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The optimal hedge ratio: A solution, a conjecture, and a challenge

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Abstract

Calculating the Optimal Hedge Ratio (OHR) is challenging for agricultural exporting countries. The alternatives have been either to rely upon closed-form solutions for the OHR that require unpalatable assumptions (quadratic utility, or CARA utility with Gaussian distributions) or to employ complicated numerical methods. This paper derives an approximate closed-form solution for “compact” distributions with “small” risks. Given empirical distributions of prices and quantities it requires simple calculations to arrive at the OHR for any desired class and calibration of risk preferences. To the extent that futures markets are unbiased, the solution also suggests that a simple minimum-variance calculation may be sufficient to calculate the OHR.

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1. Introduction

Cocoa is an important source of export revenue for countries in Sub-Saharan Africa such Cameroon, Cote d'Ivoire, Ghana, Nigeria, Togo, and Uganda [see Armah (2012), and Kamdem, Kamdem, Kamdem, and Kono (2020)]. Like prices of other agricultural commodities, those of cocoa on world markets are extremely volatile, so these countries face a great deal of price risk and production risk for their exports. Fluctuations in cocoa prices directly affect income, employment, and government revenues, so governments have searched for ways to manage these risks. Apparently, non-market risk management schemes have not worked well, so there has been a perennial search for "market-based" ways to manage price and production risk [again, see Armah (2012) and Kamdem, Kamdem, Kamdem, and Kono (2020)]. This has led to the development of a long and rich literature, starting with Johnson (1960) and McKinnon (1967), that explores the use of futures markets as a risk-management tool for agricultural commodity exports.

Since Rolfo's (1980) seminal paper, this literature has centered upon deriving and estimating the Optimal Hedging Ratio (OHR), the proportion of production that should be hedged using futures contracts. Getting a closed-form solution for the OHR requires the use of a mean-variance setup. However, mean-variance preferences can only be reconciled exactly with expected utility in two cases: (1) when utility is quadratic, and (2) when there are Constant Absolute Risk Aversion (CARA) preferences, and the underlying distribution is Gaussian. This is problematic since, as Samuelson (1970) famously observed, distributions in economics are typically not Gaussian and quadratic utility "leads to well known absurdities" [Samuelson (1970, p. 537)]. Furthermore, it is commonly thought that CARA is unrealistic since people typically exhibit decreasing absolute risk aversion (DARA). Almost all the literature accepts one or another of these assumptions as the cost of having a closed-form solution: Ederington (1979), Sy (1990), and Moosa (2003) simply assume mean-variance utility; Rolfo (1980), Armah (2012), and Kamdem, Kamdem, Kamdem, and Kono (2020) entertain quadratic utility; Ho (1984), Lence (2009), Armah (2012), Kamdem, Kamdem, Kamdem, and Kono (2020)] employ the CARA/Gaussian framework.[1]

Since the assumptions needed to generate closed-form solutions are so restrictive and unappealing, some papers have calculated the OHR numerically using more general preferences. Rolfo (1980) calculates the OHR numerically for logarithmic utility, for example. More recently Armah (2014) does so for Constant Relative Risk Aversion (CRRA) preferences, and Kamdem, Kamdem, Kamdem, and Kono (2020) for both CRRA and Expo-Power [Saha (1993)] preferences.[2]

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[1] All of these are one-period models, except for Ho (1984), who uses CARA utility in a dynamic, continuous-time model.

[2] Armah (2014) also provides a numerical solution for the Nelson and Escalante (2004) utility function, a type of two-parameter, non-expected utility function that permits risk vulnerability.

In this paper I derive an *approximate* closed form solution that holds for *any* utility function when risks are "small," in the sense of Pratt (1964). Samuelson (1970)] called such distributions "compact," and used them to effect an approximate reconciliation of mean-variance analysis with expected utility for "small" risks. As in Heany and Poitras (1991), the OHR consists of two parts: First, there is a pure hedging term that minimizes the variance of wealth; it is independent of risk preferences. Second, there is a speculative component that depends upon the expected excess return of the futures position; it does dependent upon risk preferences. The approximate solution permits the OHR to be calculated exactly, given the distributions of prices and quantities and any calibration of any desired risk preferences. This should greatly simplify the implementation of risk management policies in Cocoa producing countries.

