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Automation, fertility, and labor share in an aging, overlapping generations economy

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Abstract

This paper presents an analysis of the effect of population aging on automation in production using an overlapping generations model with endogenous fertility decisions by individuals. Population aging induces individuals to reduce the number of children they have to prepare for a longer (expected) retirement. If an extension of life expectancy substantially reduces the fertility rate, then population aging decreases the automation capital per worker. However, population aging lowers the labor share.

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1. Introduction

Increasingly automatable production processes may lead to a new industrial revolution. Do machines substitute for workers and thereby lower the labor share? This paper considers this issue in an aging, overlapping generations economy.

Prettner (2019) introduces automation into the descriptive Solow (1956) model to analyze its impact on economic growth and the labor share. In the model, automation is considered a perfect substitute for labor in goods production, although they are imperfect substitutes for conventional capital. Heer and Irmen (2019) point out that with perfect substitutability between conventional capital and automation capital as stores of value for individuals, the return rates of both types of capital and labor are identical. This amendment illustrates the interdependency between the dynamic evolutions of the two types of capital. Heer and Irmen (2019) ultimately conclude that perfect substitution between automation and labor is not sufficient for a vanishing labor share whereas the aggregate production technology becomes an AK-type production function.

In contrast to the descriptive model, Irmen (2021) instead extends the analysis by using an overlapping generations model. Irmen (2021) emphasizes the effects of population aging on the equilibrium automation, GDP growth, and labor share. Irmen (2021) shows that population aging brought about by increased longevity has different dynamic effects on automation, growth, and the labor share than those caused by a decline in fertility.[1]

However, in Irmen (2021), the fertility decisions of individuals are exogenously given and independent of life expectancy. It is widely known, however, that the fertility decisions of individuals are endogenously made depending on various economic conditions (Yakita, 2017; Doepke and Zilibotti, 2019). To address this point, this study endogenizes fertility decisions in a two-period overlapping generations model to analyze the impacts of increased longevity on automation, and the labor share. This is done to explore the hypotheses that, with a large decline in fertility caused by high longevity, automation capital per worker might become smaller to promote savings, while conventional capital becomes greater. To simplify the analysis, our model of the production sector draws upon Prettner (2019) and Heer and Irmen (2021).

The next section introduces a simple overlapping generations model populated by agents with two-period lives whose lifetimes are uncertain. Section 3 studies the impact of increased longevity on the equilibrium, and the final section concludes the paper.

2. Model

[1] The analyses conducted in Irmen (2021) are based on a task-based model developed by Acemoglu and Restrepo (2018) and others.

Time is discrete and ranges from 0 to ∞. The length of each period is normalized to unity. Individuals are identical, except for their ages. Each individual lives for two periods: young and old. When young, they work, rear children, and save for their old age. At the end of the young age, they face a survival probability of $\lambda \in (0,1]$ into old age. Individuals surviving to old age retire and consume their wealth they have amassed. We assume a competitive insurance market and risk-neutral insurance companies.

The budget constraint during the young age of generation t is

$$w_t(1 - zn_t) = c_t^1 + s_t, \tag{1}$$

where w_t is the wage rate, n_t denotes the number of children the individual has, z is per-child rearing time, c_t^1 represents young-age consumption, and s_t is lifecycle savings for possible retirement. The budget constraint of old age is

$$c_{t+1}^2 = R_{t+1}s_t, \tag{2}$$

where c_{t+1}^2 is old-age consumption and R_{t+1} denotes the return rate of saved assets. Because of lifetime uncertainty, the individual deposits their entire savings with life insurers in exchange for annuity policies. Insurance companies rent savings out as fixed capital to firms producing goods in period $t+1$. In return, firms pay a rental rate of r_{t+1} per unit of savings. Insurance companies pay an actuarially fair return, $r_{t+1}/\lambda = R_{t+1}$ per unit of savings to surviving old individuals at period $t+1$. The (expected) lifetime budget constraint becomes

$$w_t(1 - zn_t) = c_t^1 + c_{t+1}^2 / R_{t+1}. \tag{3}$$

The expected lifetime utility of generation t is assumed to be $u_t = \ln c_t^1 + \varepsilon \ln n_t + \lambda\rho \ln c_{t+1}^2$, where $\varepsilon > 0$ is the utility weight of the number of children and $\rho \in (0,1)$ is the discount factor. From the first-order conditions for utility maximization subject to (3), we obtain

$$n_t = \frac{\varepsilon}{z}\frac{1}{1+\varepsilon+\lambda\rho} (\equiv n) \quad \text{and} \quad s_t = \frac{\lambda\rho}{1+\varepsilon+\lambda\rho} w_t. \tag{4}$$

