundary conditions is that the value of the option to sell becomes smaller as g increases and that the decision maker will sell when the present value of the project cash flows falls far enough below the exit value.

If we note g^* as the critical value (or reservation value) triggering the option exercise, then several conditions follow, as in Barone-Adesi & Whaley (1987):

$f(g^*) = 1$ "value matching condition" (12)

$f'(g^*) = 0$ "smooth pasting condition" (13)

Solving (11) under conditions (12) and (13) to get the option value in the continuation region as well as the critical value, we obtain as in Clark et al (2010).[6]:

$$F = \frac{k\pi}{k+\gamma} + \frac{\gamma S}{r+\gamma} + SK_2 g^{\eta_2} \tag{14}$$

where:

$$\eta_2 = \frac{-(r-k-\delta^2/2) - \sqrt{(r-k-\delta^2/2)^2 + 2\delta^2(r+\gamma)}}{\delta^2} \tag{15}$$

$$K_2 = -\frac{k}{\eta_2(k+\gamma)} g^{*1-\eta_2} \tag{16}$$

$$g^* = \frac{\eta_2}{\eta_2 - 1}\left[\frac{r(k+\gamma)}{k(r+\gamma)}\right] \tag{17}$$

Assuming a real option to divest with two correlated Choquet-Brownian sources of ambiguity impacting asset value and exit price, the optimal strategy is summarized in (14), (15), (16), and (17). The no-ambiguity case is the special case when μ, δ, ds are as defined as in (a). In the presence of ambiguity, μ, δ, ds are otherwise defined by (b). We observe the indirect introduction in key formulas of parameters *m* and *s*, directly deriving from *c*, which summarizes the attitude of the decision maker toward ambiguity.

3. Comparative Statics Results

As usual in the real options literature, it is possible to proceed to a sensitivity analysis to explore the characteristics of the divestment rule and identify the impact of changing fundamental parameters. Key results from our comparative statics over the option value and the optimal moment of exercise of the option may be summarized as such:

- an increase in growth rate for project value, correlation rate, or discount rate, respectively α, ρ, r, delays the exercise of the option by raising the value of g^* and reduces the 'true' value of the project by lowering F (Fig 2, 4, and 6).
- an increase in growth rate for the exit value and convenience yield, respectively (π, k), accelerates the exercise of the option by lowering the value of g^* and increases the 'true' value of the project by increasing F (Fig 1 and Fig 3).
- an increase in γ also delays the moment of optimal exercise of the option (Fig 5).
- the effect of changing the volatilities σ, ω are mitigated and parameters dependent.

It appears that an increase in the degree of (positive) correlation between the two sources of ambiguity postpones the option exercise and reduces the project value *ceteris paribus*. It is also not surprising that an increase in the growth rate of the naked project value delays the option exercise, while an increase in the trend parameter for the exit value has just the opposite effect. Changes in the risk-free rate and the convenience yield also have opposite impact.

Moreover, if more frequent random Poisson type of shocks may be anticipated, then it negatively impacts the option value (due to the threat of a forced premature exit). Note also that an increase in the exit value increases the value of the assets that may be divested.

[6] See, for instance, Dixit and Pindyck (1994, p. 142-143; p. 209-210) for simple treatment for getting a solution through dynamic programming (using linear combination), as well as a description of the fundamental quadratic's intuition.

4. Conclusion

Our comparative static results concur with those in Clark et al. (2010). Regarding a change in ambiguity, the intensity of perceived ambiguity significantly affects option value and the timing of exercise, but its impacts depend on the value taken by other parameters. This is just the same as observed in the case of changing volatilities, so there is similar parameter dependence for risk and ambiguity. Of course, contrary to the case with one source of ambiguity (Kast et al., 2014), it is not possible to identify a general rule regarding the impact of ambiguity for a real option to divest with two correlated sources of ambiguity. This aligns with Trojanowska and Kort (2010), where ambiguity aversion in a real option model is shown to have an equivocal impact on the value of waiting, accelerating investment only in certain situations. On a related issue, Miao and Wang (2010) suggest reconciling some possible divergence in results by considering various definitions of the moment of resolution (or not) of ambiguity.