This solution suggests both a conjecture and a challenge.

The conjecture is that risk preferences may often be ignored in calculating the OHR. If futures markets are even roughly unbiased – as they are in the case studied by Kamdem, Kamdem, Kamdem, and Kono (2020) - then the speculative term in the OHR will be very small, leaving only the hedging term. It is of course an empirical question of whether and when futures markets are unbiased; one need not assert that McKinnon (1967) was literally correct in asserting that the objective of risk management should be to minimize the variance of wealth, and ignore risk aversion. However, looking at the minimum variance hedge might provide a good back-of-the-envelope calculation of what the OHR might be.

The challenge is to see how well the approximate solution does relative to the exact solution calculated using numerical methods. My conjecture is that it will often do quite well, at least as a first pass at the OHR. But I invite other researchers to compare the approximate solutions to the corresponding exact solutions calculated using their sophisticated numerical methods [e.g., Armah (2014) and Kamdem, Kamdem, Kamdem, and Kono (2020)].

2. The Optimal Hedge Ratio

The setting is the same as in Rolfo (1980) and Kamdem, Kamdem, Kamdem, and Kono (2020).[3] Before his harvest a commodities producer is uncertain about both the quantity $\tilde{q}$ he will sell and the price per unit $\tilde{p}$ he will receive for it. He can mitigate uncertainty about his revenue by purchasing futures contracts. The price of the product on the futures market $\tilde{p}_f$ is also uncertain before the harvest. However, he can purchase futures contracts at a futures price f that is quoted prior to the harvest. If he holds n futures contracts, then his wealth ex-ante is

$$\tilde{w} = \tilde{p}\tilde{q} - n\left(f - \tilde{p}_f\right) \tag{1}$$

[3] Armah (2014) also includes transactions costs.

Denote the means of the prices and quantity by $E\tilde{p} = \bar{p}, E\tilde{p}_f = \bar{p}_f$, and $E\tilde{q} = \bar{q}$. It should be remembered that all these variables vary with time; to simplify notation I follow Rolfo's (1980) example and suppress time indices.

Our producer derives utility $U(\tilde{w})$ from his wealth. The *only* restrictions I impose upon his preferences are that he be wealth-loving and risk-averse, $U^{/}(\tilde{w}) > 0$, $U^{//}(\tilde{w}) < 0$. As usual, absolute risk aversion is then defined as $A(\tilde{w}) = -U^{//}(\tilde{w})/U^{/}(\tilde{w})$ and relative risk aversion as $R(\tilde{w}) = -U^{//}(\tilde{w})\tilde{w}/U^{/}(\tilde{w})$. The producer chooses the number of futures contracts to maximize his expected utility:

$$\max_n EU(\tilde{w}) \tag{2}$$

Performing this maximization yields the familiar first-order condition

$$EU^{/}\left(\tilde{p}\tilde{q} - n\left(f - \tilde{p}_f\right)\right)\left(f - \tilde{p}_f\right) = 0. \tag{3}$$

As noted, before, closed-form solutions to this problem are only possible for two special cases: quadratic utility, and exponential utility (constant absolute risk aversion) combined with Gaussian wealth. Both are highly restrictive. Armah (2014) and Kamdem, Kamdem, Kamdem, and Kono (2020) therefore undertake calibrated numerical solutions for CRRA utility; Kamdem, Kamdem, Kamdem, and Kono (2020) also do so for power-expo preferences.

I will now demonstrate how to derive *approximate* closed-form solutions in the presence of "small" risks. Since the method itself is the contribution here, I will discuss the derivation in the text, but leave some of the detailed calculations for the appendix.