The number of children is constant, and hence, so is the working population growth rate. We assume that the condition $n_t \geq 1$ or $z \leq \varepsilon/(1+\varepsilon+\lambda\rho)$ is satisfied in the following.

The aggregate production technology is assumed to be a Cobb-Douglas production function, $Y_t = K_t^{\alpha}(L_t + M_t)^{1-\alpha}$ ($\alpha \in (0,1)$), where K_t and M_t denote conventional and automation capital, respectively, and L_t is labor. The profit maximization conditions yield the following conditions.[2]

$$\alpha(\frac{K_t}{L_t + M_t})^{\alpha-1} = r_{Kt},\ (1-\alpha)(\frac{K_t}{L_t + M_t})^{\alpha} = r_{Mt}, \text{ and } (1-\alpha)(\frac{K_t}{L_t + M_t})^{\alpha} = w_t, \quad (5)$$

where r_{Kt} and r_{Mt} represent the return rates on conventional and automation capital, respectively. Because of the perfect substitutability between labor and automation, we have $r_{Mt} = w_t$. The arbitrage between conventional and automation capital in the savings-asset market yields $r_{Kt} = r_{Mt}$. Therefore, the condition $r_{Kt} = r_{Mt} = w_t$ holds. From $\alpha(\frac{K_t}{L_t + M_t})^{\alpha-1} = (1-\alpha)(\frac{K_t}{L_t + M_t})^{\alpha}$, we obtain

$$\frac{\alpha}{1-\alpha} = \frac{K_t}{L_t + M_t} \text{ or } \frac{1-\alpha}{\alpha}K_t = L_t + M_t. \quad (6)$$

Inserting (6) into the production function, we obtain $Y_t = (\frac{\alpha}{1-\alpha})^{\alpha-1}K_t$, that is, an AK-type production technology. From (5) and (6), we also obtain $w_t = (1-\alpha)^{1-\alpha}\alpha^{\alpha}$.

The equilibrium condition in the labor market is given as $L_t = (1 - zn_t)N_t$, where N_t is the number of workers in period t. Using (4), we obtain $L_t / N_t = (1+\lambda\rho)/(1+\varepsilon+\lambda\rho)$. The labor supply per worker is constant for all generations.

The equilibrium condition in the capital market is given as $s_t N_t = K_{t+1} + M_{t+1}$.[3] Using (4) and (6), the condition yields

$$k_{t+1} = \alpha[(1-\alpha)^{1-\alpha}\alpha^{\alpha}\frac{\lambda\rho}{\varepsilon / z} + \frac{1+\lambda\rho}{1+\varepsilon+\lambda\rho}](\equiv k). \quad (7)$$

Conventional capital per worker becomes intertemporally constant. This implies that aggregate conventional capital grows at the working-population growth rate $n_t = N_{t+1} / N_t$. Therefore, GDP also grows at the working population growth rate.

Condition (6) implies that the per-worker automation capital, $m_t = M_t / N_t$, also remains constant and the aggregate automation capital grows at the working population growth rate. Finally, the labor share, $w_t L_t / Y_t$, remains constant at equilibrium. This

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[2] Interior solutions are assumed in this study.

[3] We assume here that capitals fully depreciate after one period. Assuming instead that the depreciation rate after one period is less than one does not alter the analysis or results as long as old age individuals consume net capital stocks.

result is not surprising because we assume away technological progress or human capital accumulation, unlike Acemoglu and Restrepo (2018).