Finally, it appears that Choquet-Brownian motions are tractable enough to be used to model ambiguity generated by multiple sources, at least when correlated. It may be of interest to use them further to explore the impact of ambiguity on models of compound or interrelated options.

Appendix. Comparative Statics

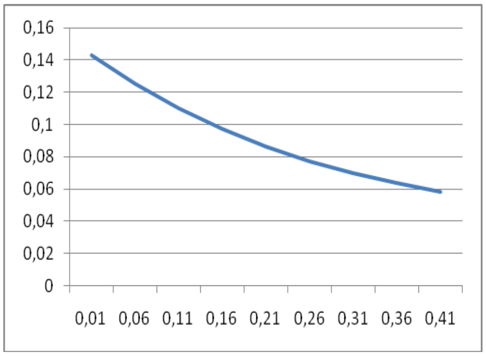

Fig 1. g* as a function of π

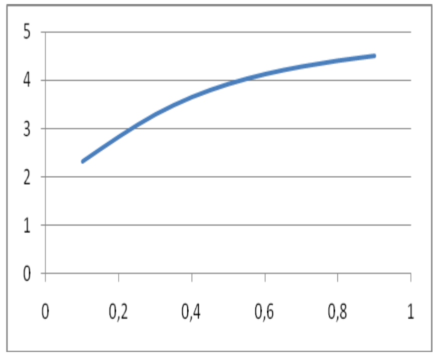

Fig 2. g* as a function of α

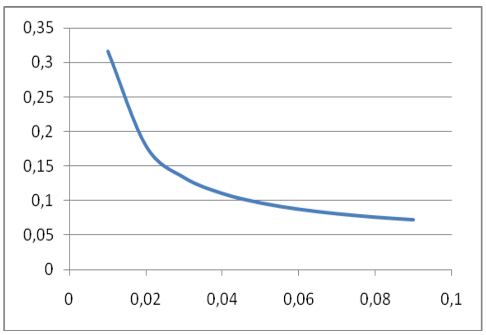

Fig 3. g* as a function of k

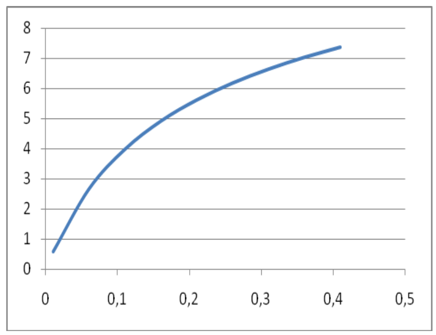

Fig 4. g* as a function of r

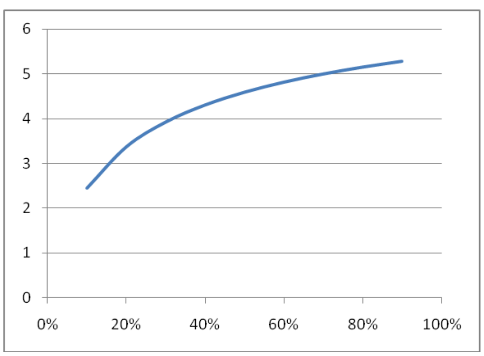

Fig 5. g* as a function of γ

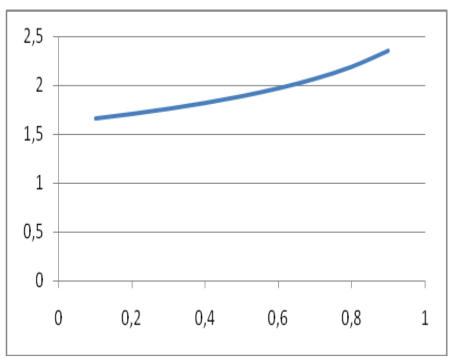

Fig 6. g* as a function of ρ

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