Let us begin by adopting some notation from Rolfo (1980) and Kamdem, Kamdem, Kamdem, and Kono (2020): Define $e^p = \frac{\tilde{p}-f}{f}, e^q = \frac{\tilde{q}-\bar{q}}{\bar{q}}$, and $e^{p_f} = \frac{\tilde{p}_f - f}{f}$; Kamdem, Kamdem, Kamdem, and Kono (2020) refer to these respectively as "spot price risk," "quantity risk," and "futures price risk." The budget constraint can then be written as

$$w = f\bar{q}(1 + e^p)(1 + e^q) - nfe^{p_f}. \tag{4}$$

This is Equation (8) in Rolfo (2020) and Equation (1) in Kamdem, Kamdem, Kamdem, and Kono (2020).[4] Finally, define $\bar{w} = f\bar{q}$, the value of wealth when the realized risks are zero. This can be roughly be interpreted as the expected value of production evaluated at the current forward rate, and so it can be interpreted as initial expected wealth.

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[4] There is another way of formulating the problem: Since f reflects expectations of $\tilde{p}$ and $\tilde{p}_f$, we could use small deviations of them from their means. In other words, instead of using e^p, e^q , and e^{p_f} we could work with small forecast errors $\frac{\tilde{p}-\bar{p}}{\bar{p}}, \frac{\tilde{q}-\bar{q}}{\bar{q}}$, and $\frac{\tilde{p}_f-\bar{p}_f}{\bar{p}_f}$. The solution would be the same. However, I will stick to the former notation to make the results more readily comparable to those of Rolfo (1980) and Kamdem, Kamdem, Kamdem, and Kono (2020).

The goal is to characterize the hedging ratio, $n/\bar{q}$, when these risks are "small" in the sense of Pratt (1964, p. 125), that is, in the limit as their variances approach zero. Intuitively, that means that the distributions of the spot and futures prices are restricted to be clustered near the futures price $(\tilde{p} \cong f, \tilde{p}_f \cong f)$ and that the distribution of quantity is clustered near its predicted value ($\tilde{q} \cong \bar{q}$). Samuelson called such distributions where the mass is "piled up" around the mean "compact." I also follow Pratt (1964) in assuming that the squares of the means of the risks are of order less than the variances and so can be ignored; in other words $E(e^p)^2 = E(e^q)^2 = E(e^{p_f})^2 \approx 0$.[5]

I employ a method like that in the two-period models in Eaton (1985) and Smith (1991). Take a Taylor series of $EU(\widetilde{w})$ around $e^p = e^q = e^{p_f} = 0$:

$$EU(\widetilde{w}) = U(\overline{w}) + U^{/}(\overline{w})E[\overline{w}(e^p + e^q) - nfe^{p_f}] + \frac{U^{//}(\overline{w})}{2}E[\overline{w}(e^p + e^q) - nfe^{p_f}]^2 \tag{5}$$

In the appendix I show that, since the squares of the means of the risks are small, expected utility reduces to

$$EU(w) = U(\overline{w}) + U^{/}(\overline{w})[\overline{w}E(e^p + e^q) - nfEe^{p_f}] + \frac{U^{//}(\overline{w})}{2}[\overline{w}^2 var(e^p + e^q) - 2n\overline{w}cov(e^p + e^q, e^{p_f}) + n^2f^2var(e^{p_f})] \tag{6}$$

Maximizing this with respect to n leads to the following result.

Proposition: *For small risks, the optimal hedging ratio (OHR) is*

$$\frac{n^*}{\bar{q}} = \frac{cov(e^p + e^q, e^{p_f})}{var(e^{p_f})} - \frac{1}{R(\overline{w})}\frac{Ee^{p_f}}{var(e^{p_f})}. \tag{7}$$

Note that the OHR is the sum of two components:[6]

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[5] In his dissertation, Fischer (1969) called them "concentrated" distributions

[6] Heany and Poitras (1991) derive the same decomposition in a different setting: Using my notation, their result (Equation 3, page 606) is that for *any* distributions of e^p, e^q, and e^{p_f} the OHR is

$$\frac{n^*}{\bar{q}} = \frac{cov(e^p + e^q, e^{p_f})}{var(e^{p_f})} - \frac{EU^{/}(\widetilde{w})}{\overline{w}EU^{//}(\widetilde{w})}\frac{Ee^{p_f}}{var(e^{p_f})}.$$

This has the same general form as Equation (7). The second, speculative terms are different in the models, however. In theirs it involves a ratio of the expectations of the first derivative and the second derivative; in mine it is the reciprocal of relative risk aversion evaluated at initial wealth. Their expression turns into mine in the limit for small risks. It would be hard to implement their formula in policy analysis because it would require the evaluation of the expectations of highly non-linear functions of random wealth; the virtue of working with small risks is that it requires the calculation of only simple functions of known, initial wealth.