3. Population aging

A rise in the probability of survival into old age increases the ratio of surviving old individuals. Therefore, increases in survival probability imply population aging. By studying the impact of an increase in the survival probability, λ, we can analyze the effects of population aging.

First, we can easily show that population aging increases conventional capital per worker ($dk / d\lambda > 0$). Second, from (4), we see that the labor supply per worker $1 - zn_t$ increases with population aging because the fertility rate decreases with population aging ($dn / d\lambda < 0$). Third, the effect of population aging on the aggregate labor supply is ambiguous because population aging increases the per worker labor supply and decreases the number of workers. Finally, the effect on automation capital per worker $m_t = \frac{1-\alpha}{\alpha} k_t - (1 - zn_t)$ is given as

$$\frac{dm_t}{d\lambda} = \alpha z[(1-\alpha)^{2-\alpha} \alpha^{\alpha-1} \frac{\rho}{\varepsilon} + \frac{dn}{d\lambda}], \quad (8)$$

the sign of which is ambiguous. If the negative effect on fertility is sufficiently large, then automation per worker decreases with population aging. Conversely, if the effect of fertility is small, population aging increases automation per worker. This last result differs from the result obtained by Irmen (2021), who assumes an exogenously given fertility rate.[4] Summarizing these results, we have the following proposition:

Proposition 1

Longer life expectancy increases the per-worker levels of conventional capital stock and market labor supply. The effect of population aging on automation per worker is ambiguous. If the increased longevity substantially decreases the fertility rate, then population aging due to longer life expectancy reduces the per-worker level of automation.

The intuition underlying these results can be explained as follows. The increased probability of surviving into old age induces individuals to prepare for a longer old age. Individuals increase market labor supply by reducing the number of children they have and saving more. Therefore, this increases conventional capital, which is imperfectly

[4] In Heer and Irmen (2019), because the labor supply is exogenously given, both conventional and automation capital moves in the same direction.

substitutable for labor. However, the accumulation of automation capital is depressed to prevent the return rate from declining because of its perfect substitutability with labor. Recall that the asset accumulation of individuals comes from wage income.[5] As such, firms will consequently control capital accumulation between conventional capital and automation to prevent the labor share from becoming too low. In particular, when declines in the fertility rate, caused by population aging, are great, increases in the market labor supply are also great. The increased labor supply likely substitutes automation to maintain the return rate constant.

The impact of population aging on the labor share can be easily shown by

$$\frac{w_t L_t}{Y_t} = 1/[(1-\alpha)^{1-\alpha}\alpha^{\alpha}\frac{\lambda\rho}{\varepsilon / z}\frac{1+\varepsilon+\lambda\rho}{1+\lambda\rho}+1]. \tag{9}$$

The denominator of the right-hand side of (9) increases with λ. Therefore, we have $d(w_t L_t / Y_t)/d\lambda < 0$. Population aging always reduces the labor share.

Proposition 2

Population aging due to longevity reduces the labor share even when automation per worker decreases.

Although population aging increases the labor supply and hence wage income per worker (with a constant wage rate), the increase in GDP is greater because conventional capital increases as well.

4. Conclusion

This paper re-examined automation analyses using a Diamond (1965)-type overlapping generations model without bequest motives, instead of the Solow model used by Prettner (2019) and Heer and Irmen (2019). We endogenize the fertility decisions of individuals, and the dynamics in this study have no transitions with a unique equilibrium. Therefore, GDP grows at the working population growth rate, and the labor share remains intertemporally constant.

Population aging lowers the labor share, the fertility rate, and GDP growth rate, but it increases the labor supply per worker, conventional capital per worker, and GDP per worker. However, when population aging substantially decreases the fertility rate, thereby

[5] Young workers cannot borrow from survived retirees on security of future capital income. This might be one of the reasons for the boundedness of the labor share away from zero in an overlapping generations model. Our economy runs counter to immiserating growth discussed by Benzell et al. (2018).

substantially increasing the labor supply per worker, per worker automation decreases.

Declaration of Conflicting Interests

The author declares no potential conflicts of interest with respect to the research, authorship, or publication of this article.

Data Availability Statement

Data sharing is inapplicable to this paper because no new data were analyzed.

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