- The first term is the minimum-variance hedging demand. In other words, it is the solution to

$$\min_{\eta} var(\widetilde{w}). \tag{8}$$

  This was the problem initially formulated by McKinnon (1967); the solution to it is independent of risk preferences. See also preferences Johnson (1960), Stein (1961), and Ederington (1979).

- The second term depends upon the expected excess return $Ee^{p_f} = Ep_f - f$ via the Sharp ratio, $Ee^{p_f}/var(e^{p_f})$. It can be thought of as the "speculative" component of the demand for futures, and depends inversely upon relative risk aversion evaluated at initial wealth, $R(\overline{w})$. Notice that if $Ep_f = f$ then the speculative component of the OHR disappears. This is a dramatic result of Beninga, Eldor, and Zilcha (1983), that if the futures market is unbiased, then the OHR reduces to the minimum-variance demand.

It must be emphasized that this is a closed-form solution that holds for *any* utility function with positive and diminishing marginal utility. It should be compared to the OHR in the two cases that have permitted solutions so far, which also exhibit the same two-part decomposition: the mean-variance model [for example, Rolfo (1980), Equations (1) and (8) and Kamdem, Kamdem, Kamdem, and Kono (2020, Equation (2)] and CARA preferences with Gaussian returns [for example, Kamdem, Kamdem, Kamdem, and Kono (2020, Equation (3)]. This approximate OHR for any desired utility can be calculated simply by calibrating the degree of relative risk aversion. There is no need for complicated numerical solutions. One need only calculate the variances and covariances of the data and then insert different calibrations of risk preferences.

3. The OHR for Different Preferences

Here I will report the OHR for a menu of different preferences; one may easily think of others. I will make some preliminary suggestions about calibrating each utility function.

3.1 Mean-Variance Utility

Here people derive utility directly from the moments of the distribution:

$$V(\widetilde{w}) = E\widetilde{w} - bvar(\widetilde{w}). \tag{8}$$

Strictly speaking this is not consistent with the model here model since these are not expected utility preferences. Of course, they can be generated from an appropriately calibrated quadratic expected utility function. I include them since they are so common in the literature [Ederington (1979), Sy (1990), Moosa (2003), Kamdem, Kamdem, Kamdem, and Kono (2020)]. The OHR in this case is

$$\frac{n^*}{\bar{q}} = \frac{cov(e^p + e^q, e^{p_f})}{var(e^{p_f})} - \frac{1}{2b\overline{w}} \frac{Ee^{p_f}}{var(e^{p_f})}. \tag{9}$$

For some description of how to calibrate this case, see the discussion of quadratic preferences below.

3.2 Quadratic Utility

This is of course the canonical case:

$$U(\widetilde{w}) = \eta \widetilde{w} - \mu \widetilde{w}^2 \tag{10}$$

The OHR here is

$$\frac{n^*}{\bar{q}} = \frac{cov(e^p + e^q, e^{pf})}{var(e^{pf})} - \frac{\eta - 2\mu\bar{w}}{2\mu\bar{w}} \frac{Ee^{pf}}{var(e^{pf})} \tag{11}$$

Note that this is observationally equivalent to the OHR with mean-variance preferences in Equation (9) when $b = \mu/(\eta - 2\mu\bar{w})$.

The quadratic case seems to hard to calibrate. Both Rolfo (1980) and Armah (2014) consider a range of values for μ from 0 to infinity.

3.3 Constant Absolute Risk Aversion (CARA) Utility

The CARA (or exponential) utility function is the other workhorse of the literature with exact solutions:

$$U(\widetilde{w}) = e^{-\lambda \widetilde{w}}. \tag{12}$$

The associated OHR is:

$$\frac{n^*}{\bar{q}} = \frac{cov(e^p + e^q, e^{pf})}{var(e^{pf})} - \frac{1}{\lambda\bar{w}} \frac{Ee^{pf}}{var(e^{pf})}. \tag{13}$$

Assuming a Gaussian distribution for wealth, this would hold exactly. However, it is important to note that this approximate solution does *not* require wealth to be Gaussian.

It is hard to calibrate the CARA case because at it is so rarely used in empirical work. Wiens (1976) uses $\lambda = .0085 - .009$, Buccola (1982), $\lambda = .0012$; Love & Buccola (1991) $\lambda = .016 - .140$, and Chavas & Holt (1990) $\lambda = 4.54 - 14.25$.

3.4 Constant Relative Risk Aversion (CRRA) Utility

This the natural benchmark since it is used so much in both theoretical and empirical work:

$$U(\widetilde{w}) = \widetilde{w}^{\alpha}. \tag{14}$$

It is not possible to derive an exact solution for the OHR in this case, but the approximate solution is:

$$\frac{n^*}{\bar{q}} = \frac{cov(e^p + e^q, e^{pf})}{var(e^{pf})} - \frac{1}{\alpha} \frac{Ee^{pf}}{var(e^{pf})} \tag{15}$$

How might this be calibrated? There is a big literature on this, but normally relative risk aversion $R = 1 - \alpha$ is thought to be between 1 and 3 or 4. For example, see Friend & Blume (1975), Epstein & Zin (1991), Szipro (1982), Hansen & Singleton (1983), Mankiw & Shapiro (1986), Love & Buccola (1991), and Chavas & Holt (1990). Kandel & Stambaugh (1991) say it is 29, but the upper limit of plausible values is usually thought to be Mehra & Prescott's (1985) 10. Gandelman & Hernández-Murillo (2015) provide a recent survey; they also put estimates of R between 1 and 3 but concentrated around 1 (log utility). However, I might recommend using something like the range of values, $R = .5,1,3,10,$ as in Smith (1999). Armah (2014) choses a reasonable value of $R = 4$.

3.5 Expo-Power Utility

Saha (1993) introduced this interesting, more flexible class of preferences:

$$U(\widetilde{w}) = \theta - e^{-\beta \widetilde{w}^{\alpha}}. \qquad (16)$$

Its OHR is:

$$\frac{n^*}{\bar{q}} = \frac{cov(e^p + e^q, e^p f)}{var(e^p f)} - \frac{1}{\alpha + \beta \bar{w}^{\alpha}} \frac{E e^p f}{var(e^p f)} \qquad (17)$$

There is a paucity of work on how to calibrate this. In very different settings, Saha, Shumway, & Talpaz (1994) use $\alpha = .3654, \beta = 2.737$, while Holt & Laury (2002) estimate $\alpha = .731, \beta = .029$. Smith & Zhang (2007) also estimate these parameters. However, their model incorporates habit formation, so the argument of the utility function is essentially the gross growth rate of consumption; the estimates may not be relevant here.

4. Conclusion and Discussion

The availability of a closed-form solution for the OHR should greatly simplify risk management in agricultural exporting countries. Given empirical data about price and quantity distributions and any desired class and calibration of risk preferences, the OHR emerges with a simple calculation. Indeed, might imagine creating an online interactive tool for, say, the OHR in cocoa producing countries: The website would collect, and solicit, data from different countries in different time countries and make it available as a data bank for public use. Alternatively, users could enter their own statistical data. The tool would then calculate the OHR for any a range of preferences, calibrated as desired.

Of course, the value of such a tool depends upon how closely the approximate solution is to the exact solution. Does it work well, and if not, when? I conjecture that the deviation will be fairly small. However, I invite other researchers to calculate the exact OHR for a menu of preferences using numerical methods and to compare them formally to the approximate OHRs.

The solution also suggests another simplification in calculating the OHR. If the futures price is an unbiased estimator of the spot price ($E e^{p_f} = 0$) then the speculative term disappears and the OHR *is* the minimum variance hedge, as shown by Beninga, Eldor, and Zilcha (1983). One would not expect this to hold exactly, since the extent to which futures markets are biased is hotly contested. However, this suggests a simple rule of thumb as a first pass at estimating the OHR: if futures markets are even close to being unbiased then the minimum variance hedge should be a good first approximation to the OHR. Furthermore, one might ask *how much* risk preferences matter for the OHR? For a marginal increase in the coefficient of relative risk aversion, for example, *how much* will the OHR fall?

Appendix

Define $\overline{w} = f\overline{q}$. This is the expected value of production evaluated at the current forward rate, and so can be interpreted as initial expected wealth. Take a Taylor series of $EU(\widetilde{w})$ around $e^p = e^q = e^{pf} = 0$:

$$EU(\widetilde{w}) = U(\overline{w}) + U^{/}(\overline{w})E[\overline{w}(e^p + e^q) - nfe^{pf}] + \frac{U^{//}(\overline{w})}{2}E[\overline{w}(e^p + e^q) - nfe^{pf}]^2 \qquad (A.1)$$

Now consider the expectations in the first and second order terms of the series. The first-order term is

$$E[\overline{w}(e^p + e^q) - nfe^{pf}] = \overline{w}E(e^p + e^q) - nfEe^{pf}. \qquad (A.2)$$

Next expand the second-order term:

$$E[\overline{w}(e^p + e^q) - nfe^{pf}]^2 = \overline{w}^2E(e^p + e^q)^2 - 2nf\overline{w}E(e^p + e^q)e^{pf} + n^2f^2Ee^{pf^2}. \qquad (A.3)$$

This can be rewritten as

$$\begin{aligned} &E[\overline{w}(e^p + e^q) - nfe^{pf}]^2 \\ &\qquad = \overline{w}^2\left[var(e^p + e^q) + \left(E(e^p + e^q)\right)^2\right] \\ &\qquad - 2nf\overline{w}[cov(e^p + e^q, e^{pf}) + E(e^p + e^q)Ee^{pf}] + n^2f^2[var(e^{pf}) + (Ee^{pf})^2] \end{aligned} \qquad (A.4)$$

Since the products of expectations are small – that is that $\left(E(e^p + e^q)\right)^2 = E(e^p + e^q)Ee^{pf} = (Ee^{pf})^2 \approx 0$ - this reduces to

$$E[\overline{w}(e^p + e^q) - nfe^{pf}]^2 = \overline{w}^2var(e^p + e^q) - 2nf\overline{w}cov(e^p + e^q, e^{pf}) + n^2f^2var(e^{pf}). \qquad (A.5)$$

Using Equations (6) and (9), Equation (5) becomes

$$EU(w) = U(\overline{w}) + U^{/}(\overline{w})[\overline{w}E(e^p + e^q) - nfEe^{pf}] + \frac{U^{//}(\overline{w})}{2}[\overline{w}^2var(e^p + e^q) - 2nf\overline{w}cov(e^p + e^q, e^{pf}) + n^2f^2var(e^{pf})] \qquad (A.6)$$

Maximizing this with respect to n yields the first-order condition

$$-U^{/}(\overline{w})fEe^{pf} - U^{//}(\overline{w})[f\overline{w}cov(e^p + e^q, e^{pf}) - nf^2var(e^{pf})] = 0. \qquad (A.7)$$

The optimal futures contract n^* is therefore

$$n^* = \frac{\overline{w}cov(e^p + e^q, e^{pf})}{fvar(e^{pf})} + \frac{U^{/}(\overline{w})}{U^{//}(\overline{w})}\frac{Ee^{pf}}{fvar(e^{pf})}. \qquad (A.8)$$

Since $\overline{w} = f\overline{q}$, however, we can solve for the optimal hedge ratio as

$$\frac{n^*}{\overline{q}} = \frac{cov(e^p + e^q, e^{pf})}{var(e^{pf})} + \frac{U^{/}(\overline{w})}{U^{//}(\overline{w})\overline{w}}\frac{Ee^{pf}}{var(e^{pf})}. \qquad (A.9)$$

Recalling the definition of relative risk aversion yields Equation (7) in the text.